

Private Polling in Elections and Voter Welfare*

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Abstract

This paper develops a model of elections in which two candidates poll voters prior to taking policy positions, providing the candidates with private information about the voters' preferred policies. In contrast to electoral competition models with symmetric information, private information may be inconsistent with existence of pure strategy equilibrium. We characterize the (essentially) unique mixed strategy equilibrium. We show that candidates who receive moderate signals adopt more extreme platforms than their information suggests, while candidates who receive more extreme signals moderate their platforms relative to their pollsters' advice. Although candidates' platforms diverge in equilibrium, we offer plausible conditions under which they do not do so by as much as voters would like. As a result, the electorate always prefers less correlation in candidate signals, as correlation reduces the choices that candidates provide, and therefore voters prefer private to public polling. For similar reasons, some noise in the polling technology raises voters' welfare; this suggests a novel justification for spending caps in election campaigns.

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1 Introduction

Since the seminal papers of Hotelling (1929), Downs (1957), and Black (1958), spatial competition models have greatly advanced our understanding of elections and campaigning. The key prediction of the Downsian model is the median voter theorem, possibly the most famous result in political economy: Given voters with single-peaked preferences over a unidimensional policy space, the unique equilibrium in an election between two office-motivated candidates who are perfectly informed about the voters' preferences is for both candidates to locate at the median voter's preferred policy. The key normative insight of this setting is that such policy convergence cannot hurt the median voter, and nor does it lower the welfare of risk averse voters with other ideologies.¹

In reality, however, candidates often differentiate their platforms. A host of researchers, including Page (1978), Merrill and Grofman (1999), Poole and Rosenthal (1997) and Budge et al. (2001), document empirically that candidates' platforms diverge from the estimated median voter's preferred policy, and yet are not too extreme.² At the same time, platform convergence is not perceived favorably by the popular press, nor by many academic scholars. To wit it, when policy platforms converge, it is often argued that there is "not enough choice" between candidates, and that "they are all the same." Indeed, a manifesto calling for "responsible parties" presented in 1950 by the Committee on Political Parties of the American Political Science Association, which included the most influential political scientists of those days, was based on the premise that office-motivated candidates do not provide the electorate enough choice.³ Because policy convergence is inherent to the basic Downsian model, and because it is in fact socially optimal in that framework, the basic model appears insufficiently rich to address these issues.

Our analysis starts from the observation that, in practice, political candidates do not know

¹To elaborate, if voters have symmetric single-peaked preferences and are risk averse, they all prefer the known median policy to facing an election with two differentiated candidates who win with positive probability, because such an election requires that the candidates' platforms are located symmetrically about the median policy.

²See, for example, the National Election Survey data estimating presidential candidates' platforms from 1964 to 1972 (Page (1978), chapters 3 and 4) and for 1984 and 1988 races (Merrill and Grofman (1999) pages 55-56). Budge et al. (2001) compare estimates of the US and British median voters based on survey data (such as the NES and British Election Survey) with estimates of candidates' platforms derived from speech and writing context analyses. They find clear evidence of divergence from the median policy, and no evidence of extremization. Poole and Rosenthal (1997) obtain similar findings using roll call voting to estimate Congress-persons' platforms (pages 62-63).

³For example, the opening statement reads, "Popular government of a nation [...] requires political parties that provide the electorate with a proper range of choice between alternatives of action." (Committee on Political Parties (1950), page 15). Practical proposals are also presented on how differentiated party platforms should be formed and how parties should insure their implementation by elected candidates (pages 50-56). Page (1978), page 21, observes that "[Many] American political scientists, most notably Woodrow Wilson and E.E. Schattschneider, have called for parties *to provide the electorate with sufficient choice.*" Our italics.

voters' policy preferences with certainty when selecting platforms. Determining the median voter's location is a difficult task in large elections, especially in the context of a complex political debate. Accordingly, candidates devote substantial resources to gathering information about voters through private polling. Eisinger (2003) finds that, since the Roosevelt administration, private polls have been an integral part of the White House modus operandi. Medvic (2001) finds that 46 percent of all spending on U.S. Congressional campaigns in 1990 and 1992 was devoted to the hiring of political consultants, primarily political pollsters. In addition, the major parties provide polling services to their candidates. Of course, this private polling information is jealously guarded by candidates and parties. Indeed, Nixon had polls routinely conducted, but did not disclose results even to the Republican National Committee; and F.D. Roosevelt described private polling as his "secret weapon" (Eisinger (2003)).

We develop a model of elections in which candidates receive private polling information on the voter's preferences. To be able to use explicit functional forms, we assume the following structure of information: Candidates receive signals about a component, β , of the median voter's preferred policy, where β is a discrete random variable; in addition to this component, the median voter's location is determined by a uniformly distributed term α uncorrelated with the candidates' signals. The final location of the median voter is then $\alpha + \beta$. One interpretation of this formulation is that voters are unwilling or unable to provide pollsters accurate summaries about all of their views, as is suggested, for instance, by the empirical work of King and Gelman (1993). Another interpretation is that candidates learn about the position initially preferred by the median voter, β , after which electoral preferences may shift by α .⁴ Before selecting a platform, each candidate receives a signal drawn from an arbitrary finite set of possible signals. Each candidate updates about the actual median policy and the platform of the opponent, then chooses a platform, and the candidate whose platform is closest to the actual median policy wins. Our construction is general with respect to correlation in polling signals, thereby capturing both private and public polling.

In a companion paper, Bernhardt, Duggan, and Squintani (2005) (henceforth BDS), we provide a general analysis of the existence and continuity properties of mixed strategy equilibria in the private polling model. BDS (2005) show that when even a small amount of private information is present, the nature of equilibrium policies changes drastically: In any pure strategy equilibrium, after receiving a signal, a candidate locates at the median of the posterior distribution over the location of the median voter, where the posterior is *conditioned on both candidates receiving that*

⁴For example, after platforms have been selected, a weakening economy may change voters' views about increased fiscal spending; or terrorist attacks may alter voters' views about civil rights restrictions.

same signal. As a consequence, the candidates' locations are biased estimates of the true location of the median voter. And if signals are self-reinforcing, in the sense that an additional observation of any given positive signal shifts the conditional distribution of the median voter to the right, then an implication is that the candidates adopt policies that are more extreme in expectation, given their private information, than the median voter.

In this paper, we give necessary and sufficient conditions for this pure strategy equilibrium to exist. As the polling technology becomes finer, so that the number of possible signals increases, this condition becomes more difficult to fulfill. Thus, we find that the strategic incentives of privately informed candidates leads them to adopt location strategies that cannot be precisely predicted, i.e., the candidates must use mixed strategies. We prove that, in the absence of the pure strategy equilibrium, there is a unique mixed strategy equilibrium in which the locations of the candidates are ordered with respect to their signals. We give the closed form solution of this equilibrium and show that candidates with sufficiently moderate signals adopt their pure strategy equilibrium platforms, locating more extremely than their information suggests, while candidates who receive more and more extreme signals mix over policy positions, tempering their positions by more and more toward the unconditional median.

Our welfare analysis allows us to make sense of the wide-spread unfavorable view of platform convergence and of the claims that office-motivated candidates do not provide voters with enough choice. The logic underlying our analysis is that because the location of the median voter is unknown, candidates cannot perfectly target the median voter's preferred policy. The platform ultimately implemented is the one closest to the median voter's position, and assuming voters' preferences are correlated with the median voter's, then the benefit of dispersion in candidate platforms is that it gives voters greater choice. The cost is that, depending on a voter's location relative to the median voter, the outcome of the election will be worse with positive probability. Because candidates care only about winning, they do not internalize these externalities, and as a result, candidates may not collectively provide enough platform dispersion from the standpoint of the electorate.

To formalize this logic, we first prove that all voters in our model have the same ex ante preferences over candidate strategies. We then give conditions on signals under which it is socially optimal for candidates locate more extremely than their equilibrium positions following some signal realization. The key condition on the information structure is the following: Given a positive signal, for example, if a more moderate signal is realized, then the probability it is also positive must be

sufficiently high. The condition is automatically realized in the two- and three-signal models, and it always holds for the most moderate signals. Whether it holds for general signals depends on whether the conditional distributions of signals are sufficiently single-peaked. Thus, we provide theoretical underpinnings for informal complaints that candidates do not provide enough choice.

A similar logic implies that greater signal correlation reduces voter welfare: Correlation reduces both the degree by which candidates “extremize” their platforms given their signals, as well as the probability that candidates receive different signals and, hence, choose distinct platforms. We then show that the effect of signal precision on welfare is non-monotonic. Increased polling accuracy raises the probability that candidates correctly identify the median voter’s preferred policy, raising the welfare from any one candidate’s platform, but it also raises the probability that the candidates adopt close platforms, reducing the choice that candidates give to the voters. The net effect of these forces is undetermined: In some cases, greater precision will raise welfare, while in others it has the opposite effect.

These two final results allow us to derive the welfare properties of private and public polling in electoral competition. Because the voters’ welfare is not monotonic in the precision of (and hence the resources devoted to) private polling, spending caps in elections may raise voter welfare even when campaign advertising is truly informative and beneficial to the electorate. We also find that public polling is always more detrimental to the electorate than private polling. Hence, sharing results from private polls would harm the electorate. This is because sharing information raises the correlation between candidates’ information, adversely reducing platform diversity. This finding provides support for polling bans provisions adopted in countries such as Canada, Italy, France, Argentina (see Emery (1994)), a support that does not rest on claims that public polling may distort elections because of bandwagon effects or because of distortions in voters’ participation.

A small number of other papers have considered aspects of elections with privately-informed candidates. Ledyard (1989) is the first to raise the issue of privately-informed candidates and considered examples exploring the effects of the order of candidate position-taking, public polls, and repeated elections. In work independent of ours, Chan (2001) studies a three-signal model that differs structurally from ours in that a valence term, common to all voters but unobserved by the candidates, is attached to each candidate. He shows existence of a pure strategy equilibrium when signals are almost uninformative, and he does not consider the form of mixed strategy equilibria when pure strategy equilibria fail to exist. In line with our finding, he gives an example in which increased signal precision lowers the welfare of the median voter. Ottaviani and

Sørensen (2002) numerically characterize a model of financial analysts who receive private signals of a firm's earnings and simultaneously announce forecasts, with rewards depending on the accuracy of their predictions. The case of two analysts can be interpreted as a model of electoral competition with privately-informed candidates. Surprisingly, they show that greater competition increases the strategic bias in forecasts: As the number of forecasters increases, forecasts become more extreme.

2 The Electoral Framework

2.1 The Model

Two political candidates, A and B , simultaneously choose policy platforms, x and y , on the real line. There is a unique median voter, whose preferred policy position is given by μ . Candidate A wins the election if his platform is closer to the median voter's preferred position than candidate B 's, i.e., if $|x - \mu| < |y - \mu|$, and A loses if he locates further away. If $|x - \mu| = |y - \mu|$, then the election is decided by a fair coin toss, so that A wins with probability one half.

Candidates do not observe μ . However, candidates can privately poll voters about their preferred policies. We assume polling generates signals about the date 1 location of the median voter, given by β , and then candidates choose platforms. The election is at date 2. Between dates 1 and 2, the median voter's preferred platform may shift, so that the median voter's final preferred position is $\mu = \alpha + \beta$. For simplicity, we assume that β is a discrete random variable with support on $b_1 < b_2 < \dots < b_N$ and that α is independently and uniformly distributed on $[-a, a]$.⁵

Polling provides candidates private real-valued signals i and j about β , drawn from a finite set I . The pooled information of the candidates is represented by a signal pair (i, j) , where the first component refers to candidate A 's signal and the second refers to candidate B 's. Candidates have a common prior over the joint distribution of α , β , and the polling signals. We assume that candidates have access to equally-informative polling technologies, in a sense that we make precise below. However, we will impose no restrictions on the correlation between candidates' signals, allowing for conditionally-independent signals and perfectly-correlated signals as special cases.

Let $P(b_k)$ denote the prior probability of b_k . Let $P(i, j|b_k)$ denote the probability of signal pair

⁵An alternative interpretation is that α simply captures an additional source of error inherent in the polling process, e.g., α may represent a component of policy preferences about which voters are unwilling to divulge information.

(i, j) conditional on b_k , so that the marginal probability of (i, j) is:

$$P(i, j) = \sum_{k=1}^N P(i, j|b_k)P(b_k).$$

We assume that $P(i, i) > 0$ for all signals i . Let $F_{i,j}(\cdot)$ denote the distribution of $\mu = \alpha + \beta$ conditional on signal pair (i, j) , and let $f_{i,j}$ be the associated density. Note that $f_{i,j}$ is strictly positive on its convex support. By equally-informative polling technologies, we mean that $P(i, j) = P(j, i)$ and $F_{i,j} = F_{j,i}$ for all signals $i, j \in I$.

Let $P(i)$ represent the marginal probability of a generic signal i , which is positive for all i , so that conditional probabilities $P(\cdot|i)$ can be defined using Bayes rule. Given signals i and j , we let $m_{i,j}$ be the uniquely-defined median of $F_{i,j}$, and we let m_i be the uniquely defined median conditional on signal i . Finally, given a subset $K \subseteq I$ of signals, let $m_{i,K}$ be the uniquely-defined median conditional on one candidate receiving signal i and the other receiving a signal in the set K . We index signals naturally, so that greater signals imply higher values of the median conditional on like signals, i.e., $i < j$ implies $m_{i,i} < m_{j,j}$, and we let $i + 1$ denote the signal following i in this ordering (similarly for $i - 1$). Assume that distinct signals determine distinct conditional medians: For all signals i and j with $i \neq j$ and for every subset $K \subseteq I$ of signals, $m_{i,K} \neq m_{j,K}$.

To simplify the analysis, we assume that $a > b_N - b_1$, so that the conditional distribution of the median given signals i and j is linear over the sub-interval $[b_N - a, b_1 + a]$:

$$F_{i,j}(z) = \frac{a - m_{i,j} + z}{2a}, \text{ for all } z \in [b_N - a, b_1 + a].$$

This assumption captures the idea that, with positive probability, the unresolved uncertainty in the median voter's position at the time of polling is sufficient to swamp policy preferences elicited through polling.

If candidate A locates to the left of B , i.e., $x < y$, then candidate A wins when $\mu < (x + y)/2$. Conversely, if $x > y$, then A wins when $\mu > (x + y)/2$. The probability that B wins is, of course, just one minus the probability that A wins. Thus, the probability that candidate A wins when A adopts platform x and receives signal i , and B adopts platform y and receives signal j , is

$$\begin{aligned} \pi_A(x, y|i, j) &= F_{i,j}\left(\frac{x + y}{2}\right), & \text{if } x < y, \\ \pi_A(x, y|i, j) &= 1 - F_{i,j}\left(\frac{x + y}{2}\right), & \text{if } y < x, \\ \pi_A(x, y|i, j) &= 1/2, & \text{if } x = y. \end{aligned} \tag{1}$$

We define a Bayesian game between the candidates in which pure strategies for the candidates are vectors $X = (x_i)$ and $Y = (y_j)$, and the solution concept is Bayesian equilibrium. The equilibrium is *symmetric* if $X = Y$.

Given pure strategies X and Y , candidate A 's interim and ex-ante expected payoffs are:

$$\Pi_A(X, Y|i) = \sum_{j \in I} P(j|i) \pi_A(x_i, y_j|i, j) \quad \text{and} \quad \Pi_A(X, Y) = \sum_{i \in I} P(i) \Pi_A(X, Y|i).$$

The ex-ante game that candidates play is a two-player, constant-sum game: For all X and Y ,

$$\Pi_A(X, Y) + \Pi_B(X, Y) = 1.$$

A consequence of the fact that the game has a constant sum is that equilibria are interchangeable in the sense that if (X, Y) and (X', Y') are equilibria, then so are (X, Y') and (X', Y) .

3 Equilibrium Analysis

Pure strategy equilibria of our model have a simple form: *if* a pure strategy Bayesian equilibrium exists, then it is unique, and after receiving a signal, each candidate locates at the median of the distribution of μ conditional on *both* candidates receiving that signal. BDS (2004) prove the following result.

Theorem 1 *If (X, Y) is a pure strategy Bayesian equilibrium, then $x_i = y_i = m_{i,i}$ for all $i \in I$.*

The intuition behind the result is simple. First, suppose that candidates A and B use identical strategies, so that $X = Y$, and suppose that both candidates receive the signal i , where for simplicity these are the only realizations for which they locate at $x_i = y_i$. If this position is not equal to the median $m_{i,i}$ conditional on two i signals, then either candidate could raise their payoff by a small move toward that conditional median: If A does this, then A 's expected payoff given signal realization i for B jumps discontinuously, but for other signal realizations for B , A 's payoff varies continuously with A 's location. Hence, the slight deviation raises A 's payoff, which is impossible in equilibrium.

Next, observe that with identical polling technologies, the electoral game is symmetric and constant-sum. Hence, the argument extends to *all* pure strategy equilibria: If (X, Y) is a pure strategy equilibrium, then symmetry implies that (Y, X) is an equilibrium, and interchangeability

implies that (X, X) is an equilibrium. Thus, by Theorem 1, we see that $x_i = m_{i,i}$ for each signal i , and an analogous argument holds for Y .

Consider the implications of Theorem 1 in a setting where there is a zero signal $i = 0$ that conveys no information about whether the median voter's location is to the left or to the right, in the sense that the posterior on β is symmetric about zero following $i = 0$. Suppose that positive signals correspond to positive conditional medians, i.e., $i > 0$ implies $m_i > 0$, and likewise for negative signals, and that informative signals are self-reinforcing, in the sense that $i > 0$ implies $m_{i,i} > m_i$ and $i < 0$ implies $m_{i,i} < m_i$. Then a corollary of Theorem 1 is that candidates tend to take policy positions that are extreme relative to the median of μ given only their own information. That is, the platforms of candidates who receive high signals tend to overshoot μ , while those who receive low signals tend to undershoot.

We now impose additional structure on the conditional medians, in order to derive necessary and sufficient condition for existence of the pure strategy equilibrium described in Theorem 1:

(A1) For all signals i and j ,

$$m_{i,j} = \frac{m_{i,i} + m_{j,j}}{2}.$$

The next result, proved in the appendix, establishes that the pure strategy equilibrium exists if and only if each signal i is a median of the probability distribution $P(\cdot|i)$. That is, the probability conditional on signal i that the other candidate receives a signal less than i is no greater than one half, and similarly for signals greater than i . This condition limits the incentive to move away from $m_{i,i}$ after signal i to compete against a candidate who receives a more moderate signal or more extreme signal.⁶

Theorem 2 *Assume (A1). A necessary and sufficient condition for the pure strategy equilibrium in which $x_i = y_i = m_{i,i}$ for all $i \in I$ to exist is that for all signals i ,*

$$\sum_{j:j \leq i} P(j|i) \geq \sum_{j:j > i} P(j|i) \quad \text{and} \quad \sum_{j:j < i} P(j|i) \leq \sum_{j:j \geq i} P(j|i). \quad (2)$$

Condition (2) is weakest when there are just two possible signals, in which case it is satisfied as long as signals are not negatively correlated. The condition is most restrictive for the “extremal” signals, for which $P(i|i) \geq \frac{1}{2}$ is implied, and its restrictiveness obviously increases with the

⁶Theorem 2 extends to the more general setting in which the candidates may have access to different polling technologies. The necessary and sufficient condition for existence of this symmetric pure strategy equilibrium consists of two sets of inequalities as in (2), one conditioning on candidate A 's signal and the other conditioning on candidate B 's.

number of signals. Thus, in elections with finely-detailed polling, it becomes important that candidate platforms are not uniquely pinned down by their polling information, as in a pure strategy equilibrium.

Because the pure strategy equilibrium does not always exist, we now analyze equilibria of the electoral game when we allow for mixed strategies for the candidates. Here, a mixed strategy is a vector $G = \{G_i\}_{i \in I}$ of cumulative distribution functions, where G_i represents the distribution over platforms adopted by a candidate after receiving polling signal i . These distributions allow us to capture the possibility that a candidate's position following a signal cannot be precisely predicted by her opponent. A mixed strategy equilibrium is a pair (G, H) of mixed strategies such that candidate A 's strategy G maximizes her expected payoff given any signal i , and similarly for candidate B 's strategy H . BDS (2004) prove that mixed strategy equilibria always exist, and that the mixed strategy equilibrium payoffs are unique and vary continuously in the model's parameters. They use this last result to prove upper hemicontinuity of equilibrium mixed strategies, and they show that the support of mixed strategy equilibria is bounded by the interval defined by the smallest and largest conditional medians.

To obtain an explicit characterization of the mixed strategy equilibria of our model, we now impose more structure on the distribution over candidates' signals.

(A2) For all signals i, k with $k < i$,

$$\sum_{j:j < \ell} P(j|k) \geq \sum_{j:j < \ell} P(j|i), \text{ for all } \ell.$$

Assumption **(A2)** is a stochastic dominance restriction on the conditional distributions of signals. It says that raising a candidate's signal leads to a first-order stochastic increase in the distribution of his opponent's signal.

(A3) For all signals i, k , with $k < i$,

$$\sum_{j:j < i} P(j|i) \geq \sum_{j:j < k} P(j|k) \quad \text{and} \quad \sum_{j:j \leq i} P(j|i) \geq \sum_{j:j \leq k} P(j|k).$$

Assumption **(A3)** says that the higher is a signal that a candidate receives, the more likely is his signal to exceed his opponent's. We use **(A2)** and **(A3)** to characterize the set C of signals i that satisfy the inequalities (2) of Theorem 2:

$$\sum_{j:j \leq i} P(j|i) \geq \sum_{j:j > i} P(j|i) \quad \text{and} \quad \sum_{j:j < i} P(j|i) \leq \sum_{j:j \geq i} P(j|i).$$

Let $\bar{c} = \max C$ and $\underline{c} = \min C$.

Proposition 1 *Under (A2), C is non-empty. Adding (A3), C is connected: Given any signal i , we have $i \in C$ if and only if $\underline{c} \leq i \leq \bar{c}$.*

Under assumption (A1), Theorem 2 implies that if all signals are in C , then the pure strategy equilibrium exists and is unique. We will prove that this qualitative result extends to the general case where C may be a proper subset of I : a candidate who receives a moderate signal $i \in C$ locates at $m_{i,i}$, while a candidate who receives a signal $i \notin C$ mixes over locations.

We search for equilibria that satisfy a simple monotonicity condition. Specifically, we assume that following any given signal, if candidates do not locate deterministically, then they locate according to a distribution with connected support; and we assume that these supports are non-overlapping and ordered according to the candidates' signals. In the following definition, we let \underline{x}_i and \bar{x}_i denote the lower and upper bounds of a candidate's mixed strategy distribution following signal i . When these bounds coincide, the distribution is degenerate on $\underline{x}_i = \bar{x}_i$, i.e., the candidate locates deterministically.

Definition 1 *A mixed strategy G is ordered if:*

- (a) *for all signals i , $\text{Supp}(G_i) = [\underline{x}_i, \bar{x}_i]$*
- (b) *for all signals i and j with $i < j$, $\bar{x}_i \leq \underline{x}_j$.*

In Theorem 3, we characterize the unique ordered equilibrium, when it exists. We establish that candidates who receive moderate signals in the central set C extremize their platforms, adopting the conditional medians $m_{i,i}$ from Theorem 1. However, candidates with more extreme signals, say $i > \bar{c}$, are more likely to face an opponent who draws a signal less than theirs, and this leads them to moderate their location. We prove that candidates who receive extreme signals outside of C adopt convex, increasing mixed strategy densities, that the supports of distributions following extreme signals admit no "gaps," and that with probability one candidates locate more moderately than the conditional medians.

Theorem 3 *Under (A2) and (A3), if there is an ordered equilibrium, then it is unique and has the following symmetric form. For all $i \in C$, candidates locate at $m_{i,i}$, and for all $i > \bar{c}$, candidates*

mix according to an increasing, convex density,

$$g_i(x) = \frac{\Phi_i}{2} \sqrt{\frac{m_{i,i} - \underline{x}_i}{(m_{i,i} - x)^3}} > 0 \quad (3)$$

on an interval $[\underline{x}_i, \bar{x}_i]$ with $\bar{x}_i < m_{i,i}$. Here,

$$\Phi_i = \frac{\sum_{j:j \leq i} P(j, i) - 1/2}{P(i, i)} > 0 \text{ and } \underline{x}_i = m_{i,i} \left[1 - \left(\frac{\Phi_i}{\Phi_i + 1} \right)^2 \right] + \underline{x}_{i-1} \left(\frac{\Phi_i}{\Phi_i + 1} \right)^2, \quad (4)$$

with $\underline{x}_{\bar{c}} = m_{\bar{c}, \bar{c}}$. These supports are adjacent, in the sense that $\bar{x}_{i-1} = \underline{x}_i$ for all $i > \bar{c}$. The associated cumulative distribution function with which candidates mix is given by

$$G_i(x) = \Phi_i \left[\sqrt{\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x}} - 1 \right]. \quad (5)$$

Further, the expected platform of a candidate with signal $i > \bar{c}$ is a weighted average of \underline{x}_i and $m_{i,i}$,

$$E[x_i] = \frac{\Phi_i}{\Phi_i + 1} \underline{x}_i + \frac{1}{\Phi_i + 1} m_{i,i}. \quad (6)$$

There is an analogous characterization of G_i for $i < \underline{c}$.

The proof in the appendix proceeds sequentially. We first prove that an equilibrium mixed strategy cannot put positive probability following signal i on any platform other than $m_{i,i}$. Given an interval on which G_i is continuous, our assumption of non-overlapping supports implies that the candidate's expected payoff conditional on signal i is differentiable over that interval. Since the candidate must be indifferent over all positions in the support of his mixed strategy, the second order condition must hold over the interval, taking the simple form

$$3g_i(x)f_{i,i}(x) + g'_i(x)(2F_{i,i}(x) - 1) = 0.$$

We solve this system of ordinary differential equations for a mixed-strategy equilibrium up to the initial condition $g_i(\underline{x}_i)$. We use this characterization to show that if $i \in C$, then the candidate necessarily places probability one on the conditional median $m_{i,i}$. Following an extreme signal $i \notin C$, the candidate has an incentive to locate to compete against opponents with more moderate signals, and this implies that candidates with more extreme signals mix according to non-degenerate distributions, moderating their positions relative to the conditional medians. We show that there can be no gaps in the supports of the candidates' mixed strategies. Finally, integration reveals the properties of the equilibrium mixed strategy stated in the theorem.

We next provide conditions under which the unique equilibrium described in Theorem 3 exists.

(A4) For all signals i and k , $P(k|k) \geq P(k|i)$.

Assumption **(A4)** simply says a candidate is most likely to receive signal k when the other candidate also receives k . Adding **(A4)** to the conditions in Theorem 3, we show that each candidate's payoffs are single-peaked in x_i around the support of his mixed strategy. Obviously, such single-peakedness is a more stringent condition than needed to ensure existence.⁷

Theorem 4 *If one candidate adopts the strategy specified in Theorem 3, then under **(A2)**–**(A4)**, following any signal i , the other candidate's payoff is weakly single-peaked in his location x_i and maximized by all $x_i \in [\underline{x}_i, \bar{x}]$.*

Together, Theorems 3 and 4 yield the following key corollary.

Corollary 1 *Under **(A2)**–**(A4)**, there exists a unique ordered equilibrium, and its closed form is specified in Theorem 3.*

Inspection of the closed-form solution in equation (5) of Theorem 3 for the equilibrium mixed strategy following signal $i > \bar{c}$ reveals several comparative statics results:

- Ceteris paribus, increasing the conditional median, $m_{i,i}$, leads to a first-order stochastic increase in the equilibrium distribution; i.e., candidates tend to locate further away from the unconditional median.
- So, too, reducing Φ_i —for example, by increasing the probability that the candidates draw the same signal, $P(i, i)$, or by reducing the probability of competing against a candidate with a more moderate signal, $\sum_{j:j < i} P(j, i)$ —leads to a first-order stochastic increase in $G_i(\cdot)$.
- In particular, because $\Phi_i \rightarrow 0$ as $\sum_{j:j \leq i} P(j, i) \rightarrow 1/2$, it follows that as the condition for candidates to adopt their pure strategy location of $m_{i,i}$ becomes close to being satisfied, candidates place almost all probability close to their pure strategy location of $m_{i,i}$.
- Under reasonable structural assumptions, increasing signal precision implies that $P(i, i)$ grows in i , and $\sum_{j:j < i} P(j, i)$ decreases, which implies a declining Φ_i . Thus, increasing signal precision leads to candidates locating more extremely.

⁷In the proof of Theorem 4, assumptions **(A3)** and **(A4)** are used only to address signals $i \notin C$. It follows that if (2) holds for all signals, then **(A2)** is sufficient for existence of the pure strategy equilibrium; assumption **(A1)** is not needed.

We now exploit the closed-form solution for G_i to derive a key characterization result: Candidates with more moderate signals expect to locate more extremely relative to their information than do candidates with more extreme signals. Specifically, we provide simple conditions under which candidates with extreme signals $i > \bar{c}$ expect to locate further away from $m_{i,i}$ as i increases.

First note that under reasonable conditions, the Φ_i coefficients, $\left[\sum_{j \leq i} P(j, i) - 1/2\right] / P(i, i)$, in our equilibrium characterization of $G_i(\cdot)$ will increase in i , for $i > \bar{c}$. To see this, note that under **(A3)**, $\sum_{j: j \leq i} P(j, i)$ rises with i : increasing A 's signal raises the probability that his signal is at least as high as B 's. Hence, as a candidate's signal grows more extreme, Φ_i rises as long as the conditional probability that the other candidate receives the same signal, $P(i, i)$, does not rise too sharply with i (and one might think that $P(i, i)$ should fall with i). We assume precisely this regularity condition.

(A5) For all signals $i > \bar{c}$, $\Phi_{i+1} \geq \Phi_i$ and for all $i \leq \underline{c}$, $\Phi_{i-1} \leq \Phi_i$.

Because Φ_i is increasing in i , for $i > \bar{c}$, it follows immediately that $G_i(m_{i,i} - x)$ rises faster with i . Hence, to prove that candidates with more extreme signals expect to locate more moderately relative to their information, we just need to show that $m_{i,i} - \underline{x}_i$ is strictly increasing in i . To do this, use the difference equation (4) describing the relationship between \underline{x}_{i+1} and \underline{x}_i to solve for

$$m_{i+1,i+1} - \underline{x}_{i+1} = m_{i+1,i+1} - m_{i,i} + \left(\frac{\Phi_{i+1}}{\Phi_{i+1} + 1}\right)^2 (m_{i,i} - \underline{x}_i), \quad \text{for } i \geq \bar{c}.$$

At $i = \bar{c}$, $\underline{x}_{\bar{c}} = m_{\bar{c},\bar{c}}$, so that $(m_{\bar{c}+1,\bar{c}+1} - \underline{x}_{\bar{c}+1}) - (m_{\bar{c},\bar{c}} - \underline{x}_{\bar{c}}) = m_{\bar{c}+1,\bar{c}+1} - m_{\bar{c},\bar{c}} > 0$. Continuing, inductively

$$\begin{aligned} & (m_{i+1,i+1} - \underline{x}_{i+1}) - (m_{i,i} - \underline{x}_i) \\ &= [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}] + \left(\frac{\Phi_{i+1}}{\Phi_{i+1} + 1}\right)^2 (m_{i,i} - \underline{x}_i) \\ & \quad - \left(\frac{\Phi_i}{\Phi_i + 1}\right)^2 (m_{i-1,i-1} - \underline{x}_{i-1}) \\ & \geq [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}] + \left(\frac{\Phi_{i+1}}{\Phi_{i+1} + 1}\right)^2 [(m_{i,i} - \underline{x}_i) - (m_{i-1,i-1} - \underline{x}_{i-1})] \\ & > [m_{i+1,i+1} - m_{i,i}] - [m_{i,i} - m_{i-1,i-1}], \end{aligned}$$

where the first inequality follows because $\Phi_{i+1} \geq \Phi_i$, and the second inequality follows from the induction hypothesis. We have shown that when $[m_{i+1,i+1} - m_{i,i}]$ is constant, $m_{i,i} - \underline{x}_i$ is strictly increasing with significant slack. Thus, a gross sufficient condition for candidates with increasingly extreme signals to locate increasingly moderately relative to their information is that the distance between successive conditional medians, $[m_{i+1,i+1} - m_{i,i}]$, not fall too quickly with i for $i \geq \bar{c}$:

Theorem 5 Under (A2)–(A5), there exists $\delta > 0$ such that if $[m_{j+1,j+1} - m_{j,j}] - [m_{j,j} - m_{j-1,j-1}] > -\delta$ for all $j \geq \bar{c}$, then $m_{i,i} - E[x_i]$ is increasing in i for $i \geq \bar{c}$.

We end this section by presenting a pair of three-signal examples. In Example 1, a moderate signal indicates that the median voter is likely to be centrally located. In this setting, we find that extreme location remains likely even when the pure strategy equilibrium does not exist. It also suggests that increased signal precision or increased signal correlation makes extreme location by candidates more likely.

Example 1: Three Informative Signals. Consider a uniform model with three equally-likely values of $b \in \{-1, 0, 1\}$, and three possible signals, $i \in \{-1, 0, 1\}$. With probability q , the candidates receive the same signal, and with probability $1 - q$ they receive conditionally-independent signals. A signal is correct with probability $p > 1/3$, and with equal probability $\frac{1-p}{2}$ either of the other signals is drawn. Numerical calculations reveal that following a high signal of 1, the probability a candidate chooses a moderate platform $x_1 \in [0, m_1]$ is bounded from above by $\frac{1}{2}(\sqrt{2} - 1) \approx 0.207$, achieving this bound when signals are uninformative, i.e., $p \downarrow \frac{1}{3}$, and signals are uncorrelated, i.e., $q = 0$. Thus, with high probability, a candidate extremizes his location, in the sense that he locates more extremely than his forecast, given his signal, of the median voter's position. Numerical calculations also reveal that extreme locations become more likely as signals become more accurate, i.e., as p rises, or as the correlation q in signals rises.

In Example 2, a moderate signal is uninformative about the median voter's location. We find that the more likely is an informative signal, the more likely candidates are to adopt more extreme platforms. However, whether increasing the informativeness of a extreme signal leads to more extreme location depends on the likelihood that an informative signal is received.

Example 2: Uninformative Central Signal. Suppose that realizations of $\beta \in \{-1, 1\}$ are equally likely and there are three possible signals, $i \in \{-1, 0, 1\}$ that are conditionally-independently distributed given β . With probability r , a candidate receives an informative signal $i \in \{-1, 1\}$ of quality q , where given β , the probability $i = \beta$ is $\frac{1+q}{2}$. If $q = 1$, then a signal of $i \in \{-1, 1\}$ is completely informative, and if $q = 0$, the signal is uninformative. With probability $1 - r$, a candidate receives signal $i = 0$, independently of β , so that this signal is uninformative. Figure 2 plots level sets of the probability of locating at least as moderately as $m_1 = q$ following signal 1, when the pure

strategy equilibrium fails to exist.⁸ Here, the higher is a level curve, the lower is the probability of a moderate location, i.e., the more likely a candidate is to choose a platform that extremizes the informational content of his signal. (The top level set represents zero probability of moderation, and the bottom level set represents moderation with probability one.) Note that increasing r (the probability of an informative signal) leads to a higher probability of extremization. In contrast, increasing q (the informativeness of signals 1 and -1) has an ambiguous effect depending on r .

4 Voter Welfare

Our analysis has revealed that private polling by candidates may induce candidates to locate extremely relative to their private information. We now investigate the properties of socially optimal platforms and the consequences of equilibrium platform choices for voter welfare. While “overshooting” by candidates causes them to bias their location away from the expected location of the median voter, it also leads to greater expected separation between the candidates and therefore to greater choice for voters. We show that, with sufficient uncertainty about the median voter’s location, the latter effect dominates: voters prefer that candidates take even more extreme positions than they do in equilibrium. We then investigate how the statistical properties of the polling technology affect candidate location and voter welfare. In particular, we first show that increased correlation in candidate signals reduces voter welfare. Finally, we illustrate how the optimal amount of noise in the polling technology from the perspective of voters depends on the correlation in signals and the amount of uncertainty about the median voter’s location.

We address these issues in the zero-symmetric setting, in which $i \in \{-K, \dots, -1, 0, 1, \dots, K\}$ and $\beta \in \{-B, \dots, -1, 0, 1, \dots, B\}$, where $P(b) = P(-b)$, and $P(i, j|b) = P(-i, -j| -b)$. We adopt the standpoint of a social planner who does not have an informational advantage over the candidates in the electoral game, where the role of the social planner is to choose strategies for the two candidates to maximize voter welfare. To maintain consistency with equilibrium choices, we restrict the social planner to symmetric and monotone strategies. That is, the social planner selects the same strategy X for the two candidates, and $x_i = x_{-i}$ for all signals i , and $x_i > x_j$ for $i > j$.

To make welfare comparisons unambiguous, we introduce voters who have the same ex-ante ordering over candidate strategies. In particular, we assume that shifts in preferences in the elec-

⁸A pure strategy equilibrium exists if and only if $r \geq 1/(1 + q^2)$; i.e., fixing the informativeness of signals, a pure strategy equilibrium exists if and only if the probability of an informative signal is high enough. Otherwise, candidates locate at $m_{0,0} = 0$ following the uninformative signal 0, and mix according to Theorem 5 upon receiving signals 1 or -1 .

torate affect all voters in a common way and that voters have quadratic utilities. Voter v 's preferred policy, θ_v , is defined relative to the median voter's preferred policy μ : $\theta_v = \mu + \delta_v$. That is, a voter's "relative ideal point", δ_v , represents the position of v 's ideal point relative to μ . A change in μ simply shifts the distribution of voter ideal points. A voter with ideal point θ receives utility $u(\theta, z) = -(\theta - z)^2$ from policy outcome z , and each voter votes for the candidate whose platform is closest to his preferred policy. We let $W_{\delta_v}(X)$ represent the utility that voter v expects to receive if candidates use strategy X . Lemma 2, proved in the appendix, implies that, up to a voter-specific constant term, all voters have identical expected utilities from candidate strategies.

Proposition 2 *For all strategies X , voter δ_v 's expected utility from X is a fixed amount δ_v^2 less than the expected utility of the median voter:*

$$W_{\delta_v}(X) = -\delta_v^2 + W_0(X).$$

A consequence of Proposition 2 is that without loss of generality we may focus on the median voter's welfare:

$$\begin{aligned} W_0(X) = & - \sum_b \sum_i P(b, i, i) \int_{-a}^a \frac{(\alpha + b - x_i)^2}{2a} d\alpha \\ & - \sum_b \sum_i \sum_{j:j>i} P(b, i, j) \left[\int_{-a}^{[x_i+x_j]/2-b} \frac{(\alpha + b - x_i)^2}{2a} d\alpha + \int_{[x_i+x_j]/2-b}^a \frac{(\alpha + b - x_j)^2}{2a} d\alpha \right] \\ & - \sum_b \sum_i \sum_{j:j<i} P(b, i, j) \left[\frac{1}{a} \int_{-a}^{[x_i+x_j]/2-b} \frac{(\alpha + b - x_j)^2}{2a} d\alpha + \int_{[x_i+x_j]/2-b}^a \frac{(\alpha + b - x_i)^2}{2a} d\alpha \right]. \end{aligned} \quad (7)$$

The extension of this expression to mixed strategies is conceptually straightforward and hence it is omitted. In what follows, we therefore drop the subscript on W .

We now establish the strict concavity of the welfare function, which allows us to characterize the social optimum via first-order conditions. The proof proceeds by decomposing the Hessian of W into the sum of two matrices, where one is diagonal with negative entries and the other is a symmetric matrix in which the diagonal entry in each row i is equal to minus one times the sum of the other entries in that row. We then prove that this latter matrix is negative semi-definite.

Proposition 3 *The welfare function $W(\cdot)$ is strictly concave.*

We next examine the local properties of the welfare function at the vector of conditional medians, $M = (m_{i,i})$. Specifically, we characterize the partial derivative of welfare with respect to candidate location following an arbitrary signal k .

Theorem 6 *The partial derivative of the welfare function at the vector of medians is given by:*

$$\begin{aligned} \frac{\partial W}{\partial x_k}(M) &= \frac{1}{a} \sum_{j:j < k} P(k, j) \left[(a + m_{j,k} - m_{k,k})^2 + \sigma_{j,k}^2 - \left(\frac{x_j - m_{k,k}}{2} \right)^2 \right] \\ &\quad - \frac{1}{a} \sum_{j:j > k} P(k, j) \left[(a - m_{j,k} + m_{k,k})^2 + \sigma_{j,k}^2 - \left(\frac{x_j - m_{k,k}}{2} \right)^2 \right], \end{aligned}$$

where $\sigma_{j,k}^2$ is the variance of β conditional on signals j and k .

Theorem 6 allows us to sign the partials of the welfare function, which indicate the voters' preferences for more moderate or extreme candidate platforms. The partial derivative is positive if

$$\begin{aligned} &\sum_{j < k} P(k, j) \left[(a + m_{j,k} - m_{k,k})^2 + \sigma_{j,k}^2 - \left(\frac{x_j - m_{k,k}}{2} \right)^2 \right] & (8) \\ &> \sum_{j > k} P(k, j) \left[(a - m_{j,k} + m_{k,k})^2 + \sigma_{j,k}^2 - \left(\frac{x_j - m_{k,k}}{2} \right)^2 \right], \end{aligned}$$

and negative if this is reversed. Under assumption **(A1)**, this inequality reduces to:

$$\sum_{j < k} P(k, j) [a^2 - a(m_{k,k} - m_{j,j}) + \sigma_{j,k}^2] > \sum_{j > k} P(k, j) [a^2 - a(m_{j,j} - m_{k,k}) + \sigma_{j,k}^2]. \quad (9)$$

To interpret inequality (9), consider a positive signal $k > 0$. This inequality always holds if there are no more than three signals, and it holds for the most extreme signal $k = K$. More generally, under reasonable conditions we have $\sum_{j < k} P(k, j) > \sum_{j > k} P(k, j)$. In particular, there are $2k$ more signals less than k than greater than k . Reinforcing this, the probability that the other candidate receives a more moderate signal than k plausibly exceeds the probability he receives an equally more positive signal, in the sense that $P(k, k - h) > P(k, k + h)$ for all integers $h \leq K - k$.⁹ Inequality (9) would then hold for a sufficiently large.

Thus, when the location of the median voter is subject to sufficient uncertainty, we find that $\frac{\partial W}{\partial x_k}(M) > 0$ for $k > 0$ and, by analogous arguments, $\frac{\partial W}{\partial x_k}(M) < 0$ for $k < 0$, i.e., voter welfare would increase if the candidates located marginally more extremely than $m_{i,i}$ following each signal $i \neq 0$. Effectively, voters gain more from the added separation in platforms when the opposing candidate receives a signal $j < k$, than they lose from the reduced separation when the opposing candidate receives a signal $j > k > 0$ and from the reduced absolute accuracy of candidate A 's platform.

We now extend Theorem 6 to analyze the global properties of voter welfare. Theorem 6 establishes conditions under which voters are better off in expectation if, following each signal $i \neq 0$,

⁹This would be the case, for example, if signals were conditionally independent and the prior distribution of β were symmetric and single-peaked with sufficiently low variance.

candidates locate marginally more extremely than $m_{i,i}$, fixing platforms at conditional medians for signals $j \neq i$. We shift focus to the voters' optimal platforms, as functions of candidate signals, and show that under the conditions of Theorem 6, the socially optimal policy locations of candidates are more extreme than the conditional medians. Let X^* be the maximizer of W , which is unique by Proposition 3.

Theorem 7 *Assume inequality (8) holds weakly for all signals $i > 0$. Then the socially optimal platforms of the candidates are more extreme than the conditional medians, in the sense that $x_i^* > m_{i,i}$ for all signals $i > 0$. Analogously, $x_i^* < m_{i,i}$ for all signals $i < 0$.*

Theorem 7 does not follow from Theorem 6 and concavity of the welfare function. Rather, we must exploit the deeper structure of W : We use the fact that the cross partial derivative of W with respect to candidate locations following any distinct positive signals i and j is positive. From Theorem 6, given candidate locations at the conditional medians, we know that an increase in the platform x_K following the most extreme signal K raises voter welfare. By the fact that cross partials are positive, this then raises the marginal welfare gains from increasing platform x_{K-1} following the next most extreme signal $K - 1$, and so on. Based on these observations, we invoke a known result on supermodular games to conclude that the unique social welfare optimum X^* is characterized by policy platforms more extreme than the conditional medians.

We now relate Theorem 7 to the equilibrium analysis of the previous section. Corollary 2 follows immediately from Theorem 1 and from the characterization in Theorem 3, which shows that the support following signal $i > 0$ over which candidates mix is bounded above by the conditional median $m_{i,i}$.

Corollary 2 *Assume that signals are self-reinforcing, in the sense that $i > 0$ implies $m_i < m_{i,i}$. When the pure strategy equilibrium exists, candidates locate too moderately relative to the social optimum following every signal $i \neq 0$. Furthermore, in the mixed strategy ordered equilibrium, under (A2) and (A3), candidates locate too moderately relative to the social optimum with probability one following every signal $i \neq 0$.*

Finally, we turn to the statistical properties of the polling technology. For simplicity, we analyze the effects of correlation on voter welfare when equilibrium is characterized by pure strategies, so that we can exploit the result in Theorem 7 that voters would be made better off if candidates located more extremely than the conditional medians. To address the impact of correlation, we

assume that with probability q both candidates receive the same signal, drawn from $P(\cdot|b)$ for each realization of b , and with probability $1 - q$ candidates receive conditionally independent signals drawn from the same distributions. We impose the further structure that if a candidate receives a given positive signal, then the median conditional on the other candidate receiving that same positive signal exceeds the median conditional on the other candidate not receiving that signal, i.e., $m_{i,i} > m_{i,-i}$. The next theorem, which is proved in the appendix, establishes that increasing signal correlation reduces voter welfare.

Theorem 8 *Assume that inequality (8) holds weakly for all signals $i > 0$ and that $m_{i,i} > m_{i,-i}$. Then increasing the correlation q in candidate signals reduces voter welfare:*

$$\frac{dW}{dq}(M) < 0,$$

where we suppress the dependence of the vector M of conditional medians on q .

The direct effect of increasing correlation is to decrease voter choice, which reduces voter welfare. The key is then to establish that the indirect effect through candidate locations is also negative. We show that $\frac{\partial m_{i,i}}{\partial q}$ is proportional to $m_{i,-i} - m_{i,i}$. Therefore, under the conditions of Theorem 8, the effect of increasing q is to moderate candidate locations, which lowers welfare by Theorem 7.

The welfare effect of increasing signal precision on welfare is more subtle: The relationship is not monotonic. To illustrate this, we calculate equilibrium welfare in a simple binary setting with shock $b \in \{-1, 1\}$ and signal $i \in \{-1, 1\}$. As above, conditional on the realization b , candidates receive the same signal with probability q , and with probability $1 - q$ they receive conditionally-independent signals. We now assume that each signal is accurate with probability p . Increasing p raises the probability that both candidates receive the “correct” signal, but it also increases the probability that candidates receive the same signal, and hence do not offer voters variety. As a result, increasing signal precision eventually reduces voter welfare. This is illustrated in Figure 3, which portrays level sets of W as functions of (p, a) when $q = 0$, as well as level sets of W in (p, q) for $a = 2$.

As long as signals are not perfectly correlated, voters prefer that there be some noise in the polling technology. The optimal precision rises with signal correlation q . When signals are perfectly correlated, there is no platform variety, so that voters are better off if candidates target the median as accurately as possible. The value of precisely targeting β decreases with a : As a rises, there is more uncertainty about the median voter’s preferred platform, raising the value of increased platform choice. Hence, the optimal precision decreases in a : Voters want less accurate polls, so that candidates are less likely to receive the same signal, and thus provide greater choice.

5 Conclusion

This paper shows how private polling radically alters the nature of the strategies of office-motivated candidates, overturning the apparently robust result of platform convergence. Specifically, in any pure strategy equilibrium, candidates' platforms over-emphasize their private information: Candidates locate at the median given that *both* receive the same signal. When candidates are not sufficiently likely to receive the same signal, equilibrium is characterized by mixed strategies. In the mixed strategy equilibrium, candidates who receive moderate signals adopt more extreme platforms than their information suggests, while candidates who receive more extreme signals moderate their platforms relative to their pollsters' advice.

Contrary to existing models of elections, some platform differentiation always increases voters' welfare. Although candidates differentiate their platforms in equilibrium, voters would prefer that candidates extremize their positions by even more. From the perspective of voters, this paper finds that there is an optimal amount of noise in the polling technology. That is, the marginal social value of better information for candidates about voters becomes negative, once polls are sufficiently accurate. This suggests a rationale for campaign spending limits—such limits reduce expenditures on polling, thereby reducing the precision of candidate's signals, and possibly raising voter welfare. So too this suggests that voters may want to give dishonest answers to political pollsters in order to add noise to their polling technology. Finally, the electorate prefers private to public polling, because the increased signal correlation due to public polling reduces the diversity of platforms that candidates provide voters.

Our analysis suggests fruitful directions for future research. First, because the strategic value of better information is always positive for candidates, it is straightforward to endogenize the choice of costly polling technologies by candidates. Second, it would be worthwhile to determine how outcomes are affected when candidates have ideological preferences, and to endogenize contributions by ideologically-motivated lobbies to fund polling by candidates. Finally, as Ledyard (1989) observes, it would be useful to uncover how equilibrium outcomes are affected when candidates sequentially choose platforms, so the second candidate can see where the first locates, and hence can unravel the latter's signal, before locating.

6 Appendix

Proof of Theorem 2: Let (X, Y) denote the pure strategy profile in which $x_i = y_i = m_{i,i}$ for all $i \in I$. This is an equilibrium if and only if for all $j \in I$, candidate A is not willing to deviate and play $z \neq m_{j,j}$. Since deviations outside $[\min m_{i,i}, \max m_{i,i}]$ are strictly dominated by either $\min m_{i,i}$ or $\max m_{i,i}$, this amounts to the requirement that for all $k \in \{j, \dots, n-1\}$, and for all $z \in (m_{k,k}, y_{k+1}]$,

$$\begin{aligned}
0 \leq & \sum_{l=1}^{j-1} P(l|j) \left[F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) \right] \\
& + P(j|j) \left[F_{j,j} \left(\frac{z + m_{j,j}}{2} \right) - F_{j,j} \left(\frac{m_{j,j} + m_{j,j}}{2} \right) \right] \\
& + \sum_{l=j+1}^k P(l|j) \left[F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) - \left[1 - F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) \right] \right] \\
& + \sum_{l=k+1}^n P(l|j) \left[F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) \right];
\end{aligned} \tag{10}$$

and that for all $k \in \{1, \dots, j-1\}$, and for all $z \in [m_{k,k}, y_{k+1})$,

$$\begin{aligned}
0 \leq & \sum_{l=1}^k P(l|j) \left[F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) \right] \\
& + \sum_{l=k+1}^{j-1} P(l|j) \left[1 - F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) \right] \\
& + \Pr(j|j) \left[F_{j,j} \left(\frac{m_{j,j} + m_{j,j}}{2} \right) - F_{j,j} \left(\frac{z + m_{j,j}}{2} \right) \right] \\
& + \sum_{l=j+1}^n P(l|j) \left[F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) \right].
\end{aligned} \tag{11}$$

By **(A1)**, it follows that $F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) = F_{j,j} \left(\frac{z + m_{j,j}}{2} \right)$ and $F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) = \frac{1}{2}$. Therefore, for all signals j, l ,

$$F_{j,l} \left(\frac{z + m_{l,l}}{2} \right) - F_{j,l} \left(\frac{m_{j,j} + m_{l,l}}{2} \right) = \frac{z - m_{j,j}}{2},$$

condition (10) reduces to

$$0 \leq \left[\frac{z - m_{j,j}}{2} \right] \left[\sum_{l=1}^j P(l|j) - \sum_{l=j+1}^n P(l|j) \right], \tag{12}$$

regardless of the identity of the player who receives signal j . Analogously, condition (11) simplifies to

$$0 \leq \left[\frac{z - m_{j,j}}{2} \right] \left[\sum_{l=j}^n P(l|j) - \sum_{l=1}^{j-1} P(l|j) \right]. \tag{13}$$

If (12) and (13) hold as strict inequalities, then X is the unique best response to $Y = X$, and hence (X, Y) is the unique equilibrium: If (X', Y') is an equilibrium, then so is (X', Y) by interchangeability, and therefore $X' = X$; by a symmetric argument, $Y' = Y$. \blacksquare

Proof of Proposition 1: Let k be the highest signal such that $\sum_{j:j>k} P(j|k) > \frac{1}{2}$. (If such a k does not exist, then the signal $\min I$ satisfies the inequalities.) Clearly, $k < \max I$. We claim that signal $k + 1$ satisfies (2). First, note that

$$\sum_{j:j \leq k} P(j|k+1) \leq \sum_{j:j \leq k} P(j|k) \leq \frac{1}{2}, \quad (14)$$

where the first inequality above follows from **(A2)** and the second follows from the definition of k . Then (14) implies

$$\sum_{j:j < k+1} P(j|k+1) \leq \sum_{j:j \geq k+1} P(j|k+1).$$

Finally, from $k+1 > k$ and the assumption that k is the highest signal such that $\sum_{j:j>k} P(j|k) > \frac{1}{2}$, we have $\sum_{j:j>k+1} P(j|k+1) \leq \frac{1}{2}$, i.e.,

$$\sum_{j:j > k+1} P(j|k+1) \leq \sum_{j:j \leq k+1} P(j|k+1).$$

Therefore, $k + 1$ satisfies (2), and $C \neq \emptyset$. We now establish that, adding **(A3)**, C is connected. Let ℓ be the lowest signal such that $\sum_{j:j<\ell} P(j|\ell) > \frac{1}{2}$. Repeating the argument above, we have $\ell - 1$ satisfies (2). By construction, $k + 1 \leq \ell - 1$. Take any i such that $k + 1 < i < \ell - 1$. If $i \notin C$, then we may assume without loss of generality that $\sum_{j:j>i} P(j|i) > \frac{1}{2}$. But then the second part of **(A3)** implies that $\sum_{j:j>k+1} P(j|k+1) > \frac{1}{2}$, a contradiction. Finally, consider i such that $i < k + 1$ or $i > \ell - 1$, and without loss of generality assume the former. If $\sum_{j:j>i} P(j|i) \leq \frac{1}{2}$, then $i < k$. But then the second part of **(A3)** implies that $\sum_{j:j>k} P(j|k) \leq \frac{1}{2}$, a contradiction. Therefore, $C = \{j \mid k + 1 \leq j \leq \ell - 1\}$ is connected. \blacksquare

Proof of Theorem 3: Let (G, H) be an ordered equilibrium. We first assume the equilibrium is symmetric, so that $G = H$. Then x is a differentiable point of a candidate's expected payoff, conditional on signal i , if and only if the following holds: for all signals j , if G_j puts positive probability on x , then $x = m_{i,j}$. At every point of differentiability $x \in \text{Supp}(G_i)$, the derivative of the candidate's expected payoff function at x , conditional on signal i , with respect to x_i is:

$$\begin{aligned} & \sum_{j:m_{j,j} < x} P(j|i) \left[-f_{i,j} \left(\frac{x + m_{j,j}}{2} \right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] \\ & + \sum_{j:x < m_{j,j}} P(j|i) \left[f_{i,j} \left(\frac{x + m_{j,j}}{2} \right) \frac{(G_j(m_{j,j}) - G_j(m_{j,j})^-)}{2} \right] \end{aligned} \quad (15)$$

$$\begin{aligned}
& + \sum_{j \in I} P(j|i) \left[\int_{-\infty}^x -f_{i,j} \left(\frac{x+z}{2} \right) \frac{g_j(z)}{2} dz + (1 - F_{i,j}(x))g_j(x) \right. \\
& \left. + \int_x^{\infty} f_{i,j} \left(\frac{x+z}{2} \right) \frac{g_j(z)}{2} dz - F_{i,j}(x)g_j(x) \right],
\end{aligned}$$

where g_i is the density of G_i , wherever it is defined. BDS (2004) show generally that the supports of equilibrium mixed strategies are bounded by the left- and right-most conditional medians.

Since $f_{i,j}$ is constant at $\frac{1}{2a}$ over the relevant range under our assumptions, (15) simplifies greatly. If g_k is defined at $x \in [\underline{x}_k, \bar{x}_k]$, then the derivative of the candidate's expected payoff, conditional on signal i , with respect to x_i is

$$\frac{1}{4a} \left[- \sum_{j:j < k} P(j|i) + \sum_{j:j > k} P(j|i) + P(k|i) \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) \right] \right]. \quad (16)$$

If $x \in (\bar{x}_{k-1}, \underline{x}_k)$, then the interior term in brackets falls out, and the derivative becomes

$$\frac{1}{4a} \left[- \sum_{j:j < k} P(j|i) + \sum_{j:j \geq k} P(j|i) \right]. \quad (17)$$

Clearly, at any point of differentiability receiving positive probability under G_i , the first order condition must be satisfied, and the derivative in (16) must be equal to zero. Similarly, the candidate must be indifferent among all locations in any interval in the support of G_i , and the first order condition must hold on such an interval.

Step 1: For all k and all z , if G_k puts positive probability on z , then $z = m_{k,k}$. Define the set $I' = \{i \in I : G_i(z) - G_i(z)^- > 0\}$. Lemma A2 of BDS (2005) establishes that for all signals $i \in I'$, z must solve

$$\sum_{j \in I} \alpha_j F_{i,j}(z) = \frac{1}{2}, \quad (18)$$

where the non-negative weights,

$$\alpha_j = \frac{[G_j(z) - \lim_{w \uparrow z} G_j(z)]P(j|i)}{\sum_{l \in I'} [G_l(z) - \lim_{w \uparrow z} G_l(z)]P(l|i)},$$

sum to one. If I' contains multiple signals, then let \underline{i} and \bar{i} denote the minimum and maximum of I' , respectively. Accordingly,

$$\sum_{j \in I} \alpha_j F_{\bar{i},j}(z) = \sum_{j \in I} \alpha_j F_{\underline{i},j}(z). \quad (19)$$

Since $F_{\bar{i},\bar{i}}(z)$ is the uniquely smallest element of $\{F_{\bar{i},l}(z) \mid l \in I'\}$ and the uniquely largest element of $\{F_{\underline{i},l}(z) \mid l \in I'\}$, condition (19) implies that $\alpha_{\bar{i}} = \alpha_{\underline{i}} = 1$, a contradiction. Therefore, $I' = \{k\}$, and condition (18) yields $F_{k,k}(z) = \frac{1}{2}$, i.e., $z = m_{k,k}$.

Step 2: If G_i is continuous in an interval $[\underline{x}_i, \hat{x}]$, then given $g_i(\underline{x}_i)$, the density g_i on $[\underline{x}_i, \hat{x}]$ is characterized by the candidates' second-order condition. Since the candidate's expected payoff is constant over the interval, it must in particular be linear over this interval, so the second-order condition must be satisfied with equality. Differentiating (16), we have

$$P(i|i)[-3g_i(x) + 4(m_{i,i} - x)g_i'(x)] = 0,$$

for all $x \in (\underline{x}_i, \hat{x})$. Since the candidate chooses the platform \underline{x}_i with zero probability, we include it in the interval as well, yielding a differential equation in g_i that is easily solved. We find that

$$g_i(x) = g_i(\underline{x}_i) \left(\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x} \right)^{\frac{3}{2}} \quad (20)$$

for all $x \in [\underline{x}_i, \hat{x}]$, with associated distribution

$$G_i(x) = g_i(\underline{x}_i)(m_{i,i} - \underline{x}_i)^{3/2} \left(\frac{2}{\sqrt{m_{i,i} - \underline{x}_i}} - \frac{2}{\sqrt{m_{i,i} - x}} \right). \quad (21)$$

Thus, the second-order condition pins down the density g_i and distribution G_i on $[\underline{x}_i, \hat{x})$ up to the initial condition $g_i(\underline{x}_i)$.

Step 3: For $i \in C$, G_i is the point mass on $m_{i,i}$. Suppose not, so $\underline{x}_i < \bar{x}_i$. As the argument is symmetric, suppose without loss of generality $\underline{x}_i < m_{i,i}$. By Step 1, G_i is continuous on $[\underline{x}_i, m_{i,i})$. The first-order condition can be written as

$$g_i(x) = \frac{\sum_{j:j < i} P(j|i) + P(i|i)(1 - 2G_i(x)) - \sum_{j:i < j} P(j|i)}{4P(i|i)(m_{i,i} - x)}. \quad (22)$$

Note that $m_{i,i} \notin [\underline{x}_i, \bar{x}_i]$, for otherwise by (21) would imply that G_i takes values greater than one in a neighborhood of $m_{i,i}$. Therefore, $\bar{x}_i < m_{i,i}$, and G_i is continuous on the entire interval $[\underline{x}_i, \bar{x}_i]$. Substituting $x = \bar{x}_i$ in (22) and using $i \in C$, we have

$$g_i(x) = \frac{\sum_{j:j < i} P(j|i) - \sum_{j:i \leq j} P(j|i)}{4P(i|i)(m_{i,i} - x)} < 0,$$

a contradiction.

Step 4: For $i > \bar{c}$, G_i is continuous. First, suppose that $\underline{x}_i = \bar{x}_i$; then from Step 1, G_i puts probability one on $m_{i,i}$. We claim that for all $j < i$, $\bar{x}_j < m_{i,i}$. If $j \in C$, then this follows from Step 3 and $m_{j,j} < m_{i,i}$. If $j > \bar{c}$ and $\bar{x}_j = m_{i,i}$, then by equation (22), g_j is negative in a neighborhood of \bar{x}_j , a contradiction, establishing the claim. Consider any $z < m_{i,i}$ such that $\bar{x}_j < z$ for all $j < i$. The derivative of the candidate's expected payoff, conditional on i , at z is given by (17), which is negative, as $i > \bar{c}$. Therefore, a sufficiently small move from $m_{i,i}$ to $z < m_{i,i}$ raises the candidate's expected payoff, a contradiction. Therefore, $\underline{x}_i < \bar{x}_i$. As in Step 3, we must have $m_{i,i} \notin [\underline{x}_i, \bar{x}_i]$, and then G_i is continuous.

Step 5: For $i > \bar{c}$, supports are adjacent, in the sense that $\bar{x}_{i-1} = \underline{x}_i$. Suppose $\bar{x}_{i-1} < \underline{x}_i$. As in Step 4, the derivative in the interval $[\bar{x}_{i-1}, \underline{x}_i]$ is given by (17), which is negative. As above, a sufficiently small move from \underline{x}_i to $z < \underline{x}_i$ raises the candidate's expected payoff, a contradiction.

Step 6: For $i > \bar{c}$, the distribution G_i and its increasing, convex density g_i are given by (3) and (5); the upper bound of the support of G_i is strictly less than $m_{i,i}$; the lower bounds of the supports are as in (4); and the expected value of x_i is given by (6). The parameters \underline{x}_i and $g_i(\underline{x}_i)$ are determined by the first-order condition, which yields

$$g_i(\underline{x}_i) = \frac{\sum_{j:j \leq i} P(j|i) - \sum_{j:i < j} P(j|i)}{4P(i|i)(m_{i,i} - \underline{x}_i)} = \frac{\Phi_i}{2} \frac{1}{(m_{i,i} - \underline{x}_i)},$$

when evaluated at \underline{x}_i . Substituting for $g_i(\underline{x}_i)$ in (20) and (21) yields the following expressions for the density g_i and distribution G_i on the support $[\underline{x}_i, \bar{x}_i]$:

$$g_i(x) = \frac{\Phi_i}{2} \left[\frac{(m_{i,i} - \underline{x}_i)^{\frac{1}{2}}}{(m_{i,i} - x)^{\frac{3}{2}}} \right] \text{ and } G_i(x) = \Phi_i \left[\sqrt{\frac{m_{i,i} - \underline{x}_i}{m_{i,i} - x}} - 1 \right].$$

The condition $G_i(x) = 1$ determines the upper bound \bar{x}^i of the support, which by Step 5 coincides with \underline{x}_{i+1} . Note that for $\underline{x}_i < m_{i,i}$, a solution to $G_i(x) = 1$ does indeed exist for all $i > \bar{c}$, since $\frac{2}{\sqrt{m_{i,i} - x}}$ goes to infinity as x increases to $m_{i,i}$. By Steps 3 and 5, $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$. Therefore, the lower bounds are pinned down recursively by the difference equation:

$$\underline{x}_{i+1} = m_{i,i} \left[1 - \left(\frac{\sum_{j \leq i} P(j,i) - 1/2}{\sum_{j \leq i} P(j,i) + P(i,i) - 1/2} \right)^2 \right] + \underline{x}_i \left(\frac{\sum_{j \leq i} P(j,i) - 1/2}{\sum_{j \leq i} P(j,i) + P(i,i) - 1/2} \right)^2,$$

with the initial condition $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$, as in (4). These observations, with an induction argument starting with $\underline{x}_{\bar{c}+1} = m_{\bar{c},\bar{c}}$, yield $\bar{x}_i < m_{i,i}$ for all $i \in I$. The expectation (6) is derived simply by integrating. That g_i is increasing and convex is apparent from the functional form of the density.

Finally, we allow $G \neq H$. Because the electoral game is symmetric and zero-sum, the strategy pair (H, G) is also an equilibrium. By interchangeability, (G, G) is a symmetric equilibrium, and the characterization above continues to hold. \blacksquare

Proof of Theorem 4: Assume each candidate uses the mixed strategy G , defined in Theorem 3, and consider any signal $i > \bar{c}$. By construction, the candidate's expected payoff conditional on receiving signal i is constant on $[\underline{x}_i, \bar{x}_i]$. We show that the candidate's expected payoff falls as we move x_i further to the left of \underline{x}_i or further to the right of \bar{x}_i . First, take k such that $k < i$ and $k \notin C$. We must show that

$$\frac{-\sum_{j:j < k} P(j|i) + \sum_{j:j > k} P(j|i)}{P(k|i)} + \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) \right] \quad (23)$$

is non-negative. By construction, at $x \in (\underline{x}_k, \bar{x}_k)$, we have

$$\frac{\left[\sum_{j:j > k} P(j|k) - \sum_{j:j < k} P(j|k) \right]}{P(k|k)} + \left[1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x) \right] = 0. \quad (24)$$

By **(A2)** and **(A4)**, we have

$$\frac{\sum_{j:j>k} P(j|i) - \sum_{j:j<k} P(j|i)}{P(k|i)} \geq \frac{\sum_{j:j>k} P(j|k) - \sum_{j:j<k} P(j|k)}{P(k|k)},$$

and since $m_{i,k} > m_{k,k}$ we have

$$1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) > 1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x),$$

which implies that (23) exceeds (16), as required.

Now take $k \in C$. Note that the candidate's expected payoff conditional on signal i is discontinuous at $m_{k,k}$: letting $\Pi(x_i|G, i)$ denote the expected payoff conditional on i from locating at x_i when the other candidate uses the mixed strategy G , we have

$$\lim_{w \downarrow m_{k,k}} \Pi_A(w|G, i) - \Pi_A(m_{k,k}|G, i) = P(k|i) \left[1 - F_{i,k}(m_{k,k}) - \frac{1}{2} \right] \geq 0,$$

where the inequality follows from $m_{k,k} < m_{i,k}$. Similarly, $\Pi_A(m_{k,k}|G, i) - \lim_{w \uparrow m_{k,k}} \Pi_A(w|G, i) \geq 0$, so the candidate's payoff function is non-decreasing at $m_{k,k}$. Over the interval $(\bar{x}_{k-1}, m_{k,k})$, the derivative of the candidate's payoff function conditional on signal i is proportional to

$$\sum_{j:j \geq k} P(j|i) - \sum_{j:j < k} P(j|i) \geq \sum_{j:j \geq k} P(j|k) - \sum_{j:j < k} P(j|k) \geq 0,$$

where the first inequality follows from **(A2)** and the second from the definition of $k \in C$.

Now take $k > i$. We must show that

$$- \sum_{j:j < k} P(j|i) + \sum_{j:j > k} P(j|i) + P(k|i) \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) \right] \quad (25)$$

is non-positive. By **(A2)**, we have

$$- \sum_{j:j < k} P(j|k) + \sum_{j:j > k} P(j|k) \geq - \sum_{j:j < k} P(j|i) + \sum_{j:j > k} P(j|i). \quad (26)$$

Because (24) holds at $x \in (\underline{x}_k, \bar{x}_k)$ and $k > \bar{c}$, the left-hand side above is negative, which implies that $1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x)$ is positive. By **(A4)** and $m_{k,k} > m_{i,k}$, we have

$$P(k|k) \left[1 - 2G_k(x) + 4(m_{k,k} - x)g_k(x) \right] > P(k|i) \left[1 - 2G_k(x) + 4(m_{i,k} - x)g_k(x) \right]. \quad (27)$$

Together, (24), (26), and (27) imply that (25) is negative, as required. \blacksquare

Proof of Proposition 2: To calculate the welfare of a voter with relative ideal point δ_v , note that for any b , if both candidates receive a signal i , then they both locate at x_i , yielding voter δ_v expected utility of $(1/a) \int_{-a}^a u(\alpha + b + \delta_v, x_i) d\alpha$. If candidates receive different signals i and j , with $i < j$, one locates at x_i , while the other locates at x_j , and the candidate closest to the median voter wins the

election. That is, the candidate at x_i wins if $\mu < [x_i + x_j]/2$, or equivalently if $\alpha < [x_i + x_j]/2 - b$; and the candidate at x_j wins if $\mu > [x_i + x_j]/2$, or equivalently if $\alpha > [x_i + x_j]/2 - b$. Thus, voter δ_v 's expected utility is

$$\frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b + \delta_v, x_i) d\alpha + \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a u(\alpha + b + \delta_v, x_j) d\alpha. \quad (28)$$

Therefore,

$$\begin{aligned} W_{\delta_v}(X) &= \sum_b \sum_i P(b, i, i) \frac{1}{2a} \int_{-a}^a u(\alpha + b + \delta_v, x_i) d\alpha \\ &+ \sum_b \sum_i \sum_{j:j>i} P(b, i, j) \left[\frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b + \delta_v, x_i) d\alpha + \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a u(\alpha + b + \delta_v, x_j) d\alpha \right] \\ &+ \sum_b \sum_i \sum_{j:j<i} P(b, i, j) \left[\frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b + \delta_v, x_j) d\alpha + \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a u(\alpha + b + \delta_v, x_j) d\alpha \right]. \end{aligned} \quad (29)$$

For any b and pair i, j , with $i < j$, we aggregate the term (28) with the corresponding term for $b' = -b$ and $i' = -i, j' = -j$, so that $j' < i'$, so as to obtain:

$$\begin{aligned} W_{\delta_v}(X) &= -\frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} -(\alpha + b + \delta_v - x_i)^2 d\alpha - \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a (\alpha + b + \delta_v - x_j)^2 d\alpha \\ &- \frac{1}{2a} \int_{[-x_i-x_j]/2+b}^a -(\alpha - b + \delta_v + x_i)^2 d\alpha - \frac{1}{2a} \int_{-a}^{[-x_i-x_j]/2+b} (\alpha - b + \delta_v + x_j)^2 d\alpha \\ &= -\delta_v^2 - 2\delta_v(b - x_i) \left[a + \frac{x_i + x_j}{2} - b \right] \frac{1}{2a} - 2\delta_v(b - x_j) \left[a - \frac{x_i + x_j}{2} + b \right] \frac{1}{2a} \\ &- \delta_v^2 - 2\delta_v(-b + x_i) \left[a - \left[-\frac{x_i + x_j}{2} + b \right] \right] \frac{1}{2a} - 2\delta_v(-b + x_j) \left[-\frac{x_i + x_j}{2} + b + a \right] \frac{1}{2a} \\ &- \frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} (\alpha + b - x_i)^2 d\alpha - \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a (\alpha + b - x_j)^2 d\alpha \\ &- \frac{1}{2a} \int_{[-x_i-x_j]/2+b}^a -(\alpha - b + x_i)^2 d\alpha - \frac{1}{2a} \int_{-a}^{[-x_i-x_j]/2+b} (\alpha - b + x_j)^2 d\alpha \\ &= -\delta_v^2 - \frac{1}{2a} \int_{[-x_i-x_j]/2+b}^a u(\alpha - b, -x_i) d\alpha - \frac{1}{2a} \int_{-a}^{[-x_i-x_j]/2+b} u(\alpha - b, -x_j) d\alpha \\ &- \delta_v^2 - \frac{1}{2a} \int_{-a}^{[x_i+x_j]/2-b} u(\alpha + b, x_i) d\alpha - \frac{1}{2a} \int_{[x_i+x_j]/2-b}^a u(\alpha + b, x_j) d\alpha. \end{aligned}$$

where we used $\int_{-a}^a \alpha d\alpha = 0$. The case for pairs $i = j$ follows with analogous manipulations. This establishes that the voter's welfare is just δ_v^2 less than the welfare of the median voter ($\delta_v = 0$). ■

Proof of Proposition 3: Differentiating the expression (29) for voter welfare, with $\delta_v = 0$, we have

$$\frac{\partial W}{\partial x_k}(X) = \sum_b \left[P(k, k, b) \frac{1}{a} \int_{-a}^a (b + \alpha - x_k) d\alpha \right]$$

$$\begin{aligned}
& +2 \sum_{j:j < k} P(k, j, b) \frac{1}{a} \int_{\frac{x_j + x_k}{2} - b}^a (b + \alpha - x_k) d\alpha \\
& +2 \sum_{j:j > k} P(k, j, b) \frac{1}{a} \int_{-a}^{\frac{x_k + x_j}{2} - b} (b + \alpha - x_k) d\alpha \Big],
\end{aligned}$$

where we note that in the expression for $W(X)$, x_k appears once when $i = k$, and in the $i \neq k$ terms it appears a second time when $j = k$. After simplifying, this becomes

$$\begin{aligned}
\frac{\partial W}{\partial x_k}(X) &= \frac{1}{2a} \sum_b P(k, k, b) [(b + a - x_k)^2 - (b - a - x_k)^2] \\
&+ \frac{1}{a} \sum_{j:j < k} \sum_b P(k, j, b) \left[(b + a - x_k)^2 - \left(\frac{x_j - x_k}{2} \right)^2 \right] \\
&+ \frac{1}{a} \sum_{j:j > k} \sum_b P(k, j, b) \left[\left(\frac{x_j - x_k}{2} \right)^2 - (b - a - x_k)^2 \right].
\end{aligned}$$

Viewing the summands above as quadratic functions of the random variable b , we use mean-variance analysis and the fact that $E[b|j, k] = m_{j,k}$ to derive

$$\begin{aligned}
\frac{\partial W}{\partial x_k}(X) &= \frac{1}{2a} P(k, k) [(m_{k,k} + a - x_k)^2 - (m_{k,k} - a - x_k)^2] \\
&+ \frac{1}{a} \sum_{j:j < k} P(k, j) \left[(m_{j,k} + a - x_k)^2 - \left(\frac{x_j - x_k}{2} \right)^2 + \sigma_{j,k}^2 \right] \\
&+ \frac{1}{a} \sum_{j:j > k} P(k, j) \left[\left(\frac{x_j - x_k}{2} \right)^2 - (m_{j,k} - a - x_k)^2 - \sigma_{j,k}^2 \right],
\end{aligned} \tag{30}$$

where $\sigma_{j,k}^2$ is the variance of b conditional on signals j and k . The cross partial with respect to x_i and x_j with $j \neq i$ is then

$$\frac{\partial^2 W}{\partial x_i \partial x_j}(X) = P(i, j) \frac{|x_i - x_j|}{2a}, \tag{31}$$

where we use the fact that strategies are monotone in signal, and the second partial with respect to x_i is

$$\begin{aligned}
& \frac{\partial^2 W}{\partial^2 x_i}(X) \\
&= \frac{1}{2a} P(i, i) [-2(a + m_{i,i} - x_i) + 2(m_{i,i} - x_i - a)] + \frac{1}{a} \sum_{j:j < i} P(i, j) \left[-2(a + m_{i,j} - x_i) + \frac{x_j - x_i}{2} \right] \\
&\quad - \frac{1}{a} \sum_{j:j > i} P(i, j) \left[-2(m_{i,j} - x_i - a) + \frac{x_j - x_i}{2} \right] \\
&= \frac{1}{aP(i)} \left[-2a + \sum_{j:j < i} P(j|i)(m_{i,j} - x_i) - \sum_{j:j > i} P(j|i)(m_{i,j} - x_i) \right] - \sum_{j:j \neq i} P(i, j) \frac{|x_j - x_i|}{2}.
\end{aligned}$$

Therefore, we may decompose the Hessian of W into two matrices, $H = D + E$, where E is a symmetric matrix such that

$$e_{i,j} = P(i,j) \frac{|x_j - x_i|}{2} \text{ for } i \neq j \text{ and } e_{i,i} = - \sum_{j:j \neq i} P(i,j) \frac{|x_j - x_i|}{2}, \quad (32)$$

and D is a diagonal matrix such that

$$d_{i,i} = \frac{1}{aP(i)} \left[-2a + \sum_{j:j < i} P(j|i)(m_{i,j} - x_i) - \sum_{j:j > i} P(j|i)(m_{i,j} - x_i) \right].$$

Because x_i is bounded above by B , it follows that

$$d_{i,i} \leq \frac{1}{aP(i)} [-2a + B + B] < 0,$$

where the inequality follows from our assumption that $a > 2B$. Thus, D is negative definite. We now argue that E is negative semi-definite. Let $t \in \mathfrak{R}^{2K+1}$ be an arbitrary vector, and note that

$$\begin{aligned} \sum_{i,j} t_i t_j e_{ij} &= \sum_i t_i^2 e_{ii} + \sum_i \sum_{j:j \neq i} t_i t_j e_{ij} \\ &= \sum_i t_i^2 \left(\sum_{j:j \neq i} -e_{ij} \right) + \sum_i \sum_{j:j \neq i} t_i t_j e_{ij} \\ &= \sum_{\{i,j\}:i \neq j} (-t_i^2 e_{ij} - t_j^2 e_{ji}) + \sum_{\{i,j\}:i \neq j} (t_i t_j e_{ij} + t_j t_i e_{ji}) \\ &= \sum_{\{i,j\}:i \neq j} e_{ij} (-t_i^2 + 2t_i t_j - t_j^2) \\ &= \sum_{\{i,j\}:i \neq j} -e_{ij} (t_i - t_j)^2 \\ &\leq 0, \end{aligned}$$

where the second equality follows from condition (32), the fourth equality follows from symmetry of E , and the final inequality uses $e_{ij} \geq 0$ when $i \neq j$. Therefore, E is negative semi-definite, and $H = D + E$ is negative definite. We conclude that W is strictly concave. \blacksquare

Proof of Theorem 6: The proof follows from expression (30) after substituting $m_{k,k}$ for x_k . \blacksquare

Proof of Theorem 7: For the zero signal and for all negative signals, fix candidate locations at their socially optimal values, i.e., for all $i \leq 0$, set $x_i = x_i^*$. Consider the welfare maximization problem with the additional constraint that candidates locate at or above the conditional medians corresponding to positive signals:

$$\begin{aligned} &\max_{x_i, i > 0} W(X) \\ \text{s.t.} & \\ &x_i \leq x_j \text{ for all } i < j \\ &x_i = x_i^* \text{ for all } i \leq 0 \\ &x_i \geq m_{i,i} \text{ for all } i > 0. \end{aligned}$$

Because the domain of this problem is convex, it has a unique solution, say \hat{X} . By inequality (??), we know that $\frac{\partial W}{\partial x_i}(M) > 0$ for all signals $i > 0$. From the expression (31), we know that the cross partials of W are positive. Therefore, fixing the candidate location at $m_{i,i}$ following $i > 0$ and fixing locations at \hat{x}_j following signals $j \neq i$, we have... ■

Proof of Theorem 8: We write $W(X; q)$ to bring out the dependence of welfare on correlation given locations X , and we write $m_{i,i}(q)$ to bring out the dependence of the conditional median on correlation. The effect of an increase in correlation q may be decomposed as follows:

$$\begin{aligned} \frac{dW}{dq}(M; q) &= \frac{\partial W}{\partial q}(M; q) + \sum_i \frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q) \\ &= \frac{\partial W}{\partial q}(M; q) + \sum_{i=1}^K \left[\frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q) + \frac{\partial W}{\partial x_{-i}}(M; q) \frac{dm_{-i,-i}}{dq}(q) \right] \\ &= \frac{\partial W}{\partial q}(M; q) + 2 \sum_{i=1}^K \frac{\partial W}{\partial x_i}(M; q) \frac{dm_{i,i}}{dq}(q), \end{aligned}$$

where the third equality follows from symmetry about zero. By Theorem 6 and condition (8), we know that $\frac{\partial W}{\partial x_i}(M; q) > 0$ for all signals $i > 0$. To see that $\frac{dm_{i,i}}{dq} < 0$, note that

$$\begin{aligned} m_{i,i}(q) &= E[b|i, i, q] \\ &= \sum_b b \left[\frac{qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)}{\sum_{b'} [qP(i|b')P(b') + (1-q)P(i|b')P(i|b')P(b')] } \right]. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dm_{i,i}}{dq}(q) &\propto \sum_b b [P(i|b)P(b) - P(i|b)P(i|b)P(b)] \sum_{b'} [qP(i|b')P(b') + (1-q)P(i|b')P(i|b')P(b')] \\ &\quad - \sum_{b'} [P(i|b')P(b') - P(i|b')P(i|b')P(b')] \sum_b b [qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)] \\ &\propto \frac{\sum_b b [P(i|b)P(b) - P(i|b)P(i|b)P(b)]}{\sum_b [P(i|b)P(b) - P(i|b)P(i|b)P(b)]} - \frac{\sum_b b [qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)]}{\sum_b [qP(i|b)P(b) + (1-q)P(i|b)P(i|b)P(b)]} \\ &\propto \frac{\sum_b b [1 - P(i|b)] P(i|b)P(b)}{\sum_b [1 - P(i|b)] P(i|b)P(b)} - \frac{\sum_b b [qP(i|b) + (1-q)P(i|b)P(i|b)] P(b)}{\sum_b [qP(i|b) + (1-q)P(i|b)P(i|b)] P(b)} \\ &= \hat{m}_{i,-i} - m_{ii} \\ &< 0. \end{aligned}$$

To see that $\frac{\partial W}{\partial q}(M; q) < 0$, note that

$$\begin{aligned} W(M; q) &= - \sum_b \sum_i [P(i|b) + (1-q)P(i|b)P(i|b)] P(b) \int_{-a}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \\ &\quad - \sum_b \sum_i \sum_{j>i} (1-q)P(i|b)P(j|b)P(b) \left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right] \end{aligned}$$

$$\begin{aligned}
& + \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \Big] \\
& - \sum_b \sum_i \sum_{j < i} (1 - q) P(i|b) P(j|b) P(b) \left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \right. \\
& \left. + \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial W}{\partial q}(M; q) &= - \sum_b \sum_i P(i|b) (1 - P(i|b)) P(b) \int_{-a}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \\
& + \sum_b \sum_i \sum_{j > i} P(i|b) P(j|b) P(b) \left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right. \\
& \left. + \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \right] \\
& + \sum_b \sum_i \sum_{j < i} P(i|b) P(j|b) P(b) \left[\int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha \right. \\
& \left. + \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \right].
\end{aligned}$$

Given signals i and $j > i$, note that $\frac{m_{i,i}+m_{j,j}}{2} - b < \alpha$ implies $(\alpha + b - m_{j,j})^2 > (\alpha + b - m_{i,i})^2$. Thus, we have

$$- \int_{-a}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \leq - \int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha - \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha.$$

Similarly, given signals i and $j < i$, $\alpha < \frac{m_{i,i}+m_{j,j}}{2} - b$ implies $(\alpha + b - m_{j,j})^2 < (\alpha + b - m_{i,i})^2$. Thus, we have

$$- \int_{-a}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha \leq - \int_{-a}^{[m_{ii}+m_{jj}]/2-b} \frac{(\alpha + b - m_{jj})^2}{2a} d\alpha - \int_{[m_{ii}+m_{jj}]/2-b}^a \frac{(\alpha + b - m_{ii})^2}{2a} d\alpha.$$

Finally, we conclude that $\frac{\partial W}{\partial q}(M; q) \leq 0$, which delivers the desired result. \blacksquare

References

- [1] Budge I., H.D. Klingemann, A. Volkens, J. Bara and E. Tanenbaum (2001): *Mapping Policy Preferences; Estimates for Parties, Electors, and Governments 1945-1998*, Oxford: Oxford University Press.
- [2] Chan J. (2001): "Electoral Competition with Private Information," mimeo.

- [3] Committee on Political Parties (1950): "Toward a More Responsible Two-Party System," *American Political Science Review* 44: supplement.
- [4] Downs A. (1957): *An Economic Theory of Democracy*, New York: Harper and Row.
- [5] Eisinger R.M. (2003): *The Evolution of Presidential Polling*, Cambridge: Cambridge University Press.
- [6] Emery C. (1994): "Public Opinion Polling in Canada" Background Paper 371E, Parliamentary Research Branch, Library of the Canada Parliament.
- [7] Gelman A. and G. King (1993): "Why Are American Presidential Election Campaign Polls So Variable When Votes are So Predictable?," *British Journal of Political Science* 23: 409-451.
- [8] Hotelling H. (1929): "Stability in Competition," *Economic Journal* 39: 41-57.
- [9] Ledyard J. (1989): "Information Aggregation in Two-Candidate Elections," in *Models of Strategic Choice and Politics*, P. Ordeshook ed., Ann Arbor: University of Michigan Press.
- [10] Medvic S. (2001): *Political Consultants in U.S. Congressional Elections*, Columbus: Ohio State University Press.
- [11] Merrill S. and B. Grofman (1999): *A Unified Theory of Voting*, Cambridge: Cambridge University Press.
- [12] Ottaviani M. and P. Sørensen (2002): "Forecasting and Rank Order Contests," mimeo.
- [13] Page B. (1978): *Choices and Echoes in Presidential Elections*, Chicago: University of Chicago Press.
- [14] Poole K. and H. Rosenthal (1997): *Congress: a Political Economic History of Roll Call Voting*, New York: Oxford University Press.