POWER

PWP-078

Linear Supply Function Equilibrium: Generalizations, Application, and Limitations

Ross Baldick, Ryan Grant, and Edward Kahn

August, 2000

This paper is part of the working papers series of the Program on Workable Energy Regulation (POWER). POWER is a program of the University of California Energy Institute, a multicampus research unit of the University of California, located on the Berkeley campus.

University of California Energy Institute
2539 Channing Way
Berkeley, California 94720-5180
www.ucei.org

Linear Supply Function Equilibrium: Generalizations, Application, and Limitations

Ross Baldick
Department of Electrical and Computer Engineering, University of Texas, Austin

Ryan Grant
Analysis Group/Economics, San Francisco, CA

Edward Kahn
University of California Energy Institute, Berkeley, CA and Analysis Group/Economics, San Francisco, CA

August, 2000

Abstract

We consider a supply function equilibrium (SFE) model of interaction in an electricity market. We assume a linear demand function and consider a competitive fringe and several strategic players all having capacity limits and affine marginal costs. The choice of SFE over Cournot equilibrium and the choice of affine marginal costs is reviewed in the context of the existing literature.

We assume that bid rules allow affine or piecewise affine non-decreasing supply functions and extend results of Green and Rudkevitch concerning the linear SFE solution. An incentive compatibility result is proved. We also find that a piecewise affine SFE can be found easily for the case where there are non-negativity limits on generation. Upper capacity limits, however, pose problems and we propose an ad hoc approach.

We apply the analysis to the England and Wales electricity market, considering the 1996 and 1999 divestitures. The piecewise affine SFE solutions generally provide better matches to the empirical data than previous analysis.

Linear Supply Function Equilibrium: Generalizations, Application, and Limitations

Ross Baldick†, Ryan Grant‡, and Edward Kahn‡‡
August, 2000

1. Introduction

This paper explores the linear version of the supply function equilibrium (SFE) model. The general SFE approach was introduced by Klemperer and Meyer (1989) and applied by Green and Newbery (1992) to the electricity industry reforms in England and Wales (E&W). Green (1996) used a linear version of this model and applied it to prospective divestitures of generation assets mandated by the regulator of the electricity industry in E&W. We offer a generalization of Green's model and extend the application to subsequent changes in the horizontal structure of the electricity market in E&W, beyond those studied by Green. Before exploring these issues, it is worthwhile addressing two threshold questions. First, what does SFE offer beyond the traditional Cournot framework? Second, why use the linear form of SFE rather than the more general formulation?

Why Not Cournot?

SFE competes with the Cournot model as a practical tool for studying oligopoly in the electricity industry. Recent reforms of the electricity industry around the world have stimulated numerous studies of oligopoly behavior in restructured electricity markets. Papers of this kind have been published reflecting issues in Scandinavia, Spain, New Zealand, and U.S. electricity markets, particularly California.[1] All of these papers rely on the Cournot framework.

SFE is attractive compared to Cournot because it offers a more realistic view of electricity markets, where bid rules typically require suppliers to offer a price schedule that may apply throughout a day, rather than simply put forth a series of quantity bids over a day. In the Cournot framework, price formation depends exclusively on the specification of the demand curve (and of the specification of a competitive fringe.) The market price is determined by the intersection of the aggregate quantity offered and the demand curve. It is notoriously difficult to specify the market demand curve in electricity, due to low short-run elasticity and inexperience with market competition in

---

† Department of Electrical and Computer Engineering, University of Texas, Austin
‡ Analysis Group/Economics, San Francisco, CA
‡‡ University of California Energy Institute, Berkeley, CA and Analysis Group/Economics, San Francisco, CA

[1] For Scandinavia, see Andersson and Bergman (1995). For Spain, see Alba *et al* (1999), Ramos *et al* (1998) and Rivier *et al* (1999). For New Zealand, see Read and Scott (1996) and Scott (1998). In all three of these countries hydro storage plays an important role. For the US, see Borenstein and Bushnell (1999). US markets typically involve network congestion issues. Network congestion is treated in an oligopoly context by Hogan (1997) and by Borenstein, Bushnell and Stoft (to appear).

electricity. As a result, price predictions from Cournot models are not particularly reliable.[2]

The SFE approach is not immune to the problem of sensitivity to specification of the market demand and this issue is discussed further below. However, the SFE model formulation at least offers the possibility of developing some insight into the bidding behavior of firms, particularly in markets where they are constrained to bid consistently over an extended period of time. One recent example of this application is the use of the SFE framework by the Market Monitoring Committee of the California Power Exchange (Bohn, Klevorick, and Stalon, 1999). Therefore, the case for the SFE approach is its realism regarding the bidding behavior of firms in electricity markets. SFE can estimate equilibrium mark-ups. Equilibrium mark-ups are less transparent in the Cournot framework than in the SFE framework.

Why Use the Linear Model?

In the SFE model, functional forms must be specified for demand, cost, and supply functions. We first discuss demand. A particularly simple form is to assume a "linear" demand function; that is, at each time the demand as a function of price has a non-zero intercept and a constant negative slope.[3] Assuming that the slope is independent of time greatly simplifies the model. Most authors use linear demand functions with demand slope independent of time.[4]

Next we consider the marginal cost as a function of production. There are a range of possible functional forms. The simplest non-trivial case is an affine function with zero intercept or, equivalently, all cost functions having the same non-zero intercept. Following the literature, we will refer to marginal costs that are affine with zero intercept as "linear" marginal costs.

In some models, there is a competitive fringe as well as strategic firms. The functional form for the strategic firms and the fringe can be different. Two further decisions concern whether or not the strategic firms are assumed to be symmetric and whether or not the strategic firms and fringe have maximum capacity limits.

Finally, consider the supply as a function of price. Again, there are a range of possible functional forms. Typical applications use a form for the supply function that is similar to the assumed form of the (inverse) marginal cost function. If the cost functions are symmetric then the equilibrium supply functions often turn out to be symmetric.

[2] For example, Frame and Joskow (1998) offer the following observation made in the context of reviewing a particular Cournot model implementation in electricity, "We are not aware of any significant empirical support for the Cournot model providing accurate predictions of prices in any market, let alone an electricity market."

[3] We follow the convention of calling this specification a "linear" demand function although it is more precisely described as "affine." In discussing cost and supply functions, we reserve the word linear for affine functions with a zero intercept.

[4] A computational advantage of the Cournot framework is that constant elasticity demand curves are straightforward to represent, as in Borenstein and Bushnell (1999), p.302.

Assuming linear demand and affine marginal costs greatly simplifies the SFE mathematics. For example, Turnbull (1983) analyzes an asymmetric two firm model with linear demand, affine marginal costs, and affine supply functions. The resulting conditions for the SFE are straightforward to solve.

Green and Newbery (1992) (GN) generalize the linear demand and linear marginal cost asymmetric two firm model by analyzing strategic firms having quadratic marginal costs. This requires numerical solution of differential equations and is undertaken in the interest of greater realism (p.941). For the duopoly structure examined, GN report results primarily for the case of symmetric strategic firms. As the structure of the electricity industry in E&W has changed, realism suggests that the symmetry assumption and the duopoly assumption both need to be dropped.

The asymmetric duopoly case is also solved by GN and by Laussel (1992). Neither of the latter two papers require linearity. However, there does not appear to be any other results on asymmmetry beyond the duopoly case for non-linear marginal costs.

The great advantage of the SFE with linear marginal costs over the more general form is the ability to handle asymmetric firms when there are more than two strategic firms. Green (1996) does not emphasize this property, but it turns out to be useful in practice. As noted above, the general SFE requires solving a set of differential equations. This is sufficiently difficult that most authors typically rely on the case of symmetric firms. For practical applications, the asymmetric case is more interesting. This motivates the use of the linear model for the asymmetric, multiple strategic firm industry we consider.

Organization of paper

The remainder of the paper is organized as follows. Section 2 introduces the affine marginal costs case. We characterize the affine SFE and prove an incentive compatibility result regarding the price intercept of the equilibrium supply bids. Section 3 introduces capacity constraints. These have been addressed for strategic firms by GN under the assumption that the cost functions are identical for each firm. We address the case of capacity constraints, both for the strategic firms and fringe, where there are asymmetric costs. This case is more realistic for the England and Wales market subsequent to divestiture. We show that this situation is very much more complex than when this situation arises in the Cournot framework. Section 4 applies these methods to recent structure and price changes in the electricity market in England and Wales. The theoretical issues discussed in Sections 2 and 3 are illustrated by the numerical examples introduced in section 4. Section 5 offers some conclusions.

2. The Affine Marginal Cost Case

As discussed above, the SFE models reported in the literature typically assume that the bidders' marginal cost functions have zero intercept, or, equivalently, assume that all have a common intercept. This makes the SFE easier to find. Occasionally authors attempt to defend the plausibility of this assumption. We argue that, at least for electricity, this assumption is neither plausible nor practically useful.

What is Gained

The requirement that marginal cost functions have zero intercept is compared to a case where they are allowed to have a positive intercept and the more realistic case of non-linear marginal costs. Figure 1 illustrates these two approximations to a typical marginal cost curve characteristic of electricity generation firms. The two approximations will be used in Section 4 below. If the marginal costs curves are equal at full production, as illustrated in figure 1, then assuming that the marginal costs pass through zero will over-estimate profits compared to the true function. The affine case will typically under-estimate profits. The line through zero is likely to be particularly unrealistic when the actual supply of a firm is from the low end of its capacity. Our examples in Section 4 will show cases of this kind.

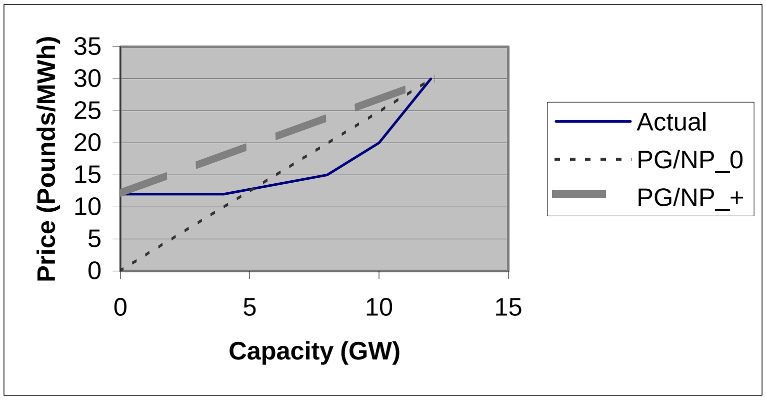


Figure 1. Linear Approximations to a Convex Marginal Cost Curve

Formulation

We begin with the same general form of the demand curve as in Green, namely:

$$D(p,t) = N(t) - \gamma p. \tag{1}$$

The underlying load-duration characteristic is specified by the function $N(t)$. Furthermore, for each time t, the demand D is "linear" in p with slope $\mathrm{d}D/\mathrm{d}p = -\gamma$. The coefficient γ is assumed to be positive.

The total cost function for firm i, C_i, $i = 1,\ldots,n$, is given as a strictly concave quadratic function of production. This form results in an affine marginal cost function for each firm:

$$\forall i,\ \forall q_i \geq 0,\ C_i(q_i) = \tfrac{1}{2}\, c_i q_i^2 + a_i q_i, \tag{2}$$

$$\forall i,\ \forall q_i \geq 0,\ \mathrm{d}C_i/\mathrm{d}q_i(q_i) = c_i q_i + a_i, \tag{3}$$

with $c_i > 0$ for each firm i for strictly convex costs. In contrast to previous analysis in the literature, we allow the a_i to be non-zero and to be specific to each firm.

We assume that the market rules specify that the supply function of each firm is affine; that is, of the form:

$$\forall i,\ q_i(p) = \beta_i (p - \alpha_i). \tag{4}$$

The parameters α_i and β_i are chosen by firm i subject to the requirement that β_i be non-negative. Strictly speaking, we should modify the supply function (4) so that it is always non-negative; however, we will initially assume that the prices are such that no supply functions ever evaluate to being negative. We will subsequently revisit this assumption and generalize the allowed form of the supply function to piecewise affine non-decreasing functions.

Solution

The basic equation governing the SFE solution is provided by Green (1996) as his equation (4), which we quote here for reference:

$$\forall i,\ q_i(p) = (p - \mathrm{d}C_i/\mathrm{d}q_i(q_i(p)))(-dD/dp + \Sigma_{j \neq i}\, dq_j(p)/dp).$$

Any solution to these coupled differential equations such that each q_i is non-decreasing over the relevant range of prices is an SFE. These equations do not involve the load-duration characteristic $N(t)$.

Substituting from (1), (3), and (4) above into Green's equation (4), noting that $\mathrm{d}q_i/\mathrm{d}p = \beta_i$, we obtain:

$$\forall i,\ \beta_i(p - \alpha_i) = (p - c_i\beta_i(p - \alpha_i) - a_i)(\gamma + \Sigma_{j\neq i}\, \beta_j). \tag{5}$$

Assuming that the bid function must be consistent across all times, this equation must be satisfied at every realized value of price p. If there are at least two distinct values of price that are realized, then (5) can only be satisfied for all the realized prices if it is identically true. That is, we can equate coefficients of like powers of p on the left and right hand side of the equation. Equating coefficients of p, we obtain Green's equation (6):

$$\forall i,\ \beta_i = (1 - c_i\beta_i)\{\gamma + \Sigma_{j\neq i}\, \beta_j\}. \tag{6}$$

Equating coefficients of the constant terms we obtain:

$$\forall i,\ -\alpha_i\, \beta_i = -(a_i - c_i\beta_i\alpha_i)\{\gamma + \Sigma_{j\neq i}\, \beta_j\}. \tag{7}$$

Both (6) and (7) must be satisfied with non-negative values of β_i for each firm i for an affine SFE to exist. Substituting from (6) for β_i for each i into the left hand side of (7) yields:

$$\forall i,\ -\alpha_i\,(1 - c_i\beta_i)\{\gamma + \Sigma_{j\neq i}\, \beta_j\} = -(a_i - c_i\beta_i\, \alpha_i)\{\gamma + \Sigma_{j\neq i}\, \beta_j\}.$$

Since the required solution must satisfy $\forall i,\ \beta_i \geq 0$ then, if $\gamma > 0$, we have that $\gamma + \Sigma_{j\neq i}\beta_j > 0$ and we can cancel this factor on both sides. Rearranging the resulting expression yields $\alpha_i = a_i$ for each firm i. Conversely:

- if $\alpha_i = a_i$ for each firm i and if (6) is satisfied with non-negative β_i,
- then (5) is satisfied and the resulting α_i and β_i specify an affine SFE.

Rudkevich (1999) shows that there is exactly one non-negative solution to (6) and presents an iterative scheme in the special case of all firms having zero intercept (that is, $\forall i,\ a_i = 0$) for finding the solution to (6). The iterative scheme begins with each firm bidding "competitively." That is, each firm i initially bids $\beta_i = 1/c_i$. Rudkevich shows that if each firm at each iteration updates its value of β_i so as to find the profit maximizing value for firm i, given the values of β_j for the other firms from the previous iteration, then the sequence of iterates converges to the optimal solution of (6).

In the affine case, we can slightly generalize Rudkevich's scheme to imagine each firm i bidding values of β_i and α_i at each iteration, with β_i and α_i chosen by firm i at each iteration to maximize profits given the most recent bids of the other firms. By the same argument as previously, the optimal value of α_i at each iteration will satisfy $\alpha_i = a_i$. Rudkevich's result therefore also provides an explanation for how the firms could arrive at the affine SFE, given that they all begin by bidding competitively.

In Appendix 1 we generalize Rudkevich's result to show that if $n < 1 + \min_i \left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2$ then Rudkevich's update is a contraction map and so the iterative scheme converges to the unique optimum from *any* starting point. The condition $n < 1 + \min_i \left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2$ is always satisfied for $n = 1,2$, but for larger values of n it depends on the cost function and demand function. If the condition is satisfied, then Rudkevich's iterative scheme converges to equilibrium even if, for example, some firms begin by bidding competitively while others begin with non-competitive bids.

The above discussion and uniqueness result are both predicated on the assumption that bids are affine. If non-affine functions may be bid then there are in general multiple SFE.

Aggregate demand and supply

Now considering the aggregate demand and price as a function of time, we have:

$$\begin{aligned}\forall t,\ N(t) - \gamma p(t) &= \Sigma_i\, \beta_i\ (p - a_i),\\ &= p(t)\, \Sigma_i\, \beta_i\ - \Sigma_i\, \beta_i a_i,\end{aligned}$$

where the sum is over all i.

So,

$$\forall t,\ p(t) = (N(t) + \Sigma_i \beta_i a_i)/(\Sigma_i \beta_i + \gamma). \tag{8}$$

Incentive compatibility

The condition $\alpha_i = a_i$ for each firm i means that it is incentive compatible for bidders to reveal the intercepts of their marginal cost functions. This generalizes a similar result in Rudkevich (1999), which was proven for the case where marginal costs had intercept $a_i = 0$. A straightforward economic interpretation of this result is that, at low output levels, the bidders have no motive to exaggerate their cost since they have no infra-marginal capacity. Interestingly, however, the result that $\alpha_i = a_i$ for each firm i does not rely on low output levels and prices ever being realized. The result only depends on their being at least two, perhaps high, prices being realized.

Low demand and price levels

As remarked, if prices fall below the value of a_i for a firm i then the affine functional form (4) will require negative production, which is outside the region of validity of the cost function specification (2). A minor generalization of the affine supply function would allow for piecewise affine, non-decreasing bid supply functions. That is, we now assume that the bid rules allow for piecewise affine, non-decreasing bid supply functions.

We can construct a candidate SFE in piecewise affine supply functions by piecing together several supply functions. In each piece, we use the optimality conditions (6) to evaluate the slope of the supply functions of the firms that are actually generating.[5] So long as the resulting composite supply functions are all non-decreasing then the candidate is truly an SFE.

We illustrate this approach for $n = 2$ firms and consider piecewise affine supply functions of a form such as:

$$\forall i, q_i(p) = \begin{cases} 0, & \text{if } p \le a_i, \\ \beta_i(p - a_i), & \text{if } a_i \le p \le p', \\ \beta_i'(p - a_i), & \text{if } p > p', \end{cases} \tag{9}$$

where $p' \ge a_i$ is an appropriate break-point and β_i and β_i' specify the slopes of the supply functions in the regions $a_i \le p \le p'$ and $p > p'$, respectively.

In general, there may be just one break-point, two break-points as in (9), or more than two break-points. The break-point at a_i for firm i reflects the fact that for prices below this level it cannot be optimal for firm i to produce anything.[6] With the two firm case as an example, let us assume that $a_1 < a_2$ and seek the location of the common break-point p' in each supply function. Notice that for prices up to a_1 there will be no production by either firm.

[5] In fact, we must modify the optimality conditions by recognizing that a piecewise affine supply function is not differentiable at its break-points. However, its directional derivatives exist and for our specification of the slopes we find that Green's equation (4) is satisfied in each direction.

[6] We are ignoring issues such as start-up costs.

For prices between a_1 and a_2, only firm 1 will generate. For this range of prices, we can consider the SFE where firm 1 is the only firm. Substituting into (6) yields

$\beta_1 = \frac{\gamma}{1+c_1\gamma}$. Also, $\beta_2 = 0$, since firm 2 is not producing in this region. These values apply for prices up to a_2. That is, the higher break-point in equation (9) is at $p' = a_2$ for this example.

At higher prices than a_2, both firms will generate and the resulting slopes of the demand functions are given by the simultaneous solutions of:

$\beta_1' = \frac{\gamma + \beta_2'}{1+c_1(\gamma+\beta_2')}$ and $\beta_2' = \frac{\gamma + \beta_1'}{1+c_1(\gamma+\beta_1')}$. By writing the conditions in this way, lemma 1 in Appendix 1 shows that $\beta_1' \geq \beta_1$ and $\beta_2' \geq \beta_2$.

Because $\beta_1' \geq \beta_1$ and $\beta_2' \geq \beta_2$, the resulting composite supply functions of the form (9) are non-decreasing, as illustrated in figure 2 for a firm *i*. In figure 2, the intercept is at a_i = £6/MWh, while the break-point is at p' = £12/MWh. In general, if there are *n* firms with different intercepts a_i then there will be *n* pieces in the supply function of each firm *i* with break-points at a_j, $j = 1,\ldots,n$. The composite supply function so defined is non-decreasing. In principle, similar analysis applies to the case where there are non-zero minimum capacity constraints for a firm under the assumption that the firm cannot choose to take all machines out of service and generate zero.

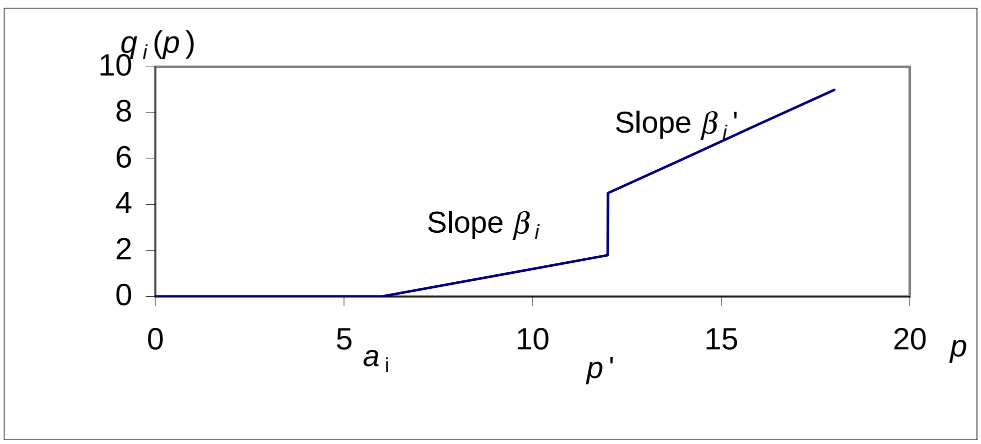


Figure 2. Illustration of piecewise affine, non-decreasing supply function.

3. Maximum Capacity Constraints

In the application discussed below, we encounter maximum capacity constraints on price-taking bidders. (Analogous issues apply in representing maximum capacity constraints on the output of strategic firms.) Green and Newbery (1992) discuss how to treat capacity constraints when cost functions are identical amongst firms. However, their arguments do not appear to generalize to the case where firms have asymmetric costs. We take a different approach here.

We use the function D_r to represent the residual demand resulting from a price taking fringe with capacity constraints. Following Bushnell (1998), if the demand is linear then the resulting residual demand D_r can be modeled as a piecewise linear function. In particular, if the marginal cost of the fringe at its full production is p' then there are γ and γ' such that:

$$-\frac{dD_r}{dp} = \begin{cases} \gamma, & \text{if } p \le p', \\ \gamma', & \text{if } p > p'. \end{cases} \tag{10}$$

The slope γ' for prices above $p = p'$ is due to the demand alone when the fringe is at capacity, while the composite slope γ for prices below $p = p'$ is due to the combined effect of demand and the competitive fringe when the fringe is also marginal. (See Bushnell (1998) section 4 for derivation of this functional form for the residual demand.) We have that $0 < \gamma' < \gamma$. That is, the combination of demand and marginal fringe capacity is more elastic than when the fringe is at its capacity.

A straightforward approach to this case would be again to posit a candidate SFE in piecewise affine functions of the form (9) as we did in the previous section. We solve for the slopes of the supply functions in each piece using (6) with the appropriate demand slope for the piece. In contrast to the case considered in the previous section, however, it will be the case that $\beta_i' < \beta_i$ because of the relationship between γ and γ'. This means that the candidate supply functions so constructed will not be non-decreasing. In particular, there is a drop in the supply at $p = p'$.

The drop in the supply function violates typical pool rules. Moreover, even if the pool rules were to allow such bids, there can be two intersections of the demand and aggregate supply curve. It is not clear that one of the intersections would be preferred by all of the firms to the other intersection. The profit function of each firm can have two local maxima, one in each of the two price regions, $p \le p'$ and $p \ge p'$. Different firms may have their global maximum in different regions. That is, the proposed supply functions may not be an equilibrium even under the (unrealistic) assumption that non-monotonic bids were allowed. A similar observation about profit functions was made in Bushnell (1998).

If we knew a priori that demand (and prices) were “low” then we could delete the part of the supply functions for $p \ge p'$. On the other hand, if we knew that the demand and prices were “high” we could delete the part of the supply function for $p \le p'$. This latter approach was taken by Green (1996) in modeling the fringe in the British pool.

However, our interest is in the case where we anticipate that price will vary from below p' to above p'. Unless the price “jumps” from below p' to above p', we are faced with deciding on a strategy for values of price near to p'. Unlike Green’s equation (4), which made no reference to the load-duration characteristic $N(t)$ of the demand, conditions for an equilibrium in piecewise affine supply functions requires knowledge of $N(t)$.

As an ad hoc approach that avoids the need to specify $N(t)$, we propose a non-decreasing approximation to the previous function. For prices $p > p'$ we maintain the functional form $\beta_i'(p - a_i)$ where the β_i' satisfy (6) for demand slope $-\gamma'$. For prices p significantly below p' we maintain the functional form $\beta_i(p - a_i)$ where the β_i satisfy (6) for demand slope $-\gamma$. For prices just below $p = p'$ we rearrange Green's equation (4) to obtain slopes β_i'' for yet another affine piece of the supply function such that the supply function is continuous at $p = p'$. That is, we posit a supply function of the form:[7]

$$\forall i, q_i(p) = \begin{cases} \beta_i(p - a_i), & \text{if } p \le p_i'', \\ \beta_i''(p - \alpha_i''), & \text{if } p_i'' < p \le p', \\ \beta_i'(p - a_i), & \text{if } p > p', \end{cases} \tag{11}$$

where:

α_i'' is the solution to $\beta_i''(p' - \alpha_i'') = \beta_i'(p' - a_i)$, so that $q_i(p)$ is continuous at $p = p'$, and

p_i'' is the solution to: $\beta_i(p_i'' - a_i) = \beta_i''(p_i'' - \alpha_i'')$, so that $q_i(p)$ is continuous at $p = p_i''$.

Figure 3 illustrates this supply function with a_i = £6/MWh, p_i'' = £10.4/MWh, and p' = £12/MWh.

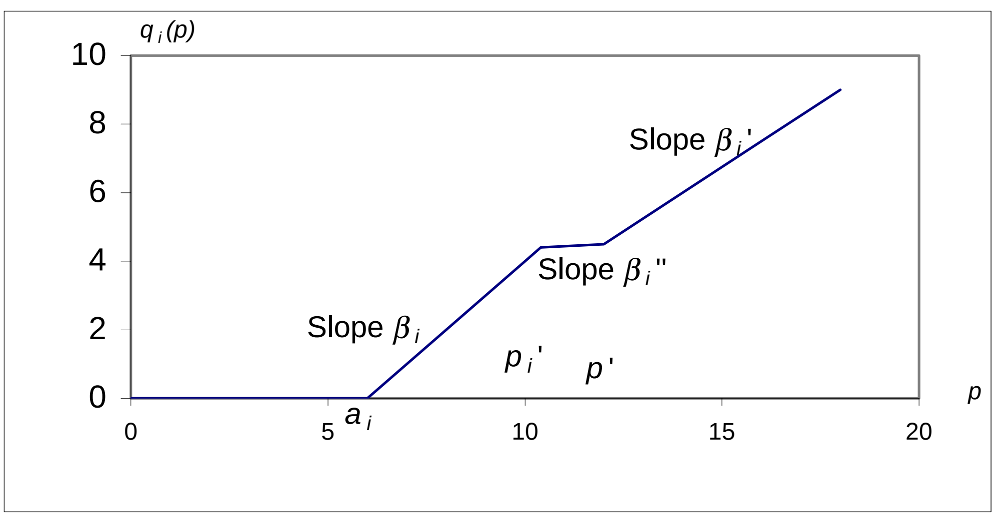


Figure 3. Illustration of supply function (11).

[7] It is also possible to consider a supply function that matches the functional form $\beta_i(p - a_i)$ at $p = p'$ by modifying the function for prices above p'. However, this results in a greater supply and it is presumably in all the strategic players interests to keep supply lower.

As discussed above, we propose to evaluate the slopes β_i" using Green's equation (4) so that the slopes of the affine supply function are optimal for prices infinitesimally below $p = p'$. However, Green's equation (4) is not conveniently expressed to evaluate the optimal slopes of supply functions because the slopes of supply functions for several firms are added together. We rearrange Green's equation (4) into the standard form of a vector differential equation: $\frac{dq}{dp} = f(q, p)$, where:

> q is a column vector of supply functions for all firms,
> $\frac{dq}{dp}$ is the derivative of the vector q, and
> f is a vector function.

To make this rearrangement, note that:

$$\begin{bmatrix} \sum_{j \neq 1} \frac{dq_j}{dp} \\ \vdots \\ \sum_{j \neq n} \frac{dq_j}{dp} \end{bmatrix} = \left[\mathbf{1}\mathbf{1}^T - \mathbf{I}\right] \frac{dq}{dp}, \tag{12}$$

where bold-face **1** is the column vector of all ones and bold-face **I** is the identity matrix. We can write down the inverse of the matrix in brackets: $\left[\mathbf{1}\mathbf{1}^T - \mathbf{I}\right]^{-1} = \frac{1}{n-1}\mathbf{1}\mathbf{1}^T - \mathbf{I}$.

Substituting into Green's equation (4) we obtain:

$$\begin{aligned} \frac{dq}{dp} &= \left[\frac{1}{n-1}\mathbf{1}\mathbf{1}^T - \mathbf{I}\right] \left[\begin{bmatrix} \frac{q_1(p)}{p - C_1'(q_1(p))} \\ \vdots \\ \frac{q_2(p)}{p - C_n'(q_2(p))} \end{bmatrix} + \mathbf{1}\frac{dD_r}{dp} \right], \\ &= \left[\frac{1}{n-1}\mathbf{1}\mathbf{1}^T - \mathbf{I}\right] \begin{bmatrix} \frac{q_1(p)}{p - C_1'(q_1(p))} \\ \vdots \\ \frac{q_2(p)}{p - C_n'(q_2(p))} \end{bmatrix} + \frac{1}{n-1}\mathbf{1}\frac{dD_r}{dp}, \end{aligned} \tag{13}$$

which is now in the standard form of a differential equation.[8] We evaluate (13) at two prices: infinitesimally above p' and infinitesimally below p'. If we consider candidate piece-wise affine supply functions that are continuous at the boundary $p = p'$ then the values of the supply function at these two prices will be the same. The only difference in (13) across the boundary from above p' to below p' is due to the change in the slope of the

[8] This form allows numerical integration to evaluate the nonlinear SFE for arbitrary demand and cost functions. In our numerical experiments, we found that nonlinear solutions are always unstable.

demand function from $-\gamma'$ to $-\gamma$. The change in slope of the supply functions across the boundary is therefore given by:

$$\beta''-\beta' = \frac{1}{n-1}\left[-\mathbf{1}\gamma + \mathbf{1}\gamma'\right] \text{ or } \forall i, \beta_i'' = \beta_i' - \frac{1}{n-1}(\gamma - \gamma'). \qquad (14)$$

Although the supply function slopes β_i'' are only valid at the price just infinitesimally below p', we nevertheless propose using this slope to "join" with the part of the supply function for low prices. If all of the slopes β_i'' in the above expression evaluate to being non-negative then we posit a supply function of the form (11). Since $\gamma > \gamma'$, we have that $\beta_i > \beta_i' > \beta_i''$ so that the situation illustrated in figure 3 shows the slopes with the correct relative magnitudes.

If any particular candidate slope β_i'' turns out to be negative then we modify (11) to be:

$$\forall i, q_i(p) = \begin{cases} \beta_i(p - a_i), & \text{if } p \le p_i'', \\ \beta_i'(p' - a_i), & \text{if } p_i'' < p \le p', \\ \beta_i'(p - a_i), & \text{if } p > p'. \end{cases} \qquad (15)$$

This makes the supply function constant between p_i'' and p' and still guarantees that the supply function is non-decreasing.

We emphasize that this set of supply functions cannot be an equilibrium in nonlinear supply functions since for prices sufficiently below p' but above p_i'' the suggested demand slopes cannot satisfy Green's equation (4). Nevertheless, we claim that the general form of this supply function is a reasonable one in practice, for three basic reasons.

The first reason is that, for the cases considered in section 4, we discretize the time dimension fairly coarsely. Under these circumstances, we find that prices near to the break-point price p' do not arise for the cases we consider. That is, the price does "jump" from well below p' to well above p' and the value of the supply function is irrelevant in the vicinity of p'.

The good fortune that prices near to p' are not realized is partially an artifact of the coarseness of our time discretization. That is, we must in general consider the case where prices are realized in the vicinity of p'. We consider how sub-optimal it is for firm i to use slope β_i'' for prices $p_i'' < p \le p'$ compared to firm i using an optimal response to the other firms, with slope say β_i^*, given that each other firm j maintains slope β_j''. We construct a simple argument that suggests that this sub-optimality is small. This is the second reason why the piecewise affine function is reasonable. This argument also suggests that although the proposed function is not an equilibrium in nonlinear supply functions it is nevertheless likely to be close to an equilibrium in piece-wise affine supply functions.

We first note that the loss in profit for firm i is quadratic in the difference $\beta_i^* - \beta_i''$. The value $\beta_i^* - \beta_i''$ is zero for price $p = p'$ and increases approximately linearly as prices decrease from $p = p'$. Assuming that the demand curve is approximately linear in t when the prices are in the range $p_i'' < p \le p'$, we can argue that the loss of profit is then approximately cubic in the duration of time that $p_i'' < p \le p'$. If this duration is relatively small then the effect on the total profits integrated over time will be small. In summary, the choice of supply function slope β_i'' is sub-optimal but the loss of profits is relatively small compared to unilaterally deviating from this strategy.

Ultimately, this argument points to the fact that in the case of capacity constraints, it is not possible to find an optimal strategy that is independent of the load-duration characteristic $N(t)$. That is, the choice of the slopes in the piecewise affine functions will be affected by the amount of time that prices are realized in the vicinity of the break-point price. For firms faced with making a decision about their supply functions, the lack of optimality may be relatively inconsequential because it is likely to persist over a relatively short time. We will see that this is true for the cases considered in section 4.

The third reason for the reasonableness of the choice of supply functions is that calculation of the optimal response by a firm *i* requires estimation of the slope of the aggregate supply function of the rest of the firms. Generalizing the discussion in Rudkevich (1999), we can imagine firm *i* fitting a piece-wise affine curve to the observed aggregate supply function. If the price p' where the fringe reaches capacity is known publicly, then it is reasonable to for firms to estimate different slopes for the parts of the supply function above and below price $p = p'$. It may even be reasonable for firms to estimate slopes for prices above p', well below p', and in the vicinity of p' as required for the functional form in (11). However, if only a few data points are observed it will not be possible to estimate parameters of functional forms with a large number of parameters. That is, piecewise affine functions with known break-points may be a practical limit to the ability to estimate functions.

4. Applications

In this section we show how to apply the previous results to modeling the price effects of structural change in the E&W electricity market. Our objective is to illustrate how the SFE enhancements described in Sections 2 and 3 improve the numerical fit of such models to actual market experience. We begin by characterizing the structural changes in the E&W electricity market. Next, we reconsider Green's forecast of the effects of divestitures made by National Power (NP) and Power Gen (PG) in 1996. This discussion illustrates the importance of fitting the demand curve properly and the role of capacity constraints. We examine the effect of the 1999 divestitures of National Power (NP) and Power Gen (PG). We show that a linear SFE that does not use positive cost intercepts for the strategic firms will predict prices that are "too low" in the post-divestiture case. We also consider two special issues in this context. First we examine the role of "earn out" payments as an explanation of post-1996 pricing behavior. Finally we illustrate the effect of fringe capacity constraints on the supply curves of strategic firms, showing that the size of this effect is small, as argued above. Table 1 summarizes the structure of the cases examined in this section.

Table 1. Cases Examined and Their Features

	Method	Structural Change	Table or Figure
1996 Divestitures	Capacity Limits Demand Curve Specification	2 firms → 3 firms	Tables 4,5,6,7
1999 Divestitures	Positive vs. Zero Intercept	3 firms → 5 firms	Tables 8,9,10
		Earn Out	Table 11
	Pasting Problem		Figure 5

The E&W Electricity Market

Market power has been a constant theme in the regulation of the electricity industry in England and Wales.[9] This has motivated regulatory intervention and responses by both incumbents and entrants. The two dominant generators, National Power (NP) and Power Gen (PG), have retired very substantial amounts of capacity, invested in new combined cycle gas turbines (CCGTs) and been required to divest generation to new entrants. Additional entry has come primarily from CCGT projects. Nuclear output also grew over time. All of this change on the supply side has occurred in the face of very sluggish growth in demand (about 1.5% per year). Table 2 summarizes the changes in capacity by ownership category over the period 1995-1999, with some comments about the previous history. This table forms the capacity basis for the LSFE estimates reported below.

Table 2. Supply Mix by Firm and Fuel Type

Category	Comment	MW 95-96	MW 98-99	MW 99-00
Nuclear	British Energy + Magnox Electric	10519	10519	10519
Interconnector	France +Scotland			3200
IPP		5000	7339	9721
Eastern			6717	6717
National Power	29 GW @ Vesting + 3 GW CCGT – retirements	23000	16236	12291
Power Gen	19 GW @ Vesting +3 GW CCGT – retirements	18845	15865	11421
AES	w/o IPPs			3945
Edison Mission Energy	w/o First Hydro			3954

[9] For recent statements of these issues see Offer (1998, 1999) and Newbery (1995, 1999) among others.

Nuclear generation expanded in 1994 when Sizewell B, the last nuclear plant constructed in E&W came into service. In 1996, the government owned Nuclear Electric was restructured into the privatized British Energy and Magnox Electric, which remains in government hands.

Independent power producers (IPPs) all rely on combined cycle gas turbine (CCGT) technology. In the first "dash for gas" from 1991-1993, IPPs contracted with regional distributors (Newbery, 1999, p.224f). By 1995 about 5000 MW of IPP capacity was operating. Table 2 shows that this nearly doubled by 1999.

Table 2 also indicates substantial reductions in capacity by NP and PG over the 1990s. When NP and PG were first created in 1989, NP had 29,486 MW and PG had 19,802 MW (Newbery, 1999, p.202). Each added about 3,200 MW of CCGT capacity and together the two firms retired more than 20,000 MW.[10] These retirements were in addition to divestitures of more than 6000 MW to Eastern in 1996 and the 1999 divestitures to AES and Edison Mission Energy indicated in Table 2.

Table 3 shows the cost and availability assumptions used in our calculations. The cost intercept is assumed to be zero for Nuclear and the Interconnector, a common intercept of £12/MWh is assumed for the generators with coal-fired plant only (Eastern, AES and EME) and £8/MWh for generators with CCGT plant (IPP, NP, and PG). These marginal cost estimates are based on Bunn and Day (1999).[11] The cost at maximum capacity follows Green (1996) for generators with thermal plant. These costs can be thought of as either open cycle turbine costs or the costs of coal plant running very few hours and recovering start up and no load costs over that short period.

The availability estimates in Table 3 for all fossil fuel generators are generic. The estimates for nuclear are set to reproduce recent production levels. The high availability for the Interconnector reflects the multiplicity of resources available from Scotland and France.

The cost and availability data in Table 3 are combined to produce the cost slope parameters c_i that are used to solve for the equilibrium prices and mark-ups. These parameters also depend upon the capacity of each firm. Appendix 2 gives the numerical values used in the results reported below.

---

[10] According to NGC (1999) there were 15,152 MW of "Disconnections" between 1991 and 1999 (see Table 3.11 and 5328 MW of "Decommissionings" (see Table 3.12) for NP and PG together.

[11] GN use £18.5/MWh as the intercept of the marginal cost function for coal plant (p.942), based on coal prices quoted in generators' prospectus. In the years since the flotation of NP and PG, coal and gas prices have declined substantially.

Table 3. Cost and Availability Parameters

Category	Availability	Cost Intercept (£/MWh)	Cost at Maximum Capacity (£/MWh)
Nuclear	83.0%	0	10.00
Interconnector	98.0%	0	10.00
IPP	85.0%	8.00	14.00
Eastern	85.0%	12.00	30.00
National Power	85.0%	8.00	30.00
Power Gen	85.0%	8.00	30.00
AES	85.0%	12.00	30.00
Edison Mission Energy	85.0%	12.00	30.00

Fitting Demand Curves

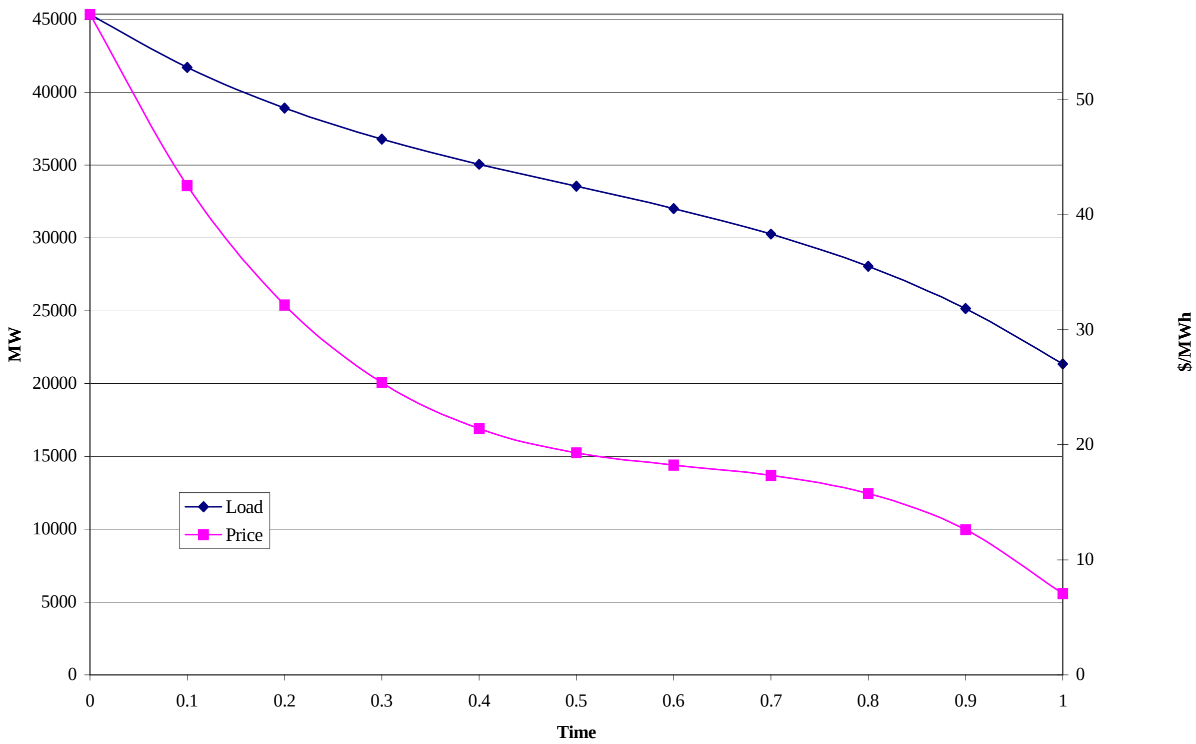
A major problem associated with practical use of both SFE and Cournot models is the representation of demand. The plausibility of price forecasts with these models depends substantially on how the demand curve is specified. Although there is little demand-side response in the E&W market, low values of demand elasticity have typically yielded poor fits to the observed data. For example, when GN used very low slopes for the linear demand curve they estimated prices that were very much higher than what was subsequently observed. Even at a high slope (-0.5 GW/(£/MWh)) the predicted prices are much higher than what was observed.

A good part of the problem of representing demand involves how the demand curve is "anchored" in price-quantity space. GN use an estimate of pre-competition marginal costs and production to anchor their demand curve. Green (1996) uses a demand curve in the form of Eq. (1) above, with N(t) specified as a cubic load duration curve and a slope parameter of -0.5 GW/(£/MWh). This value is considerably higher than what other authors use. GN use 0.25 as their "central" case; Bushnell (1998) uses 0.1; Bunn and Day (1999) use values between 0.01 and 0.10. We will follow GN and report cases for demand slopes of 0.5, 0.25 and 0.1.

Our procedure takes advantage of the Pool price history. Figure 4 shows both the load duration curve and the price duration curve for 1998-99. The prices shown in this figure are the System Marginal Price (SMP).[12] We use this data to anchor the demand curves used in our simulations. The anchor point for each of the nine periods we consider is taken to be the (p,q) pair corresponding to the deciles of the price and load duration curves. This procedure assumes that past price behavior represents a set of expectations that is familiar to strategic firms and acceptable to regulators.

[12] We ignore other price elements, i.e. the uplift and capacity charges.

Figure 4. Load & Price Duration Curves



For the conditions studied by GN and Green (1996), the price-taking producers were never marginal. Therefore, Green (1996), for example, reduces the load duration curve by his estimate of constant inframarginal production and applies the LSFE to the residual demand.[13]

By 1999, however, growth in the capacity of IPPs and the increased availability of nuclear plant made the price-takers marginal during low demand periods. Incorporating this effect introduces the need to model capacity limits explicitly. The methods discussed in Section 3 are implemented in our estimates. Examples of how this works are given below. We also discuss the plausibility of the ad hoc piecewise affine model (11).

Revisting the 1996 Divestitures
In this section, we examine the 1996 divestitures. Our chief interest is in showing how capacity constraints and demand curve representation affect numerical estimates. This is done by re-examining Green (1996), which we refer to as G96. We begin by reproducing G96 results at the same range of demand slopes used by GN. Table 4 reports the GN results and corresponding estimates from the G96 model. This table also reports the demand at each demand slope.

[13] We followed the same approach in section 3 for notational convenience. However, here we will generally report the actual demand and supply, not the residual demand.

Table 4. Duopoly Game

Demand Slope (GW/(£/MWh))	GN Price (£/MWh)	G96 Price (£/MWh)	G96 Demand (Average GW)
0.10	66.7	66.6	38.3
0.25	41.1	40.5	34.9
0.50	32.3	27.1	31.5

While the prices in GN and G96 are similar for each demand slope, GN appears to have shifted the demand curve for each demand slope case, since they report the same output in all cases. Table 4 is based on a literal interpretation of the G96 model, i.e. no shifts in the demand curve, and indicates significant demand level differences.[14] Average demand levels of 33 GW are consistent with late 1990s observations. However, both G96 prices and demand levels are substantially too high at a demand slope of 0.10, and move in more reasonable directions as the demand slope increases.

Table 5 reports tests of the G96 model where PG and NP divest 15% of their capacity to 2 firms that each bid strategically. This amounts to about 4.5 GW for NP and 2 GW for PG. Table 8 shows the same kinds of demand effects seen in Table 4. It also shows that at demand slopes of less than 0.5, the new firms would produce more than their capacity. There is no notion of capacity in these models, which amounts to the assumption of potentially infinite supply from any producer. Without an explicit mechanism to introduce maximum capacity limits, any version of the SFE approach can result in the kind of anomaly illustrated in Table 5.

Table 5. G96 Divestiture Cases

Demand Slope (GW/(£/MWh))	Divest Price (£/MWh)	Divest Demand (GW)	Maximum Output of Divested Plant (GW)
0.10	39.1	41.1	10.5
0.25	30.7	37.3	8.2
0.50	23.5	33.4	6.3

Tables 4 and 5 show that the case that is closest to observed prices and physically realizable quantities is the one that Green reports, i.e. the high demand slope case. It produces an expected price decline of about 15% compared to the corresponding duopoly case. The high demand slope is difficult to justify, however, since electricity demand is extremely inelastic in the short-run. Most studies of electricity markets using equilibrium approaches adopt a low elasticity parameterization. This mismatch between G96 and standard intuition motivates a further examination of what might improve the realism of results at lower demand slopes.

Table 6 shows the effect of introducing capacity limits and adjusting the intercept of Green's demand function. These estimates are based on 5000 MW of IPPs, 23 GW of

[14] We report the realized demand as the sum of the 15 GW inframarginal supply and the duopoly supply.

capacity for NP and 18.8 GW for NP in the duopoly case. Average and Adjusted Demand is in each case the average of the duopoly and divestiture cases.

Table 6. G96 LDC and Capacity Limits

Demand Slope (GW/ (£/MWh))	Duopoly Price (£/MWh)	Divest Price (£/MWh)	Average Demand (GW)	Adjusted Demand (GW)	Duopoly Price (£/MWh)	Divest Price (£/MWh)
G96 Load Duration Curve				Intercept Adjusted LDC		
0.10	53.9	43.9	40.1	34.6	40.9	27.4
0.25	33.3	26.9	37.5	34.5	28.7	22.9
0.50	22.6	19.8	34.4			

Comparing Tables 4, 5 and 6 requires some care. The inframarginal price taking capacity is about 1000 MW greater in the Table 6 cases than the Table 4 and 5 cases.[15] At high demand slope and without adjusting the demand curve intercept, this results in large percentage differences in the duopoly price (£27.1/MWh at a demand slope of 0.5 GW/(£/MWh) in Table 4 versus £22.6/MWh at the same slope in Table 6) and in the divestiture price (£23.5/MWh at a demand slope of 0.5 GW/(£/MWh) in Table 5 versus £19.8/MWh at the same slope in Table 6). At low demand slope (again without adjusting the demand curve intercept) the changes work differently. The duopoly price drops about 20% (from £66.7/MWh to £53.9/MWh at a demand slope of 0.10 GW/(£/MWh) and from £41.1/MWh to £33.3/MWh at a demand slope of 0.25 GW/(£/MWh)), but the divestiture price increases by about 10% (from £39.1/MWh to £43.9/MWh at a demand slope of 0.10 GW/(£/MWh) and from £30.7/MWh to £33.3/MWh at a demand slope of 0.25 GW/(£/MWh)). The divestiture price increase effect seems strongly tied to enforcing capacity limits in Table 6, because Table 5 showed the biggest problem in this regard, i.e. output greater than capacity, at low demand slope.

The objective of the Intercept Adjusted LDC cases in the right hand panel of Table 6 is to illustrate the dramatic effect on price and quantity that the shifts of the demand curve achieve. The case with demand slope of 0.25 GW/(£/MWh) involved a reduction of 4000 MW of the intercept term in Green's specification of the load duration curve. The adjustment is 7000 MW for the case of demand slope of 0.10 GW/(£/MWh). These adjustments result in average demands at virtually the same level as the case of demand slope of 0.5 GW/(£/MWh) that is shown on the left hand side of Table 6. Prices in the duopoly and divestiture cases are more reasonable with these adjustments, although still not where they should be.

A final set of tests involves using our demand curve instead of the G96 curve. It also examines both the zero and the positive intercept cases (compared to zero only in Table 6). Table 7 summarizes these tests. It should be compared to the right hand panel of Table

[15] This is due to G96 simply subtracting an estimate of inframarginal supply from the load duration curve versus explicitly representing the capacity and production of these producers.

6 to show the incremental effect of using a demand curve anchored at historical prices and quantities.

Broadly speaking, the Table 7 tests are improvements compared to the right hand panel of Table 6. For the zero intercept representation, prices decrease in all cases. With positive intercept, the prices are lower for the 0.1 GW/(£/MWh) demand slope, but they are higher for the demand slope 0.25 GW/(£/MWh) compared to the right hand panel of Table 6.

Table 7. Fitted LDC

	Demand Slope GW/(£/MWh)	Duopoly price (£/MWh)	Divest to 3 price (£/MWh)	Average GW
Zero Intercept	0.10	36.2	24.8	32.7
	0.25	26.5	21.7	33.3
	0.50	22.7	20.3	34.6
Cost Intercepts	0.10	38.4	27.2	32.6
	0.25	29.2	24.5	32.6
	0.50	25.2	23.0	33.2
Earn Out	0.10		27.3	33.1
	0.25		25.1	33.1
	0.50		23.7	33.4

Table 7 includes the effect of the earn out payments in the cost intercept case. The role of cost intercepts is discussed in more detail in the next section.

The 1999 Divestitures

In 1999, NP and PG each divested about 4000 MW of coal-fired plant. Their motivation was to meet regulatory requirements associated with their proposed vertical mergers. Table 2 shows that the U.S. firms AES and Edison Mission Energy (EME) acquired the divested plant. Both firms already had generation assets in the E&W market; IPPs for AES and both IPPs and the pumped storage plants for EME.[16] We use the LSFE framework to assess the price implications of these divestitures, and to compare the performance of the affine case with the case where the marginal cost curves must go through the origin.

Table 8 shows the results of using the data in Table 2 and two versions of the cost curve for the strategic generators with gas and coal-fired plant. The cases labeled Zero Intercept assume that all cost curve pass through the origin. The Positive Intercept cases assume the cost intercept values in Table 3. Our calculations assume that both AES and EME bid strategically. Because the price changes from divestiture are realized over time, we have used the 1998-99 level of IPP capacity (i.e 7339 MW) for the 3 firm game and the 1999-2000 level of capacity (i.e. 9721 MW) for the 5 firm game.

[16] For simplicity, the pumped storage plant is neglected in these estimates.

Table 8. Base Case Price Results (£/MWh)

	5 Firms			3 Firms		
Demand Slope (GW/ (£/MWh))	Average Price (£/MWh)	High Price (£/MWh)	Low Price (£/MWh)	Average Price (£/MWh)	High Price (£/MWh)	Low Price (£/MWh)
Zero Intercept						
0.10	15.5	27.3	7.4	23.6	51.7	8.0
0.25	16.3	31.1	7.5	20.9	41.1	8.1
0.50	17.1	34.4	7.7	19.9	38.3	8.2
Positive Intercept						
0.10	19.5	28.1	12.1	26.3	51.1	12.9
0.25	19.9	29.3	12.3	23.9	41.5	12.7
0.50	20.5	34.9	12.2	22.6	38.6	12.5

Table 8 shows for each case the time-weighted average price, the highest price and the lowest price. The price change predictions of divestiture, i.e. the difference in average prices, are greater at low elasticity than at high elasticity regardless of the cost curve characterization. As expected, the biggest differences between the positive and zero intercept representation are at the low end of the price distribution.

In reality, the SMP has dropped substantially in the 1999-2000 power year. Previously, SMP had been reasonably stable in the £23-24/MWh range over many years. The decline is projected to be about 20% from the previous level (Standard and Poor's, 2000a,b). This suggests that the estimates based on the positive intercept cases are better than those based on the zero intercept cases.

Tables 9 and 10 show the quantity behavior (MW produced per hour) of the firms for the cases with demand slope of 0.25 GW/(£/MWh) for each cost curve characterization. Table 9a shows that the fringe firms (Nuclear, Interconnector and IPP) are marginal during periods 8 and 9. This is indicated by shading the periods and indicating in bold that the fringe firm production is below the maximum levels achieved in higher demand periods. In Table 9b, the fringe firms are marginal in period 7 as well as periods 8 and 9.

Table 9a. Zero Intercept Case: 3 Strategic Firms

Time Period	Average	1	2	3	4	5	6	7	8	9
Price (£/MWh)	20.93	41.11	29.27	24.76	21.69	19.48	17.56	15.36	11.02	8.09
Nuclear	8,707	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	**8,510**
Interconnector	3,075	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136	**2,589**
IPP	5,966	6,239	6,239	6,239	6,239	6,239	6,239	6,239	**5,779**	**4,241**
Eastern	3,584	5,695	5,193	4,393	3,849	3,457	3,116	2,725	2,149	1,683
NP	6,491	10,740	9,086	7,686	6,733	6,048	5,452	4,768	4,244	3,659
PG	6,326	10,530	8,861	7,496	6,566	5,898	5,317	4,650	4,103	3,510

Table 9b. Zero Intercept Case: 5 Strategic Firms

Time Period	Average	1	2	3	4	5	6	7	8	9
Price (£/MWh)	16.29	31.12	23.24	19.41	16.80	14.92	13.29	10.96	9.34	7.54
Nuclear	8,643	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	**7,932**
Interconnector	3,039	3,136	3,136	3,136	3,136	3,136	3,136	3,136	**2,987**	**2,413**
IPP	7,655	8,262	8,262	8,262	8,262	8,262	8,262	**7,607**	**6,482**	**5,235**
Eastern	3,012	5,350	4,273	3,568	3,088	2,743	2,443	2,181	1,891	1,576
AES	1,882	3,315	2,704	2,258	1,954	1,736	1,546	1,333	1,147	944
NP	4,715	7,893	6,631	5,537	4,792	4,256	3,791	3,619	3,185	2,732
PG	4,481	7,568	6,312	5,271	4,561	4,052	3,608	3,409	2,994	2,556
EME	1,882	3,315	2,704	2,258	1,954	1,736	1,546	1,333	1,147	944

Tables 9a and 9b show that the coal based strategic firms produce even when the fringe firms are marginal. The prices in those periods, however, are below their marginal costs; i.e. the assumed intercept of £12/MWh. The zero intercept formulation forces the plants to operate at prices that are below the intercept of their true marginal cost curves. By contrast, Tables 10a and 10b show production by coal based strategic firms declining toward zero as price falls toward the marginal cost minimum.

Table 10a. Positive Intercept Case: 3 Strategic Firms

Time Period	Average	1	2	3	4	5	6	7	8	9
Price (£/MWh)	23.87	41.45	30.12	26.61	24.22	22.50	21.01	19.29	16.90	12.71
Nuclear	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732
Interconnector	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136
IPP	6,186	6,239	6,239	6,239	6,239	6,239	6,239	6,239	6,239	**5,760**
Eastern	3,005	5,695	4,993	4,026	3,367	2,894	2,482	2,009	1,351	230
NP	6,250	10,683	9,074	7,634	6,653	5,948	5,335	4,631	3,651	2,637
PG	6,106	10,502	8,859	7,453	6,495	5,807	5,209	4,521	3,565	2,542

Table 10b. Positive Intercept Case: 5 Strategic Firms

Time Period	Average	1	2	3	4	5	6	7	8	9
Price (£/MWh)	19.88	29.32	25.29	22.51	20.61	19.25	18.06	16.70	14.80	12.34
Nuclear	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732	8,732
Interconnector	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136	3,136
IPP	8,262	8,262	8,262	8,262	8,262	8,262	8,262	8,262	8,262	**7,032**
Eastern	2,296	5,086	3,903	3,085	2,528	2,128	1,780	1,380	823	114
AES	1,480	3,278	2,516	1,989	1,630	1,371	1,147	889	531	69
NP	4,636	8,324	6,751	5,664	4,923	4,391	3,928	3,396	2,656	2,051
PG	4,410	7,922	6,425	5,390	4,685	4,178	3,738	3,232	2,528	1,926
EME	1,480	3,278	2,516	1,989	1,630	1,371	1,147	889	531	69

The results in Table 8 show dramatic price effects from the 1999 divestitures, exacerbated by the additional IPP capacity. At the low demand slope preferred by many authors, i.e. 0.10 GW/(£/MWh), the average prices in the pre-divestiture case are reasonable for the zero intercept case, but the price drop prediction is too great. The positive intercept case predicts a plausible post-divestiture price but is too high pre-divestiture compared to the observed level.

The Earn Out Payments
The formulation used in Table 8 neglects a particular point about Eastern's costs that came to public attention in 1998. This is the "earn-out" payments made by Eastern to NP and PG. The plant controlled by Eastern was actually leased from NP and PG, not purchased. The lease payments were made on a variable basis at a rate of £6/MWh. This has the effect of increasing Eastern's costs by this amount. It is interesting to test how the earn-out payments affect the equilibrium price. Table 11 reports the results of re-running the 3 firm game with this change in costs.

Table 11. 3 Firm Market: Earn-out Sensitivity

	Zero Intercept			Positive Intercept		
Demand Slope	Average Price	High Price	Low Price	Average Price	High Price	Low Price
0.10	23.1	39.7	8.1	26.5	37.6	18.5
0.25	21.0	37.1	8.2	24.5	36.4	14.9
0.50	20.3	38.3	8.6	23.3	36.3	13.2

There are a number of interesting results in Table 11. First, the prices at high demand are generally lower than the corresponding cases in Table 8 . It appears that by raising Eastern's costs, it becomes less profitable for them to push up the price in this period. Second, the low end of the price distribution is only slightly higher in the zero intercept case, but significantly higher with positive intercept. The explanation of this difference is obvious. There is no way to raise Eastern's cost near zero output without having a positive intercept term. Third, the earn-out payment has ambiguous, but quite small effect in the zero intercept case. In the positive intercept case it moves in the proper direction, but even there the effect is modest. Finally, in the positive intercept case we get cases of zero production by Eastern when demand is low, but the fringe firms are still producing at full output (compare to Tables 10a and 10b). Conversely, in the zero intercept case, Eastern still produces at low demand, even when the fringe is marginal. This is the same kind of result illustrated in Tables 9a and 9b where price is below what we think is the marginal cost of a producer that is operating. As argued above, this outcome illustrates the implausibility of the zero intercept case.

Pasting Supply Curves Together
A final empirical point about the effect of capacity constraints on supply functions is illustrated in Figure 5. This figure corresponds to Table 10b. It shows the supply functions of the five strategic firms constructed according to equations (11)-(15). Figure

5 shows prices in the range £10/MWh to £30/MWh, which covers the range of realized prices. In this range, every firm except the IPP is either:

- always at capacity, or
- always marginal or off.

Consequently, the only maximum capacity reached in this range is the capacity of the IPP fringe capacity.

Recall that in (11) we posited slopes β_i'' for the supply functions for the range of prices just below where the fringe reaches capacity. These slopes were tentatively calculated according to (14). However, these slopes all turn out to be negative for the data, so the supply functions are modified to be of the form of equation (15). This accounts for the horizontal part of the supply curve in figure 5 between £12/MWh and £13/MWh,.

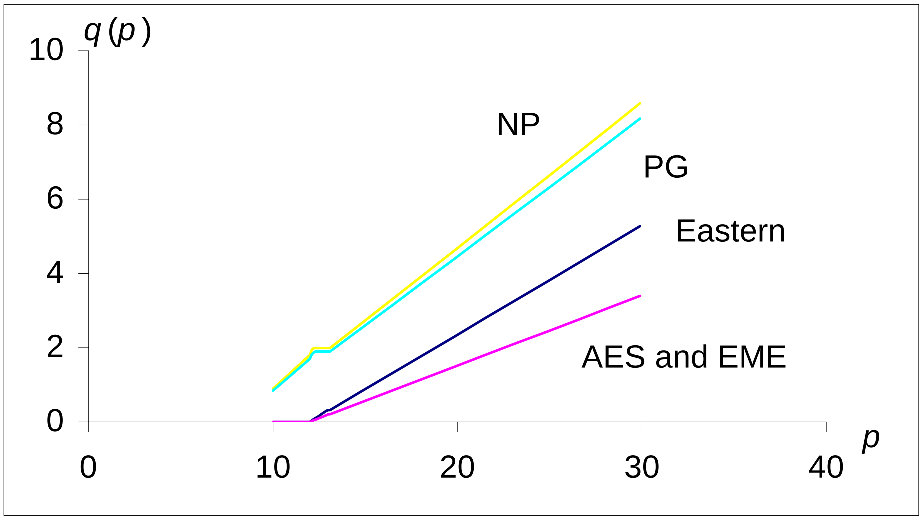


Figure 5. Piecewise affine supply curve constructed according to (15).

The supply functions are horizontal in a relatively small price range. This confirms the claim made in section 3 that the adjustment to the supply curve is relatively inconsequential in the overall supply curve.

5. Conclusions

This paper has shown that the LSFE model generalizes readily to the affine case. We also introduce capacity constraints. Capacity constraints seem essential to model cases where fringe producers set the price for any demand periods. These constraints introduce discontinuities that are in some sense more extreme than similar phenomena in the Cournot framework. We propose an ad hoc approach to pasting together the supply curve discontinuities.

These theoretical properties are illustrated in a practical setting, namely the evolution of price behavior in the E&W electricity market during periods of structural change. The affine case seems to fit the price behavior in the E&W market better than the zero intercept case.

In future work we plan to consider the effect of price caps and analyze transmission constraints. The work of Berry et al (1999) provides guidance in the consideration of transmission constraints.

6. References

Alba, J., I. Otero-Novas, C. Meseguer and C. Batlle, "Competitor Behavior and Optimal Dispatch: Modelling Techniques for Decision-Making," *The New Power Markets: Corporate Strategies for Risk and Reward*. ed. R. Jameson, London: Risk Publications, 1999.

Andersson, B. and L. Bergman, "Market Structure and the Price of Electricity: An Ex Ante Analysis of the Deregulated Swedish Electricity Market," *The Energy Journal* 16(2) 97-130, 1995.

Berry, C. A, Hobbs, B. F., Meroney, W. A., O'Neill, R. P., and Stewart, W. R., "Understanding how market power can arise in network competition: a game theoretic approach," *Utilities Policy*, 8, 139-158, 1999.

Bohn, R., A. Klevorick and C. Stalon, "Second Report on Market Issues in the California Power Exchange Energy Markets," prepared for the Federal Energy Regulatory Commission by the Market Monitoring Committee of the California Power Exchange, March, 1999.

Borenstein, S. and J. Bushnell, "An Empirical Analysis of the Potential for Market Power in California's Electricity Industry," *Journal of Industrial Economics* 47(3): 285-323, 1999.

Borenstein, S., J. Bushnell and S. Stoft, "The Competitive Effect of Transmission Capacity in a Deregulated Electricity Market," *RAND Journal of Economics* (to appear).

Bunn, D. and C. Day, "Market Structure is Fundamental," *Strategic Price Risk in Wholesale Power Markets*, London: Risk Books, 1999.

Bushnell, J. "Water and Power: Hydroelectric Resources in the Era of Competition in the Western US," University of California working paper PWP-056r, 1998

Frame, R. and P. Joskow, Testimony in State of New Jersey Board of Public Utilities Docket No. EX94120585Y and EO97070463, 1998.

Green, R. “Increasing Competition in the British Electricity Spot Market,” *Journal of Industrial Economics*, 44(2), 205-216, 1996.

Green, R. and D. Newbery, “Competition in the British Electricity Spot Market,” *Journal of Political Economy*, 100(5), 929-953, 1992.

Hogan, W., "A Market Power Model With Strategic Interaction in Electricity Networks," *The Energy Journal* 18(4) 107-142, 1997.

Klemperer, P. and M. Mayer, "Supply Function Equilibria in Oligopoly Under Uncertainty," *Econometrica* 57 (1989) 1243-77.

Laussel, D., "Strategic Commercial Policy Revisited: A Supply-Function Equilibrium Model," *American Economic Review* 82(1) 1992, 84-99.

National Grid Company (NGC). Seven Year Statement, 1999.

Newbery, D., “Power Markets and Market Power,” *The Energy Journal*, 16(3), 36-66, 1995.

Newbery, D. *Privatization, Restructuring, and Regulation of Network Utilities*. MIT Press, 1999.

Office of Electricity Regulation (OFFER), Report on Pool Prices Increases in Winter 1997/98, June, 1998.

Office of Electricity Regulation (OFFER), Rises in Pool Prices in July: A Decision Document, October, 1999.

Ramos, A., M. Ventosa and M. Rivier, “Modeling Competition in Electric Energy Markets by Equilibrium Constraints,” *Utilities Policy*, v.7, no.4 (1998) 223-242.

Rivier, M., M. Ventosa, A. Ramos, F. Martinez-Corcoles, and A. Chiarri, "A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem," Proceedings of the International Conference on Complementarity Problems, Madison, Wisconsin, 1999.

Rudkevich, A., “Supply function equilibrium in poolco type power markets: Learning all the way,” TCA Technical Report Number 0699-1701, Tabors Caramanis and Associates, June 1999.

Rudkevich, A., M. Duckworth, R. Rosen “Modeling Electricity Pricing in a Deregulated Generation Industry: The Potential for Oligopoly Pricing in a Poolco,” *The Energy Journal*, 19(3), 19--48, 1998.

Scott, T., "Hydro reservoir management for an electricity market with long term contracts," Ph.D. thesis, University of Canterbury, Department of Management, 1997.

Scott, T. and G. Read, "Modeling Hydro Reservoir Operation in a Deregulated Electricity Market," *International Transactions in Operations Research* 3(3) 243-253, 1996.

Standard and Poor's, "Tumbling Forward Power Prices Put U.K. Electricity Sector at Risk," *Infrastructure Finance Commentary*, March, 2000a.

Standard and Poor's, "AES Drax Ltd," *Infrastructure Finance,* July, 2000b.

Turnbull, S. J., "Choosing duopoly solutions by consistent conjectures and by uncertainty," *Economics Letters*, 13, 253-258, 1983.

Acknowledgment
The first author would like to thank Professor Severin Borenstein for his hospitality during a visit to the University of California Energy Institute in Summer 2000. The first author was funded in part by the National Science Foundation under grant ECS-9457133 and by a grant from Tabors Caramanis and Associates.

Appendix 1. Conditions for convergence to solution of (6)

We develop a series of results that show conditions for converging to a solution of (6). Some of the results are based on and generalize similar ideas in Rudkevich (1999). Throughout, a symbol without subscripts will stand for the vector of subscripted symbols. For example, the symbol β represents the *n*-vector with components β_i. We also make the following definitions:

$$\forall i, Z_i = \gamma + \sum_{i \neq j} \beta_j,$$

$$\forall i, \Phi_i(\beta) = \phi_i(Z_i) = \frac{Z_i}{1 + c_i Z_i}.$$

We note that any fixed point of the map Φ satisfies (6). The map Φ represents the following strategy for each firm *i*: each time the firm updates its bid supply function, it chooses the slope of its updated affine supply function slope to maximize profits, given the most recent slopes used by all the other firms. This is the same update as proposed by Rudkevich (1999). Rudkevich shows that there is a unique non-negative solution to (6) and shows that if the firms initially bid in a manner that reflect true costs then the map converges to the affine SFE slopes. Here, we consider conditions for the map Φ to be a contraction map. That is, we consider the conditions under which Φ converges to the unique solution from arbitrary initial bids having non-negative slopes.

Lemma 1.

If $\beta \geq 0$ then $\forall i, \dfrac{\gamma}{1 + c_i \gamma} \leq \Phi_i(\beta) = \phi_i(Z_i) < \dfrac{1}{c_i}$.

The functions ϕ_i are monotonically increasing in Z_i for $\beta \geq 0$.

Proof:

First, note that $(\beta \geq 0) \Rightarrow (Z_i \geq \gamma)$.

For $Z_i = \gamma, \Phi_i(\beta) = \phi_i(Z_i) = \phi_i(\gamma) = \dfrac{\gamma}{1 + c_i \gamma}$. As $Z_i \to \infty, \Phi_i(\beta) = \phi_i(Z_i) \to \dfrac{1}{c_i}$

Moreover, $\dfrac{d\phi_i}{dZ_i} = \dfrac{1}{(1 + c_i Z_i)^2} > 0$, so ϕ_i is strictly monotonically increasing in Z_i for $\beta \geq 0$.

Therefore, if $\beta \geq 0$ then $\dfrac{\gamma}{1 + c_i \gamma} = \phi_i(\gamma) \leq \phi_i(Z_i) = \Phi_i(\beta) < \dfrac{1}{c_i}$.

QED.

Lemma 1 shows that at each iteration, each updated slope β_i lies in the range $\dfrac{\gamma}{1 + c_i \gamma} \leq \beta_i < \dfrac{1}{c_i}$.

Corollary 2. Any non-negative fixed point β^* of the map Φ satisfies $\forall i, \dfrac{\gamma}{1 + c_i \gamma} \leq \Phi_i(\beta^*) < \dfrac{1}{c_i}$.

QED

Now note that:
$\Phi(\beta) = \phi((\mathbf{1}\mathbf{1}^T - \mathbf{I})\beta + \mathbf{1}\gamma)$, since $Z = (\mathbf{1}\mathbf{1}^T - \mathbf{I})\beta + \mathbf{1}\gamma$.

Therefore, by the chain rule: $\frac{d\Phi}{d\beta} = \frac{\partial \phi}{\partial Z}\frac{\partial Z}{\partial \beta} = \frac{\partial \phi}{\partial Z}\left((\mathbf{1}\mathbf{1}^T - \mathbf{I})\beta + \mathbf{1}\gamma\right)(\mathbf{1}\mathbf{1}^T - \mathbf{I})$,

and so $\left\|\frac{d\Phi}{d\beta}\right\|_2 \le \left\|\frac{\partial \phi}{\partial Z}\left((\mathbf{1}\mathbf{1}^T - \mathbf{I})\beta + \mathbf{1}\gamma\right)\right\|_2 \left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})\right\|_2$.

Lemma 3: $\left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})\right\|_2 = n-1$.

Proof:

$\forall x, \left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})x\right\|_2^2 = x^T(\mathbf{1}\mathbf{1}^T - \mathbf{I})(\mathbf{1}\mathbf{1}^T - \mathbf{I})x = x^T(\mathbf{1}\mathbf{1}^T\mathbf{1}\mathbf{1}^T - \mathbf{1}\mathbf{1}^T - \mathbf{1}\mathbf{1}^T + \mathbf{I})x = x^T((n-2)\mathbf{1}\mathbf{1}^T + \mathbf{I})x$.

Now $\mathbf{1}^T x = \sum_i x_i \le \|x\|_1$ and so $\left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})x\right\|_2^2 \le (n-2)\|x\|_1^2 + \|x\|_2^2 \le (n-2)n\|x\|_2^2 + \|x\|_2^2 = (n-1)^2\|x\|_2^2$,

where the last inequality follows from the known relationship between the 1 and 2 norms in the reals. That is, we have shown that $\left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})\right\|_2 \le n-1$. Moreover, for $x = \mathbf{1}$ we have:

$(\mathbf{1}\mathbf{1}^T - \mathbf{I})x = \mathbf{1}\mathbf{1}^T\mathbf{1} - \mathbf{1} = (n-1)\mathbf{1} = (n-1)x$, so $\left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})x\right\|_2 = \|(n-1)x\|_2 = (n-1)\|x\|_2$.

That is, $\left\|(\mathbf{1}\mathbf{1}^T - \mathbf{I})\right\|_2 = n-1$.

QED

Lemma 4: If $\forall i, \beta_i \ge \frac{\gamma}{1+c_i\gamma}$ then $\left\|\frac{\partial \phi}{\partial Z}\right\|_2 \le \max_i \frac{1}{\left(1+c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)^2}$.

Proof:

First note that $c_i Z_i = c_i\left(\gamma + \sum_{j\ne i}\beta_j\right) \ge c_i\left(\gamma + \sum_{j\ne i}\frac{\gamma}{1+c_j\gamma}\right) = c_i\gamma\left(1 + \sum_{j\ne i}\frac{1}{1+c_j\gamma}\right) \ge c_i\gamma\sum_j \frac{1}{1+c_j\gamma}$.

$$\frac{\partial \phi}{\partial Z} = \mathrm{diag}\left\{\frac{1}{(1+c_i Z_i)^2}\right\} \text{ so } \left\|\frac{\partial \phi}{\partial Z}\right\|_2 = \max_i\left\{\frac{1}{(1+c_i Z_i)^2}\right\} \le \max_i \frac{1}{\left(1+c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)^2}.$$

QED

Corollary 5: If $\forall i, \beta_i \ge \frac{\gamma}{1+c_i\gamma}$ then $\left\|\frac{d\Phi}{d\beta}\right\|_2 \le (n-1)\max_i \frac{1}{\left(1+c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)^2}$.

Corollary 6. The map Φ is a contraction mapping on $\forall i, \beta_i \ge \frac{\gamma}{1+c_i\gamma}$ if

$$n < 1 + \min_i\left(1+c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)^2.$$

Corollary 7. If $n < 1 + \min_i \left(1 + c_i\gamma \sum_j \frac{1}{1 + c_j\gamma}\right)^2$ then by the contraction mapping theorem the map Φ, when begun with an initial iterate $\beta \geq 0$, generates a sequence of iterates that converges to the unique fixed point of Φ on $\beta \geq 0$.

In summary, by Corollaries 2 and 7 and the analysis in Rudkevich (1999), there is a unique non-negative solution β^* to (6), the solution satisfies: $\forall i, \frac{\gamma}{1 + c_i\gamma} \leq \beta_i^* < \frac{1}{c_i}$, and if $n < 1 + \min_i \left(1 + c_i\gamma \sum_j \frac{1}{1 + c_j\gamma}\right)^2$ then the map Φ, when begun with an initial iterate $\beta \geq 0$, generates a sequence of iterates that converges to the unique non-negative solution of (6). For the cost slope parameters given in Table A1, this condition holds for all cases except the low slope demand slope cases of γ = 100 GW/(£/MWh).

Appendix 2. Cost slope parameters

Tables 3, 4, 5 and 6 have the same cost slopes. Because the Earn Out case (Table 6) shifts the cost curve of Eastern up by £6/MWh, the cost slope remains the same. Table A1 gives the cost slopes for these cases.

Table A1. Cost Slope Parameters for Tables 3-6

Cost slope (£/MWh/GW)	3 Zero	5 Zero	3 Positive	5 Positive
Nuclear	0.000951	0.000951	0.000951	0.000951
Interconnector	0.003125	0.003125	0.003125	0.003125
IPP	0.001907	0.001440	0.000817	0.000617
Eastern	0.004478	0.004478	0.002687	0.002687
AES	NA	0.007692	NA	0.004615
National Power	0.001852	0.002439	0.001358	0.001789
Power Gen	0.001948	0.002632	0.001429	0.001930
Edison Mission Energy	NA	0.007692	NA	0.004615

The cost slope parameters for Tables 7,8,9 are given in Table A2. The values from Table 7 come directly directly from G96 (p. 210). Table 8 uses the method recommended in G96. Finally, Table 9 uses different capacities from 3 Zero cases in Tables 3, 4 and 6. This results in different cost slope parameters.

Table A2. Cost slope parameters for Tables 7-9

	Table 7	Table 8	Table 9 Duopoly	Table 9 Divest
IPPs	NA	NA	0.002800	0.002800
National Power	0.001667	0.002000	0.001304	0.001579
Power Gen	0.002500	0.002857	0.001592	0.001781
4 GW firm		0.007500		
2 GW firm		0.015000		
Eastern				0.004478