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***Abstract*—Conjectured supply function (CSF) models of competition among power generators on a linearized DC network are presented. As a detailed survey of the power market modeling literature shows, CSF models differ from previous approaches in that they represent each GenCo's conjectures regarding how rival firms will adjust sales in response to price changes. The CSF approach is a more realistic and flexible framework for modeling imperfect competition than other models for three reasons. First, the models include as a special case the Cournot conjecture that rivals will not change production if prices change; thus, the CSF framework is more general. Second, Cournot models cannot be used when price elasticity of demand is zero, but the proposed models can. Third, unlike supply function equilibrium models, CSF equilibria can be calculated for large transmission networks. Existence and uniqueness properties for prices and profits are reported. An application shows how transmission limits and strategic interactions affect equilibrium prices under forced divestment of generation.**

***Index Terms*—Electricity competition, Electricity generation, Market models, Strategic pricing, Complementarity, Supply function models, England, United Kingdom.**

I. Introduction

THE ability to unilaterally manipulate prices—market power—is a growing concern in restructured power markets. Empirical evidence is mounting that generators have been able to raise prices well above competitive levels [7,41]. Because transmission limits can be an important source of this market power [57], many models of strategic interaction on networks have been developed (reviewed here and in [32,59]). These models can address a wide range of questions concerning industry structure and market design. For instance, models have been used to discover unanticipated ways in which market power might be exercised on networks [*e.g.*, 5,14,46], to identify locations that are particular vulnerable to market manipulation, to assess the price effects of relieving transmission constraints, and to evaluate proposed mergers.

We present a model for simulating the exercise of market power on linearized DC networks based on a flexible representation of interactions of competing generating firms. We term this representation the "conjectured supply function" (CSF) approach. A CSF represents the beliefs of a GenCo concerning how total supply from rival firms will react to price. The model can be viewed as a generalization of the Cournot models of [32,60] in that each generating company is allowed to conjecture that rival firms will adjust their supplies in response to price changes [25]—unlike the widely used Cournot approach which assumes no such adjustment. It can also be viewed as an approximation of a supply function equilibrium model, in which a first order Taylor series represents the local response of other suppliers around the equilibrium point; however, unlike SFE models, the assumed and actual responses may differ. By parametrically changing the assumed supply response, different degrees of competitive intensity can be modeled, ranging from pure (Bertrand) competition (infinitely large positive response by rivals to price increases), to oligopolistic Cournot competition (no response), and even collusion (which can be simulated by a negative quantity response to price). Positively sloped CSFs represent a degree of intensity between the Cournot and Bertrand cases. An idea similar to CSFs has been used in auction theory, in which a parameter is introduced to represent a bidder's expectations concerning how its choice of strategy will affect future bids by competing bidders [48].

The paper starts by reviewing the literature on oligopolistic price equilibrium models on power networks, showing the relationship of the CSF model to other approaches. We then present bilateral and POOLCO formulations of CSF models and summarize the properties of the equilibrium prices and profits they yield. An application is made to the England-Wales system, illustrating the advantages of the CSF approach relative to Cournot models.

II. Equilibrium Model Formulations

This section provides a review of alternative approaches to modeling GenCo interactions in oligopolistic power markets. We include overviews of: equilibrium modeling approaches; representations of GenCo strategic interactions and their application; and complementarity models using DC networks.

A. Use of Equilibrium Models for Power Markets

Most models of generator competition are based upon a general approach of defining a market equilibrium as a set of prices, producer input and output decisions, transmission flows, and consumption that simultaneously satisfy each market participant's first order conditions for maximization of their net benefits (Kuhn-Karesh-Tucker/KKT conditions) while clearing the market (supply = demand). The complete set of KKT and market clearing conditions defines a *mixed complementarity problem* (MCP) [*e.g.*, 1,12,32,51,52], which

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can also be phrased as a system of variational inequalities [18,60]. The general form of a MCP problem is as follows: find vectors x, y that satisfy the conditions $0 \leq x \perp f(x,y) \leq 0$ (read as "$x \geq 0$, $f(x,y) \leq 0$, $x^T f(x,y) = 0$") and $g(x,y) = 0$. There should be exactly as many conditions as variables.

If a market solution exists that satisfies the optimality conditions for each market player along with the market clearing conditions, it will have the property that no participant will want to alter their decision unilaterally (as in a Nash equilibrium). Although it is well recognized that no modeling approach can precisely predict prices in oligopolistic markets, there appears to be agreement that equilibrium models are indispensable for gaining insights on modes of behavior and relative differences in efficiency, price levels, and other outcomes of alternative market designs [59].

Note that the use of KKT conditions to define market equilibria means that we are assuming that each player's optimization problem is convex. This is assumption is incorrect for many power operations and planning problems. For instance, unit commitment or power plant construction involves 0-1 binary decisions [*e.g.*, 35]. As another example, nonconvex feasible regions can also occur if a generator's decision model explicitly represents how an ISO determines prices under a locational marginal pricing scheme [8,14,34,67]. In general, when nonconvexities occur, KKT conditions defining optimal solutions do not exist, and neither will market equilibria. Nonetheless, we will assume that the operations problems we simulate can be approximated as being convex, which gives us the ability to analyze large systems.

The direct solution of market equilibrium conditions by complementarity methods has important computational advantages. Large complementarity problems can be solved using GAMS-PATH [26], as well as many contemporary algorithms based on advanced nonsmooth Newton methods [23]. These algorithms permit application of strategic market models to large systems with thousands of power plants and hundreds or even thousands of transmission flowgates.

Many studies have used equilibrium models to address market power in electricity markets, with some considering competition in both energy and transmission services. The "DC" load flow approximation [58] is widely applied in such models not only because of its linearity, but also because numerical tests have found that DC congestion costs are good approximations if thermal constraints are the main concern [39]. We classify models of power markets by the clearing mechanism (centralized/POOLCO or decentralized/bilateral) and the nature of the interaction among rival generators.

Regarding market clearing mechanisms, most studies have implicitly or explicitly assumed a POOLCO-type centralized bidding process supervised by an ISO [*e.g.*, 14,46]. This process results in a set of publicly disclosed market clearing prices. There have also been studies that model bilateral trading [36] and the market power that large power traders might exercise [61]. It has been shown, however, that *if* there is perfect competition among traders so that they arbitrage away any non-cost based price differences between different locations, *then* POOLCO and bilateral trading systems yield the same prices under either perfect competition [9] or Cournot competition [44]. Thus, one would expect that mixed POOLCO-bilateral systems (*e.g.*, PJM) would also result in the same equilibria.

The other classification—the type of interaction assumed among rival generators and other players—has a crucial impact on model results. Power producers can be intensely competitive or they may collude. Seemingly arcane distinctions in assumptions concerning player interactions can result in large changes in economic equilibria and policy implications. For example, there is much debate [40,47,62] regarding the proper way to measure and analyze competition in networks and how strategic behavior by producers will manifest itself. What conclusions result depend heavily on the assumptions made. Thus, there are advantages to frameworks that can accommodate varying degrees of competitiveness.

B. Types of Strategic Interaction in Equilibrium Models

We next define several types of strategic interaction, most of them being familiar concepts from game theory and industrial organization [24,55,65]. They differ in how each generating firm f anticipates that rivals will react to its decisions concerning either prices p or quantities q. The CSF approach is designed to represent the full range of these behaviors.

The definitions below refer to competition among suppliers, so q is referred to as "sales" or "output." For the moment, we disregard the fact that demand is temporally and spatially distributed over a network. In addition, these definitions omit the effect of financial contracts upon marginal revenues [29]. Finally, these definitions assume that all players get the market clearing price; "pay your bid" (first price) auctions operate differently. Strategic models for the latter type of auctions can base revenue on the player's bid [10,53].

Types of strategic interactions that have been or could be included in power market models include:

- Pure Competition (No Market Power)/Bertrand: Just q_f in firm f 's revenue pq_f is a decision variable; the firm naively takes p as fixed. So in f's KKTs for profit maximization, marginal revenue MR (= $\partial(pq_f)/\partial q_f$) = p.
- Generalized Bertrand Strategy ("Game in Prices"): Here, $pq_f = p_f q_f(p_f, p_{-f}^*)$, where p_f is f 's decision variable, p_{-f} is the vector of prices offered by other firms, and q_f is a function of all prices. The asterisk on p_{-f} * indicates that f acts as if its rivals' prices won't change in reaction to changes in f 's prices. For a homogeneous good, f can sell as much q_f as it wants to (up to the market demand) if $p_f \leq$ lowest delivered price among rival producers; otherwise, $q_f = 0$. But for heterogeneous goods (such as "green" and "non-green" power), there may be nonzero cross price elasticities, and $q_f(p_f, p_{-f})$ takes on other forms.
- Cournot Strategy ("Game in Quantities"): Revenue $pq_f = p(q)q_f = p(q_f + q_{-f}^*)q_f$, where $p(q)$ is the inverse market demand function and q_{-f} is the quantity supplied by firms other than f. The asterisk means that f acts as if it believes that q_{-f} is fixed. Thus, f's first order conditions will have the following marginal revenue term:
$$MR = \partial(pq_f)/\partial q_f = p + (\partial p/\partial q)(1 + \partial q_{-f}^*/\partial q_f)q_f$$
$$= p + (\partial p/\partial q)(1 + 0)q_f = p + (\partial p/\partial q)q_f$$
- Collusion: If f colludes with another supplier, then they might maximize their joint profit. This makes the cooperative game theory assumption of "transferable utility";

i.e., side payments without transaction costs are possible. Other assumptions yield other collusive models.

- Stackelberg: Stackelberg models define a "leader" whose decisions correctly take into account the reactions of "followers", who do not recognize how their reactions affect the leader's decisions. *E.g.*, let firm f be the leader, and suppliers other than f be followers whose supply response to p is correctly anticipated to be $q_{-f}^{True}(p)$. Then f's revenue can be expressed as $p[q_f + q_{-f}^{True}(p)]q_f$. Stackelberg games in which f is a leader and its followers are instead customers for its output or suppliers of its inputs have other formulations. Often, $q_{-f}^{True}(p)$ is nonsmooth because it results from solving equilibrium conditions.
- General Conjectural Variations: $pq_f = p[q_f + q_{-f}(q_f)]q_f$; output $q_{-f}(q_f)$ from firms other than f is assumed to be a function of q_f. The marginal revenue term for f becomes:

$$MR = \partial(pq_f)/\partial q_f$$
$$= p + (\partial p/\partial q)(1 + \partial q_{-f}/\partial q_f)q_f = p + (\partial p/\partial q)(1+\theta)\, q_f$$

  where θ is the constant "conjectural variation" (CV). If $\theta = 0$, the Cournot game results. Meanwhile, $\theta = -1$ yields the pure competition game, while $\theta = +N$ can represent collusive behavior (quantity matching) when there are N+1 identical producers. If θ equals the actual "local" response of rivals, then this is a "consistent conjectures" model [11]; however, a theoretical criticism has been that unless peculiar informational assumptions are made, the Cournot CV is the only one that can be consistent [16]. The CV approach has been criticized in the industrial economics literature not only because it is a static model that is often used in an *ad hoc* way to analyze games that are actually dynamic (repeated), but also because of theoretical difficulties involved in empirical estimation of θ when marginal cost data is absent. However, recent theoretical work [13] shows that some CV models are the reduced form of equilibrium strategies in games involving repeated play—such as daily power auctions. Further, if credible cost data can be obtained (which is easier in power generation than in other industries), then θ can be estimated [69].
- Conjectured Supply Function (CSF): In this case, output by rivals is anticipated (perhaps incorrectly) to respond to *price* according to function $q_{-f}(p)$; as a result, $pq_f = p[q_f + q_{-f}(p)]q_f$. (In contrast, CV models posit a response to *quantity*.) This can be viewed as generalizing Stackelberg models in that the conjectured response may not equal the true response $q_{-f}^{True}(p)$. The CSF model also superficially resembles the SFE method, described next. The CSF approach has not previously been used in market power simulations; however, it has several advantages that make it worth considering. One is that $q_{-f}(p)$ might be modeled as a smooth function, simplifying calculation of equilibria. We discuss other advantages below. A drawback of CSF models is that they suffer the same theoretical limitations as the CV model.
- Supply Function Equilibria (SFE) [43]: In this game, the decision variables for each firm f are the parameters φ_f of its bid function $q_f(p|\varphi_f)$. This function describes how much q_f that f says it is willing to supply at a given price p. A market clearing mechanism (*e.g.*, the late California PX) then determines p, and sets $q_f = q_f(p|\varphi_f)$. As a result, the revenue term in f's profit function is $p[q_f(p|\varphi_f) + \Sigma_{g \neq f}\, q_g(p|\varphi_g^*)]\, q_f(p|\varphi_f)$. The asterisk in φ_g^* indicates that f treats bid functions from other firms as if they are fixed. SFE models were originally developed to address situations in which supplier response to random or varying demand conditions is considered.

We now define the equilibrium of a game involving the above strategies. Some of the games are Nash games [24,65]:

Let $X_f \in \underline{\mathbf{X}}_f$ be strategies under control of firm f; $\underline{\mathbf{X}}_f$ the space of feasible strategies for f; $X_{-f} = \{X_g, \forall g \neq f\}$; and $\Pi_f(X_f, X_{-f})$ the payoff to f given the decisions of all firms. Then $\{X_f^*, \forall f\}$ is a Nash Equilibrium in X if:

$$\Pi_f(X_f^*, X_{-f}^*) \geq \Pi_f(X_f, X_{-f}^*) \quad \forall X_f \in \underline{\mathbf{X}}_f, \forall f$$

For Cournot games, $X_f = q_f$; for generalized Bertrand games, $X_f = p_f$, and for supply function equilibria, $X_f = \varphi_f$. Important questions include whether equilibria exist in pure strategies and are unique, and how they can be calculated.

In contrast to Nash games, what we call a "generalized equilibrium" occurs if either: (a) f's feasible strategies depend on actions of other firms (*i.e.*, $\underline{\mathbf{X}}_f = \underline{\mathbf{X}}_f(X_{-f})$, called a "generalized Nash Equilibrium" in [62]), and/or (b) f anticipates that rival reactions will depend in a predictable way upon X_f (*i.e.*, $X_{-f} = X_{-f}(X_f)$). The CSF game is of type (b) [48], as are games involving Stackelberg players and conjectural variations.

$\{X_f^*, \forall f\}$ is a Generalized Equilibrium in X if:

$$\Pi_f(X_f^*, X_{-f}(X_f^*)) \geq \Pi_f(X_f, X_{-f}(X_f)) \quad \forall X_f \in \underline{\mathbf{X}}_f(X_{-f}), \forall f,$$
$$\text{and } X_{-f}^* = X_{-f}(X_f^*), \forall f$$

C. Applications of Alternative Interactions to Power Markets

Most of the above types of games have found application to power markets. Collusion has been modeled, for example, as a cooperative Nash bargaining game [3] and as cooperative limit-pricing, in which existing firms collude to prevent new firms from entering [36]. Such limit-pricing is credited with keeping a lid on prices in the UK [69]. Meanwhile, Stackelberg models have represented interactions between large power producers ("leaders") and one or more "followers" (smaller generators and/or the ISO) [*e.g.*, 33,34,50,67].

At the other extreme, the most intense competition results from Bertrand games [36,37,70], in which each firm chooses a single price for each generator or each area served, and believes that other firms will not change their prices in response. If there are no capacity limits and transmission costs, price then falls to marginal cost—the competitive result. But where there are such constraints or costs, the generalized Bertrand model results, and prices can rise above marginal cost and even fluctuate without end [8,37]. In the latter case, the equilibrium is a mixed strategy (probabilistic) one.

A less intense form of competition is Cournot competition, where firms instead choose quantity to generate or to sell as if rivals will not alter their quantities. Its simplicity and, in many cases, ease of computation have made the Cournot conjecture a popular game concept in power market models [*e.g.*, 2,6,14,17,51,52,68,71]. Another argument in its favor is that markets involving long-term commitments to capacity may show Cournot-type behavior in the long run, even if the firms compete on price in the short run [70]. Variants on the

Cournot theme include assumptions that each rival plant will hold its output fixed, that power sold by rivals to each area in a region is fixed, and that power flows induced by rivals are fixed. For example, Oren [46] shows that under Cournot assumptions about power flows, generators who recognize transmission limits can chose outputs to prevent congestion so as to avoid paying congestion charges. Stoft [63] instead models a market in which rival sales to each area are assumed fixed [see also 56]. He shows that markets with apparently competitive HHIs can yield prices well above competitive levels. Because Cournot models assume that rivals do not respond to price changes, the results are exquisitely sensitive to the elasticity and form of the market demand curve. As demand elasticities in power markets are now low (in part because of residual regulation and the lack of real-time pricing), Cournot prices tend to be very high and uncertain.

However, it has been argued that the Cournot and Bertrand assumptions may be inappropriate for POOLCO-type auctions, in which every firm bids a supply function for each generator or for their entire output. In this case, the decision variable is the bid function's parameters φ. Therefore, SFE has been chosen as the basis of many power market models [5,20,22,27,30,34,53,54,66,67,70]. The resulting equilibria generally represent an intermediate level of competition, lying between the Bertrand and Cournot results. But sometimes equilibria are not unique, and a large range of outcomes is possible; in general, the Cournot equilibrium will be their upper bound [4,30,43,64]. A drawback of SFE models is that equilibria are difficult to calculate; indeed, none may exist [5]. Thus, most SFE studies have been designed for very simple systems (*e.g.*, 1 to 4 nodes). Alternatively, when larger networks are considered, the model searches over only a handful of strategies to find the optimal strategy for each of two firms [22], or bids are restricted to a linear function with either fixed slope or intercept [34,67]. A fundamental problem is that the optimization problem faced by each firm is nonconvex, and can possess multiple local optima.

To our knowledge, there are no published power market models based on the general conjectural variations or CSFs. The major reasons appear to be the conceptual simplicity of Cournot models and the perceived appropriateness of SFE models for POOLCO markets. However, the two latter models also have serious limitations that make it worthwhile to consider alternative approaches. First, as indicated earlier, Cournot models do not give meaningful equilibria when price elasticities are low or zero—as they often are for short-run power demands, ancillary services, and short-run supplies of transmission capacity. It is not reasonable, for example, to assume that a supplier will be able push prices arbitrarily high without any response whatsoever from rival suppliers. (Conjectural variation models share this problem with Cournot models when price elasticities are very low or zero; unless $\theta \le -1$, equilibrium prices will be very high or infinite.) Another criticism of Cournot models is that they usually predict that mergers will be unprofitable for the merged firms [21].

In contrast to Cournot models, CSF models give modelers the flexibility to consider more realistic supply responses. Unfortunately, any particular supply response assumption will be somewhat arbitrary (although empirical estimation from prices, outputs, and marginal costs is possible [25]). It might therefore be argued that one may as well use Cournot models with artificially high (and also arbitrary) elasticities to simulate more intensive competition. But that approach distorts demand by decreasing it when prices are high, when in actuality demand would not change; economic and environmental market outcomes are therefore also misrepresented by the Cournot model. CSF models, in contrast, do not distort consumption in this manner. We suggest that the rival supply response implicitly assumed by each f be treated as a parameter that can be varied to explore how market power might be manifested and distort outcomes. At a minimum, it is worthwhile to simulate a range of response assumptions (including Cournot) to check whether alternative rival responses might qualitatively alter the conclusions (in the manner of [48]).

SFE models have different limitations than Cournot models. Equilibria for SFE models have proven difficult to calculate for large systems with transmission networks and significant number of generators with limited capacity. The reasons, which were referred to above, are that the generating firm's optimization problem on a network is inherently nonconvex (and hence a challenge to solve) and, further, equilibria may not exist. Unless strong restrictions are placed on the form of the bid functions (such as linear with only the slope or intercept being a variable), modelers have been forced to make unrealistic assumptions such as all firms having identical marginal cost functions. Therefore, although the asserted realism of the SFE conjecture makes it attractive for markets without significant transmission constraints, it is not a practical modeling method if realistic details on demand, generation, and transmission characteristics are desired. In contrast, complementarity models based on the Cournot conjecture have been solved for very large systems, and those models can be modified to represent strategic interactions based on conjectural variations or CSFs.

D. Complementarity Models on DC Networks

Solutions to many of the equilibrium models mentioned above are obtained either by exhaustive enumeration of combinations of strategies ("payoff matrices") which are then examined for Nash equilibria [*e.g.*, 17], or by closed-form solution of simple equilibrium models [*e.g.*, 8]. Neither approach can be used for large-scale models with many players and transmission limits or other constraints. Numerical methods are necessary. Numerical solution of equilibrium conditions stated as a MCP is the basis of several power market models [*e.g.*, 1,12,51,52], including the CSF models of this paper.

Previously, [32,60] have presented complementarity-based models of markets for energy and transmission services in which: (a) generators behave strategically in the energy market (Cournot); (b) transmission capacity is rationed competitively (*a la* Hogan [38]/Schweppe [58] or Chao-Peck [15]); (c) power flows over a linearized DC network; and (d) no arbitragers exist to erase non-cost-based differences in prices at different locations. These models (like [68]) can also include the possibility of generation capacity expansion. Existence and uniqueness of market equilibria can be proven [18,44,60]; these results are made possible by the assumption that generators are price takers with respect to transmission prices. (If generators instead recognize that their decisions are constrained by transmission limits, pure strategy equilibria will, in general, not exist [*e.g.*, 8]. This assumption means that the market for transmission is incomplete [18].)

Other models consider arbitragers/marketers. In [61], generators are competitive, but a small set of Cournot arbitragers wield oligopsonist market power when buying from generators along with oligopolistic market power when selling to power consumers. Versions of this model with thousands of variables have been solved for large systems in the EU. In contrast, the arbitraged bilateral model in [32] represents Cournot generators, with the assumption that low barriers to entry for arbitragers imply that they behave competitively. A large scale version of the latter model has been solved for the Eastern Interconnection [31], considering 2728 plants, 829 producers, and 814 transmission flowgates. Metzler *et al.* [44] prove that this model is equivalent to Cournot competition in a POOLCO system.

The CSF models of this paper can be viewed as generalizations of the bilateral (with and without arbitrage) and POOLCO models in [32,44] to the case of non-Cournot strategic interactions. These models are introduced next.

III. Conjectured Supply Function Models

The CSF model for a bilateral market has four components:

A. Sets of KKT conditions for profit-maximizing GenCos, one set for each GenCo, based on the CSF assumption. These generation firms directly contract to sell power to consumers or load-serving entities; buyers of power are price-takers and are modeled as demand curves.
B. A set of KKT conditions representing a transmission services provider (ISO) who maximizes the value of transmission services provided (*i.e.*, a Schweppe-Hogan allocator of transmission capacity) or, equivalently, an efficient Chao-Peck market for flowgates.
C. A set of KKT conditions for arbitragers who maximize profit from buying power at one location and selling it others. This set of conditions can be omitted if a bilateral model without arbitrage is to be simulated. A without-arbitrage model can yield price differences between nodes that deviate from the cost of moving power between those nodes.
D. Market clearing conditions that ensure that: the amount of transmission services demanded by generators and arbitragers equals that provided by the ISO; the amounts of power that each generating firm anticipates will be sold by other GenCos and arbitragers equal the amounts they actually sell; and the prices anticipated by different participants are consistent.

Each component is summarized in turn below. Then in Section III.E, we present a POOLCO version of the CSF model.

A. Generating Firm Model

A generating firm f participating in a bilateral market is modeled as having two basic decision variables: its generation x_{fih} [MW] from generators h at network nodes i, and its sales s_{fi} [MW] to consumers, load-serving entities, or arbitragers at node i. In addition, the price p_{fi} [\$/MWh] it anticipates at each node i is a variable. (Note that p_{fi} has subscript f. However, in equilibrium, the p_{fi} for all f must be equal at each i.)

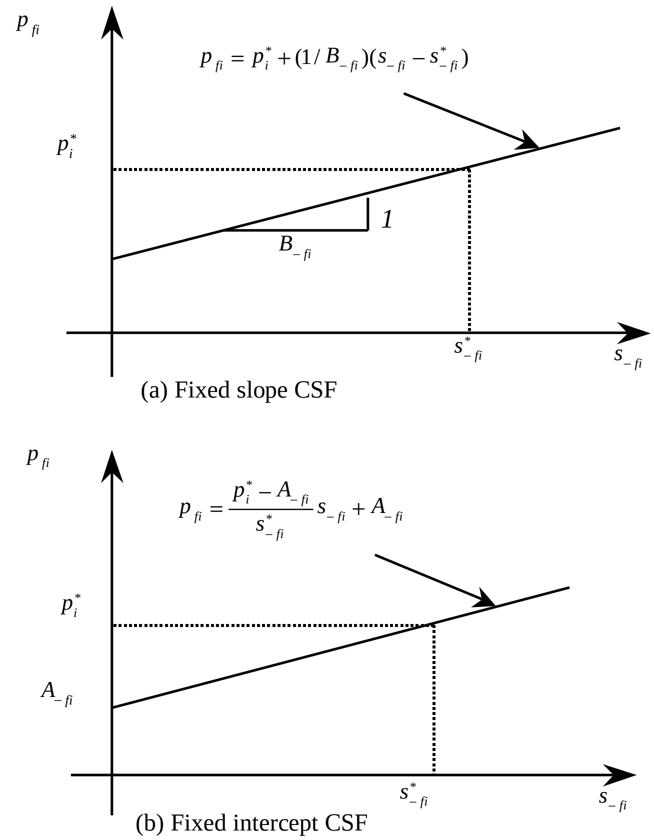

Fig. 1. Alternate Forms for Conjectured Supply Functions

The anticipated sales s_{-fi} [MW] by its rival firms ($s_{-fi} = \Sigma_{g \neq f}\, s_{gi}$) is also a variable in the CSF GenCo model, which distinguishes it from the Cournot model. Thus, total sales to consumers at i equal $q_i = s_{fi} + s_{-fi} + a_i$, where a_i [MW] is the net amount of power sold by arbitragers at i. Total sales are related to price through a demand function $q_i(p)$. In our application, demand is assumed to be affine: $q_i(p_{fi}) = Q_{io} - (Q_{io}/P_{io})p_{fi}$, with Q_{io} and P_{io} being the MW quantity and \$/MWh price intercepts, respectively. In most power market applications, this demand function will be quite inelastic.

Finally, a crucial relationship in the CSF model is the conjectured (rival) supply function itself, $s_{-fi}(p_{fi})$. It represents how f anticipates that total sales by rivals to i will depend on price. If $s_{-fi}(p_{fi}) = s_{-fi}^*$, a constant from the point of view of f, then the CSF model reduces to a Cournot model [32]. More generally, we assume that $s_{-fi}(p_{fi})$ is affine. Given these variables and relationships, firm f's problem is:

$$\begin{array}{llll}
\text{MAX } \Pi_f = \Sigma_i\, (p_{fi} - w_i^*)s_{fi} - \Sigma_{i,h}\, (C_{fih} - w_i^*)g_{fih} & & & \\
\text{subject to: } \underline{\text{CSFs:}} & s_{-fi} = s_{-fi}(p_{if}) & & \forall i \\
\underline{\text{Demand functions:}} & s_{fi} + s_{-fi} + a_i^* = q_i(p_{fi}) & & \forall i \\
\underline{\text{Generation limits:}} & g_{fih} \le G_{fih} & (\mu_{fih}) & \forall i,h \\
\underline{\text{Energy balance:}} & \Sigma_i\, s_{fi} = \Sigma_{i,h}\, g_{fih} & (\theta_f) & \\
 & \forall s_{fi},\, g_{fih} \ge 0 & &
\end{array}$$

Coefficient C_{fih} [\$/MWh] is the marginal cost of generator ih owned by f, while G_{fih} [MW] is the upper bound for generation from that unit. w_i^* [\$/MWh] is the price of transmission services from the assumed network hub to node i. The asterisk (*) on this and other variables indicates that although this quantity is a variable from the market model's point of view, it is viewed as fixed (exogenous) by firm f. Meanwhile, μ_{fih} [\$/MWh] is the dual multiplier for the generator capacity constraint, while θ_f [\$/MWh] is the dual for the energy balance, interpretable as f's marginal cost at the hub of the linearized DC network. We omit the dual variables for the demand functions and CSFs because the reduced model we actually solve (see below) eliminates those equations.

Two versions of the affine CSFs are considered here, resulting in two distinct models. The first assumes that the *slope* of $s_{-fi}(p_i)$ is constant (Fig. 1a):

$$s_{-fi}(p_{fi}) = s_{-fi}^* + B_{-fi}\,(p_{fi} - p_i^*)$$

where B_{-fi} is the assumed rate of change in rival supply per unit price, and (s_{-fi}^*, p_i^*) are a supply-price pair that the function passes through, and which GenCo f views as fixed. But from the point of view of the market, (s_{-fi}^*, p_i^*) are actually variables that in equilibrium equal the actual amounts supplied by other firms and price, respectively. This condition is imposed by the market clearing conditions of Section III.D.

In the second CSF version, each f instead assumes that the *intercept* of the function is constant (Fig. 1b):

$$s_{-fi}(p_{fi}) = (p_{fi} - A_{-fi})\, s_{-fi}^* / (\, p_i^* - A_{-fi})$$

where A_{-fi} is the assumed price intercept of the CSF for f at i.

In an equilibrium, then, price at each i will be p_i^*, and each GenCo anticipates that if price deviates from this level, then rival GenCos will change their supply from s_{-fi}^* according to the CSF assumed. Note that p_i^* and s_{-fi}^* are *not* assumptions, but are instead equilibrium values of variables.

Varying degrees of competitiveness in the market can be simulated by different values of A_{-fi} or B_{-fi}. For instance, high values of either parameter would imply more horizontal CSFs in Fig. 1 (as long as $A_{-fi} < p_i^*$); each firm f would then believe that rivals will be quick to jump in with more supply if f attempts to raise prices by restricting its output. This provides more incentive to cut prices, and the equilibrium will be closer to perfectly competitive levels than it would be otherwise. Indeed, $B_{-fi} = \infty$ yields Bertrand behavior and the competitive price p = marginal cost.

On the other hand, setting low values for the parameters yields less intensive competition; either $A_{-fi} = -\infty$ or $B_{-fi} = 0$ will result in vertical CSFs in Fig. 1, equivalent to the Cournot model. The CSF approach also gives the modeler flexibility to allow different firms to have different expectations. For example, some firms might compete intensely (which can be simulated by setting their A_{-fi} or B_{-fi} to relatively high levels), while other firms in the same market might be more inclined to attempt to manipulate prices (so their A_{-fi} and B_{-fi} might be set to relatively low levels). As suggested *supra*, these parameters can be estimated [25].

Once selected, the functions $s_{-fi}(p_{fi})$ and $q_i(p_{fi})$ can be used to eliminate variables p_{fi} and s_{-fi} from the model. Price can then be expressed as a function of the variables $\{s_{-fi}^*, p_i^*, s_{fi}, a_i^*\}$, along with parameters $\{Q_{io}, P_{io}\}$ and either A_{-fi} or B_{-fi}, depending on which CSF is used. The above GenCo profit maximization model for the bilateral market then reduces to:

$$\begin{aligned}
\text{MAX}\ & \Sigma_i\,[p_{fi}(s_{-fi}^*, p_i^*, s_{fi}, a_i^*) - w_i^*]s_{fi} - \Sigma_{i,h}\,(C_{fih} - w_i^*)g_{fih} \\
\text{s. t.:}\ & g_{fih} \le G_{fih} \qquad (\mu_{fih}) \quad \forall\ i,h \\
& \Sigma_i\, s_{fi} = \Sigma_{i,h}\, g_{fih} \qquad (\theta_f) \\
& \forall s_{fj},\, g_{fih} \ge 0
\end{aligned}$$

The KKT conditions of this model for the primal variables (s_{fi}, g_{fih}) and dual variables (μ_{fih}, θ_f) define a mixed complementarity problem that is either linear (if the fixed slope CSF is used) or nonlinear (in the case of a fixed intercept CSF). The nonlinearity in the latter case arises because the $p_{fi}(s_{-fi}^*, p_i^*, s_{fi}, a_i^*)s_{fi}$ term in the objective function is not the simple quadratic function obtained for the fixed slope case. Explicit expressions for $p_{fi}(s_{-fi}^*, p_i^*, s_{fi}, a_i)$ and the KKT conditions are in [49] (and are also available from the authors).

In real markets, there are many GenCos (*e.g.,* twenty three in the England-Wales case studied below), which yields a very large model because of the need to keep track of sales by each firm at each node. However, the model can be simplified by treating the smaller firms as price-takers rather than strategic firms. When there is arbitrage, a price-taking firm will anticipate that it can maximize its profit by selling the entire output of each of its generators at its bus at the prevailing price, which it takes as fixed; this is because any additional revenues it might anticipate from selling at a higher price elsewhere will, in equilibrium, be exactly offset by the transmission cost to that point. The price-taking and arbitrage assumptions allow the sales variables to be eliminated, resulting in the following model for price-taking firms:

$$\begin{aligned}
\text{MAX}\ & \Pi_f = \Sigma_{i,h}\,(p_i^* - C_{fih})g_{fih} \\
\text{s. t.:}\ & g_{fih} \le G_{fih} \qquad (\mu_{fih}) \quad \forall\ i,h \\
& \forall g_{fih} \ge 0
\end{aligned}$$

B. ISO Model

The derivation of this model is presented in [32]. It represents the efficient rationing of transmission capacity. (Other formulations of the transmission pricing problem are possible, such as zonal or uniform pricing [68].) There are two types of variables: y_i (the MW of transmission service provided from the hub to i) and λ_k (the dual variable upon the flow constraint for flowgate k). The model maximizes the value of services $\Sigma_i\, w_i^*\, y_i$ subject to a DC load flow, yielding KKTs:

$$\text{for } y_i,\ \forall i: \quad w_i^* - \Sigma_k\, PTDF_{ik}\, \lambda_k = 0 \qquad \text{(ISO1)}$$

$$\text{for } \lambda_k,\ \forall k: \quad 0 \le \lambda_k \perp (\Sigma_i\, PTDF_{ik}\, y_i - T_k) \le 0 \qquad \text{(ISO2)}$$

where $PTDF_{ik}$ is the power transmission distribution factor [MW/MW] for flowgate k resulting from an injection at the hub and withdrawal at node i, and T_k is the MW flowgate limit. The duals can be viewed as Chao-Peck flowgate prices, and the w_i^* as the difference between spot prices at the hub and i under locational marginal pricing [18].

C. Arbitrager Model

In equilibrium, arbitrage will eliminate any price differences between nodes that are not based on cost, implying that [32]:

$$p_{hub}^* + w_i^* = p_i^* \qquad \forall i \ne hub \qquad \text{(A1)}$$

D. Market Clearing Conditions

Market clearing conditions ensure that supplies of transmission services equal demand, and that prices and rival supplies anticipated by each f equal the actual equilibrium amounts:

$$y_i = \Sigma_f\, s_{fi} + a_i^* - \Sigma_{f,h}\, g_{fih} \qquad \forall i \qquad \text{(MC1)}$$

$$p_{fi}(s_{-fi}^*, p_i^*, s_{fi}, a_i^*) = p_i^* \qquad \forall i,f \qquad \text{(MC2)}$$

$$s_{-fi}^* = \Sigma_{m \ne f}\, s_{mi} \qquad \forall i,f \qquad \text{(MC3)}$$

Gathering together the KKT conditions for each generator f, along with conditions (ISO1,2), (A1), and (MC1-3), defines a mixed complementarity problem. The problem can be simplified by using several of the equality conditions to eliminate some variables (similar to [32]). The resulting reduced complementarity model can then be solved for the equilibrium solution, including prices (p_i*, w_i*) quantities (s_{fi} , s_{-fi}*, g_{fih} , a_i*, y_i), profits (Π_f), and dual variables (μ_{fih} ,θ_f , λ_k). The insights gained by comparing the values of these variables under alternative market designs, industrial structures (e.g., numbers of firms), and physical system designs (e.g., transmission capacity) can be useful for market designers, regulators, and market participants.

E. A POOLCO Market Model

A POOLCO market model is developed here analogous to the POOLCO Cournot model in [32]. Each generator sells its entire production at its node (so $s_{fi} = \Sigma_h g_{fih}, \forall i,f$) and each firm f anticipates how the ISO will alter accepted bids and power transfers so that locational marginal pricing (LMP) relationship is maintained (equivalent to equation (A1)). Thus, the CSF POOLCO producer model is at the same time simpler and more complex than the bilateral model presented above. It is simpler in that the s_{fi} variables can be eliminated, but it is more complex because the GenCo model now includes (A1) as a constraint along with arbitrage a_{fi} as an endogeous decision variable that adjusts so that (A1) is satisfied. The reduced form of that model is:

$$\text{MAX } \Pi_f = \Sigma_{i,h} [p_{fi}(s_{-fi}^*, p_i^*, \Sigma_n g_{fin}, a_{fi}) - C_{fih}] g_{fih}$$

s.t.: <u>LMP:</u> $p_{fi}(s_{-fi}^*, p_i^*, \Sigma_n g_{fin}, a_{fi})$

$$-p_{f,hub}(s_{-f,hub}^*, p_i^*, \Sigma_n g_{f,hub,n}, a_{f,hub}) - w_i^* = 0 \quad \forall i \neq hub$$

<u>Arbitrage balance:</u> $\Sigma_i a_{fi} = 0$

$$g_{fih} \le G_{fih} \qquad (\mu_{fih}) \qquad \forall\, i,h$$

$$\forall g_{fih} \ge 0$$

The POOLCO market clearing conditions also differ:

$$y_i = a_i \qquad \forall i \qquad \text{(MC1')}$$

$$p_{fi}(s_{-fi}^*, p_i^*, \Sigma_h g_{fih}, a_i) = p_i^* \qquad \forall i,f \qquad \text{(MC2')}$$

$$s_{-fi}^* = \Sigma_{m \neq f} \Sigma_h g_{fih} \qquad \forall i,f \qquad \text{(MC3)}$$

$$a_i = a_{fi} \qquad \forall i,f \qquad \text{(MC4)}$$

(MC1') results from the fact that "arbitrage" (actually, POOLCO) flows are the only ones in the system. (MC2') and (MC4) are necessary because prices and arbitrage are defined for each firm, but in equilibrium must be equal across firms.

The POOLCO market model is obtained by gathering the KKT conditions for the GenCo model, along with (MC1'-4) and the transmission conditions (ISO1,2). The arbitrage condition (A1) is automatically satisfied by definition of the producer models, and does not need to be included explicitly. The resulting mixed complementarity model can be simplified by using equality conditions to eliminate variables; for instance, the LMP equality can be used to eliminate the a_{fi}.

IV. CSF Model Properties

Here we examine three questions concerning theoretical properties of the solutions. The results are summarized below; proofs are available in [44,49]. As pointed out above, a key assumption underlying these results is that generators are price takers with respect to transmission.

The first question concerns the relationship of the solutions of the various models. As noted, the bilateral models with and without arbitrage in general yield different prices. So do the fixed intercept and slope CSFs (except in the extreme cases where the parameters A_{-fi} and B_{-fi} are chosen to represent either pure or Cournot competition).

However, it can be shown that the POOLCO model and bilateral model with arbitrage yield identical profits and total sales for each f and the same p_i* for the CSF (constant slope) model. This result was previously known for the cases of perfect competition [9] and the Cournot model [44], but now is partially generalized to the CSF case. Thus, in theory, the amount of GenCo market power in the POOLCO and bilateral models is the same, as long as sufficient arbitrage exists.

Pang *et al.* [49] show an additional equivalence result: that a CSF bilateral model with arbitrage can be calculated in either of two ways, either with the arbitrage condition (AC1) external to the producers' models, or with the arbitrage condition explicitly recognized by generators and incorporated in their constraint set (as in the POOLCO model). The solutions to the two models yield identical profits, total sales, and prices in the fixed slope models (which can be demonstrated using the same arguments as in [44]). Thus, the model that is easiest to solve can be used, which is the POOLCO model (as it has fewer variables). Finally, it has been shown [49] that any solution to the external arbitrage/fixed intercept model also solves the internal arbitrage/fixed intercept model.

The second question concerns whether solutions to the market equilibrium problem exist. For the fixed slope CSF model, solutions exist under very mild conditions; this can be proven in the same way that existence is proven for the Cournot model in [44] using results from linear complementarity theory. The third question, solution uniqueness, is addressed in the same way for the fixed slope CSF model. As in the case of its Cournot counterpart [44], linear complementarity theory can be used to show that profits, prices, and total firm sales are unique for that model.

Answering the second and third question for the intercept CSF model is more complicated because the market equilibrium conditions in that case define a *non*linear complementarity problem, for which fewer theoretical results exist. Yet it is possible to show that if the fixed intercepts A_{-fi} are below a computable bound, then a solution will exist for the fixed intercept model [49]. Furthermore, prices and each firm's total sales and profits will be unique.

V. Application to the England-Wales Market

As an illustration, we apply the CSF model to the England and Wales (E&W) system. Oligopoly models have been used previously to assess the impacts of market power, market structure, and divestments on E&W prices [*e.g.*, 10,19,27,30,42], but transmission constraints were not considered. Here, using the simplified model of the E&W transmission network developed by Green [28], we compare results from the fixed intercept and variable slope models with the Cournot model. At present, congestion management in the E&W market is not performed in the manner assumed by the models developed here. However, there has been an ongoing review of these arrangements by the UK Office of Gas and Electricity Market (OFGEM) [45]. The models developed here could be used to provide insight into possible outcomes of adopting the proposals that OFGEM had been considering.

A. No Transmission Constraints

Our first analysis disregards transmission constraints to show the general nature of the FCM solution. Fig. 2 shows our estimate of the E&W marginal cost curve in 2000 (the step function). This curve includes 53 power plants (including imports from Electricite' de France and Scottish power) owned by 23 different companies. For our investigation of the no transmission constraints models, we examine the possible effect of the 1996 and 1999 E&W generation divestments upon equilibrium prices using the fixed intercept FCM model. In each of those years, the Office of Electricity Regulation responded to concerns about the exercise of market power [e.g., 69] by requiring the two largest suppliers in the system (National Power and PowerGen) sell off portions of their generation assets.

A load of 52,000 MW is considered with zero price elasticity (the vertical dashed line). In that case, Fig. 2 shows that pure competition (P = marginal cost) gives a price of 15 £/MWh. However, because there are a few very large generation firms in E&W, prices that would be projected by most oligopoly models would be higher than that. As an extreme case, Cournot prices would be infinite because of the zero price elasticity; thus, that model is of dubious relevance.

On the other hand, the CSF approach gives equilibrium prices that are generally more consistent with those actually experienced (on the order of a few tens of percent above marginal cost [69]). In that model, we assume that the seven largest GenCos behave strategically (*i.e.*, their models include CSFs), while the others are price takers. We executed the fixed intercept model assuming that $A_{-fi} = 0$ for all strategic f for the model's single node $i = 1$. This level was chosen because an affine approximation to the actual marginal cost curve for the market would have an intercept of approximately that value. Thus, we are assuming that each firm acts as if it believes that its rivals will have supply curves with an elasticity of 1. The CSF model was solved three times, once for each ownership structure. The resulting equilibrium prices are 23 £/MWh for the pre-1995 market concentration, 19 £/MWh after the 1995 divestment, and 17 £/MWh after the further divestment in 1999. Thus, decreasing market concentration in that manner should be expected to significantly decrease the amount by which the companies would raise price above marginal cost. These results are summarized graphically in Fig. 2 by showing the total conjectured supply function resulting from each solution (derived from the calculated s_{-fi}*, p_i*); the effect of decreasing market concentration is to shift that curve downwards in a more competitive direction. Equilibrium prices are shown as the intercept of the CSFs with the vertical demand curve.

B. Transmission-Constrained Model

We now apply the fixed intercept and variable slope CSF models and the Cournot model using a 13 area approximation of the E&W transmission grid (Fig. 3) with the year 2000 asset ownership structure. Eight GenCos are assumed to act strategically and 14 are price takers. The strategic firms are mostly the GenCos that own multiple coal and gas fired generating plants, while independent power producers and nuclear plants are price takers. In total there we model 56 plants and 21 flowgates between the 13 nodes. Because of transmission constraints in the Midlands region, we anticipate significant price differences between northern (N1-N7) and southern nodes (N8-N13). For the fixed intercept model, we examine four intercepts: $A_{-fi} = 0$, $A_{-fi} = -10$, $A_{-fi} = -100$ and $A_{-fi} = -1000$ [£/MWh]. Four slopes are considered in the fixed slope model: $B_{-fi} = 1000$, $B_{-fi} = 100$, $B_{-fi} = 10$ and $B_{-fi} = 1$ [MW / (£/MWh)]. These values range from very competitive to uncompetitive conditions, respectively.

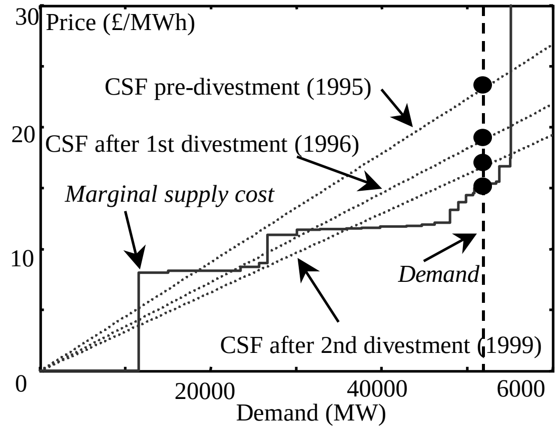

Fig. 2. E&W System Marginal Cost Curve and Conjectured Supply Functions (1995, 1996, 1999) for 52 GW Load (Dots are Calculated Equilibria)

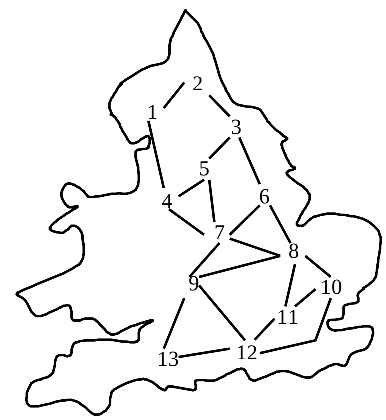

Fig. 3. E&W 13 Node System

Mean prices for the northern (N1-N7) and southern nodes (N8-N13) are calculated from the solutions of the Cournot, fixed intercept and fixed slope models. These results, as well as those for the perfectly competitive outcome, are shown in Table I for the three different price elasticities.

The influence of network constraints on power flowing from generating plants in the north of the country to load in the south is evident in these results (higher prices in the south than the north). Although such constraints would not occur for all load levels (they are more likely during high load network maintenance conditions), their influence on an efficient congestion-pricing regime can be pronounced, as seen here.

TABLE I.
COMPARISON OF COMPETITIVE, COURNOT, AND CSF PRICES FOR E&W UNDER ALTERNATIVE ELASTICITIES

Elasticity; Prices in North, South	Competitive Solution	Cournot Solution	Fixed Slope (B_{-fi} MW/($/MWh), Fig. 1a)				Fixed Intercept (A_{-fi} $/MWh, Fig. 1b)			
			1000	100	10	1	0	-10	-100	-1000
E = 0.1: P_N =	14.66	27.11	15.10	16.94	23.78	26.68	15.81	16.43	20.81	25.79
P_S =	16.78	30.04	17.39	19.88	26.70	29.61	18.88	19.42	24.15	28.72
E = 0.01: P_N =	14.66	42.02	15.11	17.17	30.96	40.17	15.83	16.45	23.69	36.89
P_S =	16.78	46.08	17.41	20.63	34.63	44.24	19.61	20.54	27.39	40.81
E = 0.01: P_N =	14.66	163.99	15.13	17.55	48.61	129.56	15.88	16.88	29.85	91.56
P_S =	16.78	172.56	17.42	21.43	55.76	136.86	19.84	21.09	33.68	98.85

The other notable feature of these results is the influence of price elasticity. For the low price elasticity scenario shown in Table I (a realistic scenario for markets for power), it can be seen that that the fixed slope and fixed intercept models (for a low B_{-fi} and a high A_{-fi} respectively) produce more reasonable price levels than does the Cournot model (this has also been observed by [25]). Thus, the combination in an oligopoly model of a transmission network and conjectured supply functions has the prospect of facilitating a fuller examination of relevant policy questions.

VI. CONCLUSION

The conjectured supply function approach to modeling oligopolistic competition on power networks is more flexible than the Cournot assumption. Meanwhile, the CSF model is computationally feasible for large systems, unlike supply function equilibrium models. A drawback of this flexibility is that there are more behavioral parameters in the model, and these parameters are not directly observable. Two approaches to dealing with this difficulty are to estimate the parameters empirically [25] (which can reveal who is competing more intensely within a market), or to vary them parametrically to assess how and where prices are affected as the degree of competitiveness in a market changes.

Future work should consider the effects of long-term contracts upon short-term operations [29]. The model should also be extended to multiple time periods [52]. This would allow modeling of, *e.g.*, energy storage, hydropower [12], and ramp rate limits. However, 0-1 unit commitment decisions cannot be modeled in complementarity problems.

The CSF approach can also be extended to other power-related markets for which the Cournot approach is inappropriate. An example is ancillary services markets, in which the demand for, *e.g.*, operating reserves or installed capacity is fixed and has essentially no price elasticity. A CSF approach can be used to explore how designs and interconnections of these markets affect the ability of generation firms to exercise market power in them. Another example is the effect of generation decisions on transmission prices. Present models make either of two extreme assumptions: either that generators act as if they cannot affect those prices (including this model and [32,60]), or that infinitesimal changes in demands for transmission services can result in large changes in transmission prices [46]. A CSF approach could represent a more realistic intermediate case between these extremes.

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