

# Information-based trade<sup>1</sup>

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## **Abstract**

We study the possibility of trade for purely informational reasons. We depart from previous analyses (e.g. Grossman and Stiglitz 1980 and Milgrom and Stokey 1982) by allowing the final payoff of the asset being traded to depend on an action taken by its eventual owner. We characterize conditions under which equilibria with trade exist. We demonstrate that our model also applies to a portfolio allocation setting, and relate our conditions for trade to standard measures of asset risk.

*JEL codes:* D8, G1.

# 1 Introduction

Following Grossman and Stiglitz (1980) and Milgrom and Stokey (1982), economists have reached a consensus that under many circumstances it is impossible for an individual to profit from superior information.<sup>1</sup> This result is often described as the “no trade” or “no speculation” theorem. The underlying argument is, at heart, straightforward. If a buyer is prepared to buy an asset from a seller for price  $p$ , then the buyer must believe that, conditional on the seller agreeing to the trade, the asset value exceeds  $p$  in expectation. But conversely, knowing this the seller is at least as well off keeping the asset.

This insight has had enormous consequences for financial economics. Almost all observers of financial markets regard trade for informational reasons — *information-based trade* — as a key motive for trade. To generate information-based trade, the vast majority of papers studying financial markets introduce “noise traders” who trade for (typically exogenous) non-informational reasons.<sup>2</sup> Provided strategic agents are unable to observe the volume of noise trader activity information-based trade is possible. However, the modeling device of noise traders has often been criticized, as is well-illustrated by Dow and Gorton’s (2008) survey. Moreover, a significant amount of trade takes place directly between relatively sophisticated parties — a setting that lies outwith the standard noise-trader framework.<sup>3</sup>

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<sup>1</sup>See also Kreps (1977), Tirole (1982), Holmström and Myerson (1983) and Fudenberg and Levine (2005).

<sup>2</sup>See, for example, Kyle (1985), and Glosten and Milgrom (1985).

<sup>3</sup>For example, many trades occur in “upstairs” markets, i.e., are trades in which “buyers and sellers negotiate in the ‘upstairs’ trading rooms of brokerage firms” (Booth *et al*, 2002). Identifying upstairs trades is relatively hard, but using detailed data from Finland Booth *et al* report that upstairs trades account for 50% of total volume. In the last few years “dark liquidity pools” (Liquidnet and Pipeline are well-known examples) have captured a significant share of trade volume, particularly for midcap stocks, and as is the case for upstairs markets are used only by relatively sophisticated traders.

In this paper we develop a distinct and hitherto neglected reason for trade between differentially informed parties. In many cases the holder of the asset must make a decision that affects its value. If better information leads to superior decisions, then the information released in trade is socially valuable. This possibility, which is implicitly ruled out in Milgrom and Stokey's otherwise general framework, is enough to generate trade even without noise traders.

#### AN EXAMPLE

The intuition for our results is best illustrated by an example. A risk neutral agent (the *seller*) owns an asset that he can potentially trade with a second risk neutral agent (the *buyer*). The asset's payoff depends on two factors: an underlying but currently unobservable fundamental  $\theta \in \{a, b\}$ , and what the eventual asset owner chooses to do with the asset. The best action for the asset owner to take depends on  $\theta$ . If  $\theta = a$  the best action is  $A$ , and the asset is worth 2 if this action is taken. If  $\theta = b$  the best action is  $B$ , and the asset is worth 1 if it is taken. The asset is valueless if any action other than the ( $\theta$ -contingent) best action is taken. The buyer and seller have the same "skill" in taking actions  $A$  and  $B$ , so that the action-contingent asset payoffs for *both* parties are as given above. We discuss various interpretations below; an immediate one is that the asset is a debt claim and the action is a restructuring decision (e.g., liquidation vs. reorganization).

The unconditional probability of fundamental  $a$  is  $1/2$ . Both the buyer and seller receive partially informative signals about the true fundamental  $\theta$ . Conditional on the fundamental the signals are distributed independently and identically. Specifically, if the true fundamental is  $a$  (respectively,  $b$ ) then each party observes signal  $s^a$  (respectively,  $s^b$ ) with probability  $3/4$ .

Consider the following trading game: after observing their signals, the buyer and seller simultaneously announce whether they are prepared to trade the asset at an

(exogenously fixed) price  $p = 0.8$ . We claim the following is an equilibrium: the buyer offers to buy independent of his signal, and the seller offers to sell if and only if he observes signal  $s^b$ .

First, consider the situation faced by the seller. If he ends up with the asset, he must decide what to do using only his own information. As such, if he sees signal  $s^a$  and does not sell, his expected payoff is  $3/2$ , while if he sees signal  $s^b$  and does not sell his expected payoff is  $3/4$ .<sup>4</sup> Consequently, after signal  $s^b$  the seller prefers to sell at a price  $p = 0.8$  rather than keep the asset; and after signal  $s^a$ , prefers to keep the asset rather than sell at this price.

Next, we show the buyer is prepared to buy at price  $p = 0.8$ . Note that in equilibrium the buyer learns the seller's signal when he acquires the asset, since in equilibrium the seller only accepts the buyer's offer when he observes signal  $s^b$ . So on the one hand, if the buyer observes signal  $s^a$  he regards  $\theta = a$  and  $\theta = b$  as equally likely, since he knows the seller saw  $s^b$ . Consequently he will choose action  $A$ , giving an expected payoff of  $2 \times 1/2 = 1$ . On the other hand, if the buyer observes signal  $s^b$ , then given the seller also observed signal  $s^b$  the buyer's probability assessment that  $\theta = b$  is  $9/10$ .<sup>5</sup> Given this, he chooses action  $B$ , yielding an expected payoff of  $1 \times 9/10 = 9/10$ . In both cases, the buyer's expected payoff exceeds the price  $p = 0.8$ . As such, the behavior described is indeed an equilibrium.

In this example both parties are strictly better off under the trade. Moreover, they are both better off even after conditioning on any information they acquire in equilibrium. The reason this is possible is that the asset value endogenously depends

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<sup>4</sup>Note that since  $3/4 \times 1 > 1/4 \times 2$ , action  $B$  is the better action to take if the only information available is that one of the signals is  $s^b$ .

<sup>5</sup>Specifically, the buyer's posterior belief is given by:

$$\Pr(b|s^b s^b) = \frac{\Pr(b) \Pr(s^b|b)^2}{\Pr(a) \Pr(s^b|a)^2 + \Pr(b) \Pr(s^b|b)^2} = \frac{(\frac{3}{4})^2}{(\frac{1}{4})^2 + (\frac{3}{4})^2} = \frac{9}{10}.$$

on the information possessed by its owner. In the example, trade transfers the asset from the seller when he observes signal  $s^b$  to the buyer. Trade creates value because it leads to a better decision after the signal pair  $s^b s^a$ . Specifically, after these signals the seller would take action  $B$  because he observes only signal  $s^b$ ; while the value-weighted best action is  $A$ , and the buyer takes this action. In essence, trade transfers the asset from an agent who is likely to make the wrong decision to one who is more likely to make the right decision. In contrast, in Grossman and Stiglitz (1980) and Milgrom and Stokey (1982) asset holders have no decision to make since the final asset payoffs are exogenous to the information possessed by its owner.

## APPLICATIONS

A number of different situations are captured by this model:

1. Most directly, the asset is a controlling equity stake in a firm; or (as noted above) a debt claim that needs restructuring.
2. The asset is a large but non-controlling block of shares in a firm with an upcoming shareholder vote;<sup>6</sup> or one of several debt claims in a firm with an upcoming bankruptcy vote.
3. The asset is an equity or debt claim with no direct decision rights, but the holder must still decide how to allocate the remainder of his portfolio. Specifically, suppose now that the seller and buyer are risk averse, and that the asset's return distribution differs across fundamentals  $a$  and  $b$ . Depending on his beliefs about the fundamental the asset holder chooses different portfolio allocations. Thus

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<sup>6</sup>While private benefits such as synergies can also explain trade of a controlling equity stake, this explanation is less readily applicable in the case of non-controlling blocks. That is, while the owners of such blocks can affect a firm's decisions by choosing how to vote, it is less clear how they can derive substantial private benefits (at least without engaging in self-dealing).

in place of an action directly affecting the asset's payoff this setting features an action (the portfolio choice) that affects the asset holder's utility. We return to this application in much greater detail in Section 5 below.

#### PAPER OUTLINE

We describe our relation to the existing literature immediately below. In Section 2 we present our general model, which closely resembles the example above but with the binary action set and signal space replaced with an arbitrary action set and continuous signal space. In Section 3 we derive conditions that are required for trade to occur *regardless* of the trading mechanism used. In Section 4 we derive necessary and sufficient trade conditions by studying one very simple trading mechanism. In Section 5 we apply our results to the case in which agents trade an asset over which they have no direct control, but instead choose portfolio allocations. We relate our previously established trade conditions to standard measures of asset risk. Section 6 concludes.

#### RELATED LITERATURE

A number of classic papers (notably, Hirshleifer 1971) note the distinction between information in an exchange economy and information in a production economy. However, the literature on the possibility of trade between differentially and privately informed parties has focused almost exclusively on information in an exchange economy. In particular, the seminal papers of Grossman and Stiglitz (1980) and Milgrom and Stokey (1982) show that under many circumstances trade is impossible in such an environment. Milgrom and Stokey's "no trade" or "no speculation" result rests on two assumptions: Pareto optimality of the initial allocation, and concordancy of beliefs, in the sense that agents agree on how to interpret future information. A subsequent literature has explored conditions under which the "no trade" conclusion

does not hold. The literature is too large to adequately survey. Representative approaches include departing from the common prior assumption, as in Morris (1994) and Biais and Bossaerts (1998), and thus breaking belief concordancy; departing from Pareto optimality, as in Dow and Gorton (1995), who assume that some agents can trade only a subset of assets; and introducing multiple trading rounds, as Grundy and McNichols (1998)<sup>7</sup> do when they show that both belief concordancy and Pareto optimality may fail at the intermediate date of a three-period model.<sup>8</sup>

None of the above papers study the possibility of trade for purely informational reasons in an economy in which asset owners must decide how to use their assets. To the best of our knowledge the only previous consideration of this case is a chapter of Diamond's (1980) dissertation.<sup>9</sup> He derives conditions under which a rational expectations equilibrium (REE) with trade exists when there are two types of agents: one type is uninformed, while the other type observes a noisy signal. The main differences between our paper and his are that (i) we study trade between agents who both possess information, (ii) we show that as a consequence, information is never fully revealed, and (iii) instead of restricting attention to the competitive (REE) outcome, in the spirit of Milgrom and Stokey (1982) we allow for all possible trading mechanisms. Moreover, Diamond's assumption that one side of the trade is completely uninformed means that assets always flow from the less to the more informed party.<sup>10</sup> In contrast, when both parties to the trade have some information, assets can flow to

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<sup>7</sup>Related, see also Coury and Easley (2006).

<sup>8</sup>One can also avoid the no-trade conclusion by using non-standard preferences: see, e.g., Halevy (2004).

<sup>9</sup>Less closely related is a recent working paper of Tetlock and Hahn (2007), who show that a decision maker would be willing to trade and act as a loss-making market maker in "weather" securities (or more generally, securities whose value is exogenous to the decision).

<sup>10</sup>Diamond does consider an equilibrium in which uninformed agents end up holding the asset. However, to support the equilibrium he must assume that uninformed agents learn only from the price at which the trade takes place, and not from the volume of trade.



the party with lower quality information.

Finally, papers such as Dow and Gorton (1997) and Guembel and Goldstein (2006) study models in which investors trade with the understanding that the equilibrium price affects real decisions and hence the profitability of their trades. However, these models rely on noise-traders to generate trade, and say nothing about the possibility of trade when there is no exogenous source of noise.

In this paper we analyze the degree to which efficiency gains arising from additional information make information-based trade possible. However, before proceeding to the details of our analysis, we wish to make the following clear: we are *not* arguing that trade is a superior mechanism relative to other alternatives. Instead, we view trade as a particular information-sharing mechanism that deserves focused attention: it is widely observed, has long interested economists, and has many appealing features.

## 2 The model

Our model is closely related to the opening example. As in the example, there are two risk neutral agents,<sup>11</sup> who we refer to as a seller (agent 1) and a buyer (agent 2). The seller owns an asset. The payoff from the asset depends on the combination of the action taken by the asset-owner and the realization of some fundamental  $\theta \in \{a, b\}$ . Neither agent directly observes the fundamental  $\theta$ , but before meeting, both agents  $i = 1, 2$  receive noisy and partially informative signals  $s_i$ . Whereas in the example signals were binary, in our main model they have full support in  $\mathbb{R}$ .

The eventual asset owner must decide what action to take. Regardless of whether the asset-owner is agent 1 or 2, the range of available actions is given by a compact set  $\mathcal{X}$ , with a typical element denoted by  $X$ . (In the opening example,  $\mathcal{X}$  is simply

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<sup>11</sup>Our analysis also covers the case of agents with constant absolute risk aversion preferences. We consider this case in Section 5 below.

the binary set  $\{A, B\}$ .) We write  $v(X, \theta)$  for the payoff when action  $X$  is taken and the fundamental is  $\theta$ , where  $v(\cdot, \theta)$  is continuous as a function of  $X$ . We emphasize that the asset payoff is independent of the identity of the asset-owner — both agents 1 and 2 are equally capable of executing all actions in  $\mathcal{X}$ .

#### PRE-TRADE INFORMATION

The information structure of the economy is described by a probability measure space  $(\Omega, \mathcal{F}, \mu)$ , where  $\Omega = \{a, b\} \times \mathbb{R}^2$  and  $\mathcal{F}$  is the  $\sigma$ -algebra  $\{\{a\}, \{b\}, \{a, b\}\} \times \mathcal{B}^2$ . (Throughout, we denote the Borel algebras of  $\mathbb{R}$  and  $\mathbb{R}^2$  by  $\mathcal{B}$  and  $\mathcal{B}^2$  respectively.) We write a typical state as  $\omega = (\theta, s_1, s_2)$ , where  $\theta$  is the fundamental,  $s_1$  is the signal observed by the seller (agent 1) and  $s_2$  is the signal observed by the buyer (agent 2). For  $i = 1, 2$  and  $\theta = a, b$  let  $\eta_i^\theta : \mathcal{B} \rightarrow \mathbb{R}$  be the conditional distribution of  $s_i$  given  $\theta$ . We write  $F_i^\theta$  for the associated distribution functions, and make the following distributional assumptions. (I) The signals  $s_1$  and  $s_2$  are conditionally independent given  $\theta$ . (II) For  $i = 1, 2$  and  $\theta = a, b$  the conditional distribution  $\eta_i^\theta$  has full support; and has a density, which we denote  $f_i^\theta$ . (III) For  $i = 1, 2$  signal  $s_i$  satisfies the strict monotone ratio likelihood property (MLRP), i.e.,  $L_i(s_i) \equiv \frac{f_i^a(s_i)}{f_i^b(s_i)}$  is strictly increasing in  $s_i$ . Moreover, we assume that the likelihood ratio is unbounded, i.e.,

$$L_i(s_i) \rightarrow 0, \infty \text{ as } s_i \rightarrow -\infty, +\infty. \quad (1)$$

That is, there are extreme realizations of each agent's signal that are very informative — even if an agent's signal is generally uninformative. (We stress that none of the results of Section 3 depend on either the existence of densities or the assumption of unbounded likelihood ratios. See also the discussion on page 15.)

Agent  $i$  directly observes only his own signal. Formally, the information of agents  $i = 1, 2$  before trade is given by the sub  $\sigma$ -algebras  $\mathcal{F}_1 = \{a, b\} \times \mathcal{B} \times \mathbb{R}$  and  $\mathcal{F}_2 = \{a, b\} \times \mathbb{R} \times \mathcal{B}$ .

## TRADE

An allocation in our economy is a pair of mappings  $\kappa : \Omega \rightarrow \{1, 2\}$  and  $\pi : \Omega \rightarrow \mathbb{R}$  where  $\kappa$  specifies which agent owns the asset, and  $\pi$  specifies a transfer from agent 2 to agent 1. Since neither agent observes the fundamental  $\theta$  both  $\kappa$  and  $\pi$  must be measurable with respect to the  $\sigma$ -algebra  $\{a, b\} \times \mathcal{B}^2$ . Let  $(\hat{\kappa}, \hat{\pi})$  denote the initial allocation, in which agent 1 owns the asset and no transfer takes place:  $(\hat{\kappa}, \hat{\pi}) \equiv (1, 0)$ .

A *trade* is an allocation  $(\kappa, \pi)$  with  $\kappa(\omega) = 2$  with strictly positive probability. To rule out trades in which both parties are exactly indifferent between trading and not trading the asset we assume that whenever the asset changes hands its final value is reduced by  $\delta > 0$ .

## POST-TRADE INFORMATION

After trade, agent  $i$ 's information is given by a  $\sigma$ -algebra  $\mathcal{F}_i^{\kappa, \pi} \subset \mathcal{F}$ , where  $\mathcal{F}_i \subset \mathcal{F}_i^{\kappa, \pi} \subset \{a, b\} \times \mathcal{B}^2$ . That is, each agent remembers his own signal, and learns at most the other agent's signal.

Each agent observes the outcome of the trade, and updates his information accordingly. Formally,  $\kappa$  and  $\pi$  are  $\mathcal{F}_i^{\kappa, \pi}$ -measurable. Moreover, in principle it is possible that the trade mechanism entails the release of additional information to agent  $i$ . In this case, the  $\sigma$ -algebra generated by  $(\kappa, \pi)$  would be a strict sub-algebra of  $\mathcal{F}_i^{\kappa, \pi}$ .

An important object in our analysis is the probability that an agent attaches to fundamental  $a$  (or  $b$ ) conditional on some information. Notationally, for any  $\sigma$ -algebra  $\mathcal{G}$  let  $Q(\omega; \mathcal{G})$  denote the conditional probability of  $\{a\} \times \mathbb{R}^2$  in state  $\omega$  relative to  $\mathcal{G}$ .

## ENDOGENOUS ASSET VALUES

The eventual asset owner must select an action  $X \in \mathcal{X}$  without knowing the realization of fundamental  $\theta$ . For each candidate action  $X$  he can evaluate the expected payoff under that action. We denote this expected payoff by  $V(q; X) \equiv qv(X, a) + (1 - q)v(X, b)$ , where  $q$  denotes the probability the agent places on fundamental  $a$ . Since the asset owner chooses the action with the highest expected payoff, his valuation of the asset is given by

$$V(q) \equiv \max_{X \in \mathcal{X}} V(q; X). \quad (2)$$

Note that  $V$  is continuous over  $[0, 1]$ , and hence bounded.<sup>12</sup>

## EX POST INDIVIDUALLY RATIONAL TRADE

Our primary goal is to characterize when trade can — and cannot — occur for purely informational reasons. The answer to this question clearly depends to some extent on the institutional environment. However, it is also clear that we want our results to be as independent as possible of *a priori* assumptions about the trading environment.

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<sup>12</sup>We have assumed that the fundamental  $\theta$  is binary-valued. The significance of this assumption is that it allows us to define the asset value  $V$  as a function of a one-dimensional summary statistic, namely the probability  $q$  that the fundamental is  $a$ . That is, uncertainty is unidimensional. Unidimensionality greatly facilitates the derivation of sufficient conditions for trade in Section 4. We conjecture the necessary conditions of Section 3 would extend to more general state spaces.

It should also be noted that it is possible to obtain a similarly tractable unidimensional framework with a richer set of fundamentals, though at the cost of introducing more assumptions on the asset payoff functions  $v(X, \theta)$ . For example, one could allow the fundamental  $\theta$  to be drawn from an arbitrary subset of  $\mathbb{R}$ , but restrict the asset payoff to take the form  $v(X, \theta) = K(X) + M(X)\theta$  for an arbitrary pair of continuous functions  $K$  and  $M$ . In this case, the expected asset payoff given action  $X$  is a linear function of the expected value of  $\theta$ , and so one can define an analogous function to  $V$  that depends only on a one-dimensional variable (i.e., the expected value of  $\theta$  as opposed to the probability of  $a$ ).

To meet these objectives, we begin by establishing necessary conditions for trade to occur in a very wide class of trading mechanisms. The only condition we impose is that trades must be *ex post individually rational*. That is, both agents 1 and 2 must prefer the post-trade outcome to the original allocation (in which agent 1 owns the asset), even after conditioning on the information they acquire in equilibrium. This condition must be met state-by-state. We adopt this requirement for two reasons. First, it is a demanding condition to satisfy, and so biases our analysis against generating trade. Second, it is used in many prior analyses. In particular, it is equivalent to Milgrom and Stokey's (1982) requirement of common knowledge of gains from trade;<sup>13</sup> and is part of the definition of a rational expectations equilibrium.

Formally, a trade  $(\kappa, \pi)$  is *ex post* individually rational (IR) if

$$\pi(\omega) \geq V(Q(\omega; \mathcal{F}_1^{\kappa, \pi})) \quad (3)$$

$$V(Q(\omega; \mathcal{F}_2^{\kappa, \pi})) - \delta - \pi(\omega) \geq 0 \quad (4)$$

almost always when the buyer gets the asset, i.e., for almost all  $\omega$  such that  $\kappa(\omega) = 2$ . Note that the information used by agent  $i$  to evaluate the trade is  $\mathcal{F}_i^{\kappa, \pi}$ , i.e., the information of agent  $i$  after trade. The analogous conditions for states  $\omega$  in which no trade occurs are that  $\pi(\omega) \geq 0$  and  $0 \geq \pi(\omega)$  almost always when the seller keeps the asset, i.e.,  $\kappa(\omega) = 1$ . It follows trivially that *ex post* IR implies that no money changes hands in almost all states  $\omega$  in which the seller keeps the asset, i.e.,  $\pi(\omega) = 0$  when  $\kappa(\omega) = 1$ .

For any allocation let  $\Omega^T$  denote the states in which the buyer acquires the asset

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<sup>13</sup>In Milgrom and Stokey, agent  $i$  evaluates the trade according to "his information at the time of trading, including whatever he can infer from prices or from the behavior of other traders" (page 19). We take this information to include at least the information revealed by the post-trade allocation. In Milgrom and Stokey's framework, there would still be no trade even if one instead assumed that agent  $i$  possessed coarser information. In contrast, in our model coarsening the information that agent  $i$  uses to evaluate trade affects trade opportunities.

(i.e.,  $\kappa(\omega) = 2$ ) and the *ex post* IR conditions (3) and (4) hold. Note that  $\mu(\Omega^T) > 0$  in any *ex post* IR trade.

#### PARETO OPTIMALITY OF THE ORIGINAL ALLOCATION

Milgrom and Stokey’s “no speculation” theorem establishes that trade cannot occur purely for information-based reasons. Of course, this in no way affects the possibility of trade for risk-sharing reasons. As such, Milgrom and Stokey’s result is predicated on the Pareto optimality of the pre-trade state-contingent allocation.

In our setting, both agents are risk neutral, and are equally capable of executing any action  $X \in \mathcal{X}$ . As such, the *only* possible motivation for trade is the differential information of the two parties. Formally, since risk-sharing motivations are absent, any state-contingent allocation is Pareto optimal. Of course, this ignores the fact that agents 1 and 2 potentially have different information, and so take different actions. However, trade motivated by such considerations is precisely information-based trade, and is the main object of our analysis.

### 3 Necessary conditions for trade

In this section we establish necessary conditions for trade to take place, all of which must hold *regardless* of the trading mechanism employed. First:

**Proposition 1.** *No ex post IR trade exists if  $V$  is monotone.*

All proofs are in the appendix. The intuition is most easily understood by considering again the opening example, which is displayed graphically in Figure 1. Recall that in the example trade occurs whenever the seller sees signal  $s^b$ . The buyer’s valuation when trade occurs is driven by the probability he places on fundamental  $a$ , i.e.,  $\Pr(a|s^b s^a)$  or  $\Pr(a|s^b s^b)$ , depending on his own signal realization. Trade is possible

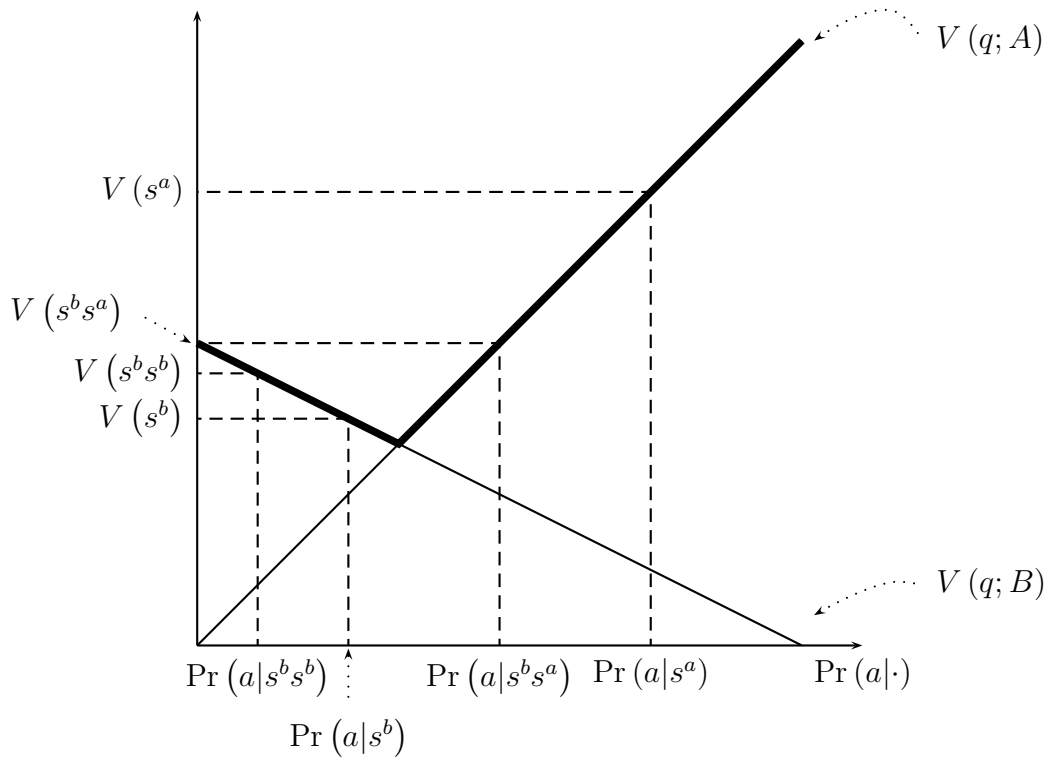


Figure 1: The graph displays  $V(q; X)$  for the opening example: the action set is  $\mathcal{X} = \{A, B\}$  and both the buyer and seller observe signals drawn from  $\{s^a, s^b\}$ . The bold line is the upper envelope of these two functions, and corresponds to the function  $V(q)$ .

because the value of the asset given these probabilities exceeds the value of the asset when the probability of fundamental  $a$  is  $\Pr(a|s^b)$ , which is the information the seller has. Since  $\Pr(a|s^b s^a) > \Pr(a|s^b) > \Pr(a|s^b s^b)$ , trade is clearly only possible in this example if  $V$  is non-monotone. Proposition 1 extends this observation to our main model, and to any trading mechanism.

As an immediate corollary of Proposition 1 we obtain:

**Corollary 1.** *Trade is possible only if (i) there is no dominant action, i.e.,  $\nexists X \in \mathcal{X}$  such that  $v(X, \theta) \geq v(X', \theta)$  for all  $X' \neq X$  and  $\theta = a, b$ ; and (ii) there is no dominant fundamental, i.e.,  $\nexists \theta \in \{a, b\}$  such that  $v(X, \theta) \geq v(X, \theta')$  for all  $X \in \mathcal{X}$  and  $\theta' \neq \theta$ .*

A second key property of the trade equilibrium in the opening example is that the buyer learns the seller's signal when trade occurs. The fact that the buyer learns everything about the seller's signal is an artifact of the binary nature of signals in the example. In general, however, a necessary condition for trade is that the buyer learns *something* about the seller's signal when trade occurs:

**Proposition 2.** *There is no ex post IR trade in which the buyer learns nothing whenever he acquires the asset.*

Proposition 2 says that trade is not possible if it does not convey some information to the buyer. This conclusion is very much in line with the existing no-trade literature. At the same time, and as our opening example makes clear, trade is at least sometimes possible if it enables the buyer to learn the seller's signal.

To understand Proposition 2, it again helps to reconsider the opening example. Suppose that trade occurred in this example *without* the buyer learning anything about the seller's signal. For specificity, suppose further that trade only occurs when the buyer sees signal  $s^a$ .<sup>14</sup> Recall that agents always learn at least the information

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<sup>14</sup>Similar arguments apply for the cases of trade following signal  $s^b$ , and trade after both buyer



revealed by the trade allocation. Consequently, for the buyer not to learn anything trade must occur after both signal pairs  $s^a s^a$  and  $s^b s^a$ . So when trade occurs, the buyer places probability  $\Pr(a|s^a)$  on the fundamental being  $a$ ; while the seller places probability  $\Pr(a|s^a s^a)$  or  $\Pr(a|s^b s^a)$  on the fundamental being  $a$ , depending on his own signal realization.

To see why this information is inconsistent with trade, look again at Figure 1. The asset value  $V$  is single-troughed as a function of the probability  $q$  of fundamental  $a$ . Since the buyer does not learn the seller's signal, at one state in which trade occurs the seller places a higher probability on fundamental  $a$  than does the buyer, while in another state the seller places a lower probability on fundamental  $a$ . Specifically,  $\Pr(a|s^a s^a) > \Pr(a|s^a) > \Pr(a|s^b s^a)$ . Given the shape of  $V$  it follows that at least one of  $V(\Pr(a|s^a s^a))$  and  $V(\Pr(a|s^b s^a))$  exceeds  $V(\Pr(a|s^a))$ . But in words, this comparison says that at at least one of  $s^a s^a$  and  $s^b s^a$  the seller's valuation exceeds the buyer's valuation — contradicting the trade conditions.

The key step in this argument is the shape of the  $V$  function. Since  $V$  is the upper envelope of functions  $V(q; X)$ , each of which is linear,  $V$  itself is convex. An immediate consequence is:

**Lemma 1.**  *$V$  is a single-troughed function.*

The proof of Proposition 2 follows from the shape of  $V$ , and is along the same lines as the above discussion of the example. The main complication in the formal proof is the need to form conditional probabilities for arbitrary information possessed by the seller. At the same time, the proof is simplified somewhat by our assumption of unbounded likelihood ratios (see (1)). We emphasize, however, that (as the example illustrates) this property is not essential for the result, and a proof for the case of bounded likelihood ratios is contained in an earlier working paper.

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signal realizations.

For our next two results, it is useful to separate the benefits and costs of trade. *Ex post* IR implies

$$\int_{\Omega^T} (V(Q(\omega; \mathcal{F}_2^{\kappa, \pi})) - V(Q(\omega; \mathcal{F}_1^{\kappa, \pi}))) \mu(d\omega) \geq \mu(\Omega^T) \delta. \quad (5)$$

The lefthand side is the benefit of trade. Since the buyer's information in state  $\omega$  is different from the seller's, he potentially takes a different action. This causes the value of the asset when owned by the buyer to potentially diverge from the value of the asset owned by the seller, in spite of their equal ability to execute all actions  $X \in \mathcal{X}$ . The righthand side is the direct cost of trade, i.e., the trade cost  $\delta$  multiplied by the probability of trade occurring.

An almost immediate consequence of (5) is:

**Proposition 3.** *Suppose an ex post IR trade exists. Then there exists a non-null subset of the trade set  $\Omega^T$  in which the buyer's action differs from the action the seller would take if he controlled the asset in the same state.*

Note that Proposition 3 is also a corollary of Milgrom and Stokey's main result.

Proposition 3 says that trade is associated with a change in action. Two possible applications include the role of vulture investors in debt restructuring, and corporate raiders. With regard to the former, it is widely perceived that vulture investors' behavior in restructuring negotiations differs from that of the original creditors (see, e.g., Morris 2002). With regard to the latter, there is evidence that large scale layoffs and divestitures follow takeovers (see, e.g., Bhagat *et al* 1990).

A second implication of inequality (5) is that as the seller's information becomes infinitely accurate the probability of trade converges to zero. The reason is that as the seller's information quality grows his own signal almost perfectly reveals the fundamental  $\theta$  in most states, and so the seller takes the full-information optimal action. This effectively eliminates the gains from trade. Since the direct costs of trade are fixed by the parameter  $\delta$ , the probability of trade must approach zero.

Formally, the seller’s signal is high quality if the likelihood ratio  $L_1$  of the signal is either very low or very high with high probability:

**Proposition 4.** *Consider a sequence of economies, indexed by  $n$ , that are identical apart from the conditional distribution of the seller’s signal,  $\eta_{1(n)}^\theta$ , along with a corresponding sequence of ex post IR trade sets  $\Omega_{(n)}^T$ . Suppose the quality of the seller’s signal becomes arbitrarily good as  $n \rightarrow \infty$ , in the sense that for any  $\varepsilon > 0$  and  $\theta = a, b$*

$$\eta_{1(n)}^\theta (\{s_1 : L_{1(n)}(s_1) \in [\varepsilon, 1/\varepsilon]\}) \rightarrow 0.$$

*Then the probability of trade converges to zero, i.e.,  $\mu(\Omega_{(n)}^T) \rightarrow 0$ .*

Similar to Proposition 4, one can show that if the buyer’s signal becomes arbitrarily uninformative then the probability of trade likewise converges to zero.

Recall that Proposition 2 says that the buyer must learn something about the realization of the seller’s signal if trade is to occur. However, the buyer must have information of his own to complement information he acquires from the seller. That is, if instead the seller’s information is much more informative than the buyer’s, the buyer’s information adds almost nothing and the above observations imply that the probability of trade is very low.

## 4 Sufficient conditions for trade

Proposition 1 establishes that trade is possible only if the asset value  $V$  is non-monotone in the probability of fundamental  $a$ . Whether this condition is also sufficient to generate trade depends, in part, on the trading mechanism used. In this section we show that non-monotonicity of  $V$  is sufficient for trade in at least some trading mechanisms. We do so using a constructive proof for arguably the simplest mechanism possible: (1) a non-strategic third-party — a “broker” — sets a price  $p$ , and then (2)

the buyer and seller simultaneously and publicly announce whether they wish to trade at price  $p$ .

**Proposition 5.** *Suppose  $V$  is non-monotone and the third-party posted price mechanism is used. Choose any price  $p \in (\min V, \min \{V(0), V(1)\} - \delta)$ . There exists an equilibrium of the following form: the seller offers to sell when he sees a signal  $s_1 \in S_1^T \equiv [\underline{s}_1, \bar{s}_1]$ , and the buyer offers to buy if he sees a signal  $s_2 \in S_2^T \equiv \mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)$ . The ex post IR constraints are satisfied in equilibrium.*

Together, Propositions 1 and 5 establish that non-monotonicity of  $V$  is both *necessary* and *sufficient* for trade.

In the equilibrium of Proposition 5, the buyer offers to buy whenever his signal is either high or low, that is, when it is relatively informative of the fundamental. Given that  $V$  is non-monotone and single-troughed (see Figure 1), the buyer's valuation of the asset is relatively high at such signals. Similarly, the seller offers to sell when he sees an intermediate signal, that is, a signal that is relatively uninformative about the fundamental. Given the shape of  $V$  the seller's valuation is relatively low at such signals.

In equilibrium, trade transfers control of the asset from an agent who has received an uninformative signal to one who has received an informative signal. Moreover, because of its contingent nature trade also reveals information about the agents' signals to each other. Specifically, when the seller retains the asset he learns whether or not the buyer's signal is in  $S_2^T$ ; and when the buyer acquires the asset, he learns that the seller's signal is in  $S_1^T$ .

Proposition 5 establishes the existence of a continuum of equilibria, indexed by the trade price  $p$ . Comparing the lowest price  $p = \min V$  to the highest price  $p = \min \{V(0), V(1)\} - \delta$ , the buyer's demand (i.e., the probability of accepting the price) decreases (from 1 to 0), while the seller's supply increases (from 0 to 1). That is, the comparative static across equilibria generates an downwards sloping demand curve

and an upwards sloping supply curve.<sup>15</sup>

An important implication of the no-trade theorems established in the existing literature is that economic agents would not spend resources to acquire information. In contrast, our next result shows that this is not true in our model.<sup>16</sup> The key reason is, of course, that information is valuable. The non-trivial aspect of the result consists of showing that an agent's information is valuable above-and-beyond the information he acquires from the other agent in the course of trade.

**Proposition 6.** *Suppose the buyer and seller must each incur a cost  $k > 0$  in order to observe their signals. Fix any price  $p \in (\min V, \min \{V(0), V(1)\} - \delta)$ . Provided the information acquisition cost  $k$  is sufficiently small there exists an equilibrium of the third-party posted price mechanism in which both the buyer and seller acquire their signals and trade occurs with positive probability.*

#### ALTERNATE MECHANISMS

The third-party posted price mechanism we considered above describes many trading environments well. For example, both buyers and sellers take the price as exogenous when they submit market orders; in upstairs trades, in which the upstairs broker proposes the price; and in crossing networks (POSIT is a well-known example) in which the price is determined elsewhere.<sup>17</sup>

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<sup>15</sup>Because of the interdependency between the buyer and seller, more conditions would be required to establish that the demand (respectively, supply) curve is monotonically downwards (respectively, upwards) sloping.

<sup>16</sup>See, for example, Berk (1997) and Jackson and Peck (1999) for models in which agents pay to acquire a costly signal even in the absence of exogenous noise.

<sup>17</sup>In the third-party posted price mechanism, we also assumed that the offers to buy and sell are public. The reason we make this assumption is so that the equilibrium conditions from the mechanism coincide with our *ex post* IR conditions. In practice, the seller clearly learns whether the buyer offers to buy if he offers to sell. Likewise, the buyer learns whether the seller offers to sell if

Nonetheless, in at least some situations it is the trading parties themselves who set the price. We have also analyzed trade possibilities in one such mechanism, in which (1) the buyer proposes a price  $p \in P$ , where  $P$  is finite set of possible offers,<sup>18</sup> and (2) the seller accepts or rejects. We establish that trade in this mechanism is necessarily more complicated than in the third-party posted price mechanism. Specifically, if trade takes place, it must do so at a price that partially depends on the buyer's signal  $s_2$ . However, our main conclusion — that trade is possible even absent noise traders — remains valid. Details are available in an appendix posted on the authors' webpages.

## 5 Portfolio selection

In our basic model agents choose an action that directly affects the payoff produced by the asset. Clearly in many circumstances agents trade assets over which they have little or no direct control. Even in such settings, however, an agent who owns an asset must still decide how to allocate the remainder of his portfolio, and if the agent is risk averse this decision affects the agent's utility from holding the asset. When agents are unsure about how asset returns are distributed, gains from trade arise for the same reasons as in our basic model, as we now demonstrate.

In order to remain close to our basic model, we adopt what is essentially a partial equilibrium approach: we consider a many-investor economy with a risk free security he offers to buy. It is irrelevant what the buyer learns if he does not offer to buy. As such, the only remaining question is whether the seller learns whether the buyer offers to buy if he *does not* offer to sell. For many upstairs trades it is reasonable to suppose that the seller learns the buyer's announcement when he does not offer to sell. That is, for many stocks only a few individuals hold a large block, and so the holder of a block learns whether or not there is buying interest from whether or not an upstairs broker contacts him.

<sup>18</sup>The assumption that the offer set  $P$  is finite ensures that the action set is finite. As is well-known, equilibrium existence is not guaranteed in games with infinite action spaces.

and  $n + 1$  risky securities, and consider when information-based trade of just one of these securities — security 1 — occurs. Accordingly, we assume that the fundamental  $\theta$  has no effect on how an investor *without* security 1 would choose to allocate his wealth across the risk free security and securities  $2, \dots, n + 1$ .<sup>19</sup> We then endow a single investor, agent 1, with security 1. From our analysis of necessary conditions for trade, we know that the only potential buyers of security 1 are investors who observe an informative signal about the fundamental  $\theta$ . To keep our analysis close to the basic model we assume there is just one such investor, agent 2.<sup>20</sup> We analyze under what conditions agent 2 buys security 1 from agent 1 — that is, when information-based trade of security 1 occurs. (We assume for now that security 1 is indivisible, and return to this point in the conclusion.)

All investors in the economy are free to take any position in the risk free security, and any position  $\psi \in \Psi \subset \mathbb{R}^n$  in securities  $2, \dots, n + 1$ . Importantly, agents 1 and 2 are free to rebalance their portfolios after they trade (or do not trade) security 1.<sup>21</sup>

All investors have a common utility function  $u$ . The initial wealth of agents 1 and 2 is  $W_1$  and  $W_2$  respectively. As in the basic model, we typically refer to agent 1 as the seller and agent 2 as the buyer. We normalize both the return on the risk free security and the price of each risky security  $2, \dots, n + 1$  to unity. We denote the payoff of security 1 by  $R$ , and the vector of excess returns (over the risk free security) of securities  $2, \dots, n + 1$  by  $r$ . Both  $R$  and  $r$  are stochastic, with distributions that potentially differ across fundamentals  $a$  and  $b$ .<sup>22</sup>

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<sup>19</sup>This assumption implies that aggregate demands for the risk free security and securities  $2, \dots, n + 1$  are independent of perceptions about the fundamental. Hence the prices of these securities are likewise independent of perceptions about the fundamental.

<sup>20</sup>Allowing for multiple buyers would not fundamentally change our analysis.

<sup>21</sup>By footnote 19, the prices of the risk free security and securities  $2, \dots, n + 1$  are unaffected by investors' beliefs about what agents 1 and 2 have observed.

<sup>22</sup>If the distribution of  $r$  differs across fundamentals  $a$  and  $b$ , it does so in a way that leaves the optimal portfolio allocation of any investor without security 1 unchanged.

The expected utility of an agent with wealth  $W$  who owns security 1 and believes that the probability of fundamental  $a$  is  $q$  is given by

$$U(W, q) \equiv \max_{\psi \in \Psi} E[u(W + R + r\psi) | q].$$

Likewise, the expected utility of an agent who does not own security 1 is

$$\bar{U}(W, q) \equiv \max_{\psi \in \Psi} E[u(W + r\psi) | q].$$

For use below, note that  $U$  is convex in  $q$  (the argument is the same as for Lemma 1); while  $\bar{U}$  is linear in  $q$  because by assumption the fundamental  $\theta$  has no effect on portfolio choice of an agent without security 1.

Exactly as in our basic model, an allocation is described by the pair of mappings  $\kappa$  and  $\pi$ . We continue to assume a small cost  $\delta$  is associated with trading security 1. Analogous to before, the *ex post* IR conditions are

$$\begin{aligned} \bar{U}(W_1 + \pi(\omega), Q(\omega; \mathcal{F}_1^{\kappa, \pi})) &\geq U(W_1, Q(\omega; \mathcal{F}_1^{\kappa, \pi})) \\ U(W_2 - \pi(\omega) - \delta, Q(\omega; \mathcal{F}_2^{\kappa, \pi})) &\geq \bar{U}(W_2, Q(\omega; \mathcal{F}_2^{\kappa, \pi})) \end{aligned}$$

in almost all states in which the buyer gets the security (i.e.,  $\omega$  such that  $\kappa(\omega) = 2$ ), and

$$\begin{aligned} U(W_1 + \pi(\omega), Q(\omega; \mathcal{F}_1^{\kappa, \pi})) &\geq U(W_1, Q(\omega; \mathcal{F}_1^{\kappa, \pi})) \\ \bar{U}(W_2 - \pi(\omega), Q(\omega; \mathcal{F}_2^{\kappa, \pi})) &\geq \bar{U}(W_2, Q(\omega; \mathcal{F}_2^{\kappa, \pi})) \end{aligned}$$

in almost all states in which the seller keeps the security (i.e.,  $\omega$  such that  $\kappa(\omega) = 1$ ). Note that exactly as in the basic model, this second pair of inequalities implies that  $\pi(\omega) = 0$  for almost all states in which the seller keeps the security.

One complication of this framework relative to our basic model is that wealth effects may lead the buyer and seller to choose different portfolios, even if they have exactly the same information.<sup>23</sup> In contrast, in our basic model agents with the same

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<sup>23</sup>Even if  $W_1 = W_2$ , so that the two agents have the same initial wealth, the buyer's wealth when he acquires the security is lower.



information always choose the same action. To avoid this complication we assume that both agents share the same constant absolute risk aversion (CARA) utility,  $u(x) = -e^{-\gamma x}$ , where  $\gamma > 0$ . In this case, there exist negative-valued functions  $v$  and  $\bar{v}$  such that  $U(W, q) = e^{-\gamma W} v(q)$  and  $\bar{U}(W, q) = e^{-\gamma W} \bar{v}(q)$ . After substitution, the *ex post* IR conditions for states in which the buyer gets the security become

$$e^{-\gamma \pi(\omega)} \leq \frac{v(Q(\omega; \mathcal{F}_1^{\kappa, \pi}))}{\bar{v}(Q(\omega; \mathcal{F}_1^{\kappa, \pi}))}$$

$$\frac{v(Q(\omega; \mathcal{F}_2^{\kappa, \pi}))}{\bar{v}(Q(\omega; \mathcal{F}_2^{\kappa, \pi}))} \leq e^{-\gamma(\pi(\omega) + \delta)}.$$

Taking logs and defining  $\mathcal{V} \equiv -\frac{1}{\gamma} \ln \frac{v}{\bar{v}}$  generates precisely equations (3) and (4), with  $V$  replaced by  $\mathcal{V}$ . Because  $U$  is convex in  $q$  and  $\bar{U}$  is linear in  $q$ ,  $\mathcal{V}$  is single-troughed, just as  $V$  is (see Lemma 1).<sup>24</sup> Consequently our prior analysis applies, and trade is possible if and only if  $\mathcal{V}$  is non-monotone. Writing  $\psi_\theta$  for the optimal portfolio of an agent who holds security 1 and knows the fundamental is  $\theta$ , and  $\bar{\psi}$  for the optimal portfolio of an agent without security 1, we have:

**Proposition 7.** *Trade in security 1 is possible if and only if  $\mathcal{V}$ , or equivalently  $v/\bar{v}$ , is non-monotone. This condition is satisfied if and only if*

$$\frac{E[-e^{-\gamma(r\psi_b + R)} | a]}{E[-e^{-\gamma(r\psi_b + R)} | b]} > \frac{E[-e^{-\gamma r \bar{\psi}} | a]}{E[-e^{-\gamma r \bar{\psi}} | b]} > \frac{E[-e^{-\gamma(r\psi_a + R)} | a]}{E[-e^{-\gamma(r\psi_a + R)} | b]}. \quad (6)$$

An immediate implication of Proposition 7 is that trade is possible only if  $\psi_a \neq \psi_b$ , i.e., the fundamental actually affects the portfolio decision of an agent holding security 1.

## TRADE AND CAPM BETAS

For the remainder of this section we assume that returns are normally distributed. Notationally, write  $\mu_\theta$  for the  $1 \times n$  vector of mean returns for securities  $2, \dots, n+1$

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<sup>24</sup>The details of this argument are in the proof of Proposition 7.

in fundamental  $\theta$ , and  $\Sigma_\theta$  for the corresponding variance-covariance matrix. The optimal portfolio of an agent without security 1 is thus<sup>25,26</sup>

$$\bar{\psi} = \gamma^{-1} \Sigma_\theta^{-1} \mu'_\theta.$$

Likewise, let  $\nu_\theta$  and  $\zeta_\theta^2$  denote the mean and variance of the payoff of security 1 in fundamental  $\theta$ , with  $\Sigma_{1\theta}$  the  $1 \times n$  covariance vector of security 1 with securities  $2, \dots, n+1$ . Hence an investor with security 1 who knows  $\theta$  chooses the portfolio

$$\psi_\theta = \bar{\psi} - \beta_\theta,$$

where  $\beta_\theta = \Sigma_\theta^{-1} \Sigma'_{1\theta}$ . That is, an agent with security 1 picks a portfolio that combines the position he would choose if he did not own security 1,  $\bar{\psi}$ , with the variance-minimizing hedge of that security,  $-\beta_\theta$ .

Consequently, trade is possible only if  $\beta_a \neq \beta_b$ . Moreover, if  $n = 2$  so that there is just one risky security — the “market” —  $\beta_\theta$  is simply the CAPM beta in fundamental  $\theta$ . In this case, trade is possible only if the CAPM beta of security 1 differs across fundamentals.

## VOLATILITY AND VOLUME

Empirically, stock return volatility and trading volume are positively correlated, both at an aggregate level and at the level of individual stocks.<sup>27</sup> Our framework easily

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<sup>25</sup>To ensure that the portfolio allocation of an investor without security 1 is independent of fundamental  $\theta$ , we assume  $\Sigma_a^{-1} \mu'_a = \Sigma_b^{-1} \mu'_b$ .

<sup>26</sup>The expressions for  $\bar{\psi}$  and  $\psi_\theta$  are standard; they can also be easily derived by differentiating equation (21) in the appendix.

<sup>27</sup>See, e.g., the survey by Karpoff (1987). The typical study in this literature relates volume to volatility measured over a trailing window. Our model predicts correlation between volume and perceived future volatility. To the extent to which volatility is persistent the two correlations will be similar. Moreover, *implied* aggregate volatility from options markets is also correlated with aggregate volume (details are available upon request from the authors).

delivers just such a positive correlation, as the following simple parameterization makes clear.

Assume that the only impact of the fundamental is on the volatility of security 1, i.e.,  $\zeta_a = \zeta - \varepsilon$  and  $\zeta_b = \zeta + \varepsilon$  for some  $\zeta$  and  $\varepsilon \in (0, \zeta)$ . Thus (provided the unconditional probabilities of fundamentals  $a$  and  $b$  are approximately equal) an increase in  $\varepsilon$  increases the unconditional variance of security 1. All other properties of return distributions are unaffected by the fundamental:  $\mu_\theta$ ,  $\Sigma_\theta$ ,  $\nu_\theta$ , and the correlation of security 1 with other securities, are equal across fundamentals  $\theta = a, b$ . Finally, assume that security 1 is positively correlated with the other securities.

A straightforward application of Proposition 7 (see appendix for details) implies that *ex post* IR trade of security 1 is possible if and only if  $\varepsilon$  is sufficiently large.<sup>28</sup> Intuitively, the difference in the volatility of security 1 across fundamentals affects the optimal hedge for that security. When this effect is big enough, a seller with an uninformative signal realization is unable to hedge security 1 effectively. There is then a gain to trading the asset to the buyer if the latter has seen a more informative signal, and is thus in a better position to incorporate security 1 into his portfolio.

Because both the unconditional volatility of security 1 and trade volume are increasing in  $\varepsilon$ , this parameterization matches the empirical observation that the two quantities are positively correlated.

## 6 Conclusion

In this paper we have shown that if asset payoffs are endogenously determined by the actions of agents, then trade based purely on informational differences is possible. This conclusion stands in sharp contrast to the existing literature, which takes asset values as exogenous. Trade transfers control of the asset from an agent who has

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<sup>28</sup>We assume here that the distributions of  $r$  and  $R$  are such that trade is possible at  $\varepsilon = \zeta$ .

received an uninformative signal to one who has received an informative signal. Even without the presence of noise traders, agents in our model would be prepared to spend resources to acquire information; and this information is subsequently partially revealed by trade.

Our analysis generates a number of empirical implications. One general implication is that trade affects the action taken: when the buyer acquires the asset, he (at least sometimes) takes an action that is different from the one the seller would have taken. As we discussed, this implication is consistent with both firm policy after takeovers and with different creditor behaviors in debt restructuring, though other explanations are certainly possible. Other implications depend more on the specific application. In particular, the application of our model to the trade of non-controlling shares implies that trade volume may increase with volatility.

Clearly many avenues for future research exist. A fuller analysis of price negotiation between the buyer and seller is one important topic. Another is the extension of our portfolio allocation application. In particular, we currently assume that security 1 is indivisible, which ensures that a single agent holds security 1 in any Pareto optimal allocation. If instead security 1 were divisible the Pareto optimal allocation would generally entail multiple agents holding a strictly positive quantity of security 1. Analyzing such a framework would allow one to move beyond establishing the *possibility* of trade, as we do here, and to address the *quantity* and *direction* of trade.

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## A Appendix

**Proof of Proposition 1:** Without loss, suppose  $V$  is weakly increasing, and suppose that contrary to the claimed result the set trade set  $\Omega^T$  is non-null. Let  $P$  be the set of prices at which trade occurs, and for each  $p \in P$  let  $\Omega^T(p)$  be the subset in which trade occurs at price  $p$  and the *ex post* IR conditions hold, so  $\Omega^T = \cup_{p \in P} \Omega^T(p)$ .

We claim that for some  $p \in P$  there exists  $\omega^* \in \Omega^T(p)$  such that  $Q(\omega^*; \mathcal{F}_2^{\kappa, \pi}) \leq Q(\omega^*; \mathcal{F}_1^{\kappa, \pi})$ . Since  $V$  is weakly increasing, this claim implies that

$$V(Q(\omega^*; \mathcal{F}_2^{\kappa, \pi})) \leq V(Q(\omega^*; \mathcal{F}_1^{\kappa, \pi})).$$

However, *ex post* IR implies

$$V(Q(\omega^*; \mathcal{F}_1^{\kappa, \pi})) \leq p < p + \delta \leq V(Q(\omega^*; \mathcal{F}_2^{\kappa, \pi})),$$

giving the required contradiction.

To prove the claim, suppose to the contrary that  $Q(\omega; \mathcal{F}_1^{\kappa, \pi}) < Q(\omega; \mathcal{F}_2^{\kappa, \pi})$  for all  $p \in P$  and  $\omega \in \Omega^T(p)$ . By the definition of conditional probability, for  $i = 1, 2$ ,

$$\int_{\Omega^T} Q(\omega; \mathcal{F}_i^{\kappa, \pi}) \mu(d\omega) = \mu(\Omega^T \cap \{a\} \times \mathbb{R}^2),$$

and so

$$\int_{\Omega^T} (Q(\omega; \mathcal{F}_2^{\kappa, \pi}) - Q(\omega; \mathcal{F}_1^{\kappa, \pi})) \mu(d\omega) = 0.$$

This gives a contradiction, since by supposition  $Q(\omega; \mathcal{F}_2^{\kappa, \pi}) - Q(\omega; \mathcal{F}_1^{\kappa, \pi}) > 0$  and  $\Omega^T$  has strictly positive measure.  $\blacksquare$

**Proof of Proposition 2:** We establish Proposition 2 by contradiction. Suppose to the contrary that an *ex post* IR trade  $(\kappa, \pi)$  exists in which the buyer learns nothing whenever he acquires the asset. That is, trade occurs over  $\Omega^T$ , where  $\mu(\Omega^T) > 0$ , and  $\{F \cap \Omega^T : F \in \mathcal{F}_2^{\kappa, \pi}\} = \{F \cap \Omega^T : F \in \mathcal{F}_2\}$ .

Choose an integer  $n$  such that  $\Omega_n^T \equiv \Omega^T \cap \{a, b\} \times \mathbb{R} \times [n, n+1]$  has strictly positive mass. Since the buyer learns nothing when he acquires the asset,  $Q((\theta, s_1, s_2); \mathcal{F}_2^{\kappa, \pi}) = Q((\theta, s_1, s_2); \mathcal{F}_2) = \Pr(a|s_2)$  for all  $(\theta, s_1, s_2) \in \Omega_n^T$ . Let  $\underline{q} = \Pr(a|s_2 = n)$  and  $\bar{q} = \Pr(a|s_2 = n+1)$ , so that  $Q(\omega; \mathcal{F}_2^{\kappa, \pi}) \in [\underline{q}, \bar{q}]$  for  $\omega \in \Omega_n^T$ .

We claim that

$$\inf_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) < \underline{q} < \bar{q} < \sup_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi}). \quad (7)$$

This implies the result, as follows. *Ex post* IR for the buyer and single-troughedness of  $V$  (see Lemma 1) together imply that

$$\pi(\omega) \leq V(Q(\omega; \mathcal{F}_2^{\kappa, \pi})) - \delta \leq \max\{V(\underline{q}), V(\bar{q})\} - \delta$$

for all  $\omega \in \Omega_n^T$ . Single-troughedness of  $V$  and (7) imply that

$$\max\left\{V\left(\inf_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi})\right), V\left(\sup_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi})\right)\right\} \geq \max\{V(\underline{q}), V(\bar{q})\}.$$

But then since  $V$  is continuous there must exist  $\omega \in \Omega_n^T$  such that  $V(Q(\omega; \mathcal{F}_1^{\kappa, \pi})) > \pi(\omega)$ , contradicting *ex post* IR for the seller.

To complete the proof we must establish (7). Suppose that contrary to (7),  $\inf_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) \geq \underline{q}$ . From the definition of conditional probability, for any  $s_1$

$$\int_{\Omega^T \cap (\{a, b\} \times (-\infty, s_1) \times \mathbb{R})} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) \mu(d\omega) = \mu(\Omega^T \cap (\{a\} \times (-\infty, s_1) \times \mathbb{R})).$$



Since  $\Omega_n^T \subset \Omega^T$  and by supposition  $Q(\omega; \mathcal{F}_1^{\kappa, \pi}) \geq \underline{q}$  over  $\Omega_n^T$ ,

$$\begin{aligned} & \int_{\Omega^T \cap (\{a, b\} \times (-\infty, s_1) \times \mathbb{R})} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) \mu(d\omega) \\ & \geq \int_{\Omega_n^T \cap (\{a, b\} \times (-\infty, s_1) \times \mathbb{R})} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) \mu(d\omega) \\ & \geq \underline{q} \mu(\Omega_n^T \cap (\{a, b\} \times (-\infty, s_1) \times \mathbb{R})). \end{aligned}$$

So

$$\underline{q} \leq \frac{\mu(\Omega^T \cap (\{a\} \times (-\infty, s_1) \times \mathbb{R}))}{\mu(\Omega_n^T \cap (\{a, b\} \times (-\infty, s_1) \times \mathbb{R}))}.$$

Since after trade the buyer learns nothing,  $\Omega^T \in \mathcal{F}_2$  and so is of the form  $\{a, b\} \times \mathbb{R} \times S_2^T$ , where  $S_2^T \in \mathcal{B}$ . Note that  $\eta_1^\theta(S_2^T)$  and  $\eta_1^\theta(S_2^T \cap [n, n+1])$  are both strictly positive for  $\theta = a, b$ , since  $\mu(\Omega^T) > 0$ . So the last inequality rewrites to

$$\underline{q} \leq \frac{\Pr(a) F_1^a(s_1) \eta_1^a(S_2^T)}{\Pr(a) F_1^a(s_1) \eta_1^a(S_2^T \cap [n, n+1]) + \Pr(b) F_1^b(s_1) \eta_1^b(S_2^T \cap [n, n+1])}.$$

But since the likelihood ratio is unbounded (see (1)) the righthand side converges to 0 as  $s_1 \rightarrow -\infty$ , giving a contradiction and thus showing  $\inf_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi}) < \underline{q}$ . A parallel argument implies  $\bar{q} < \sup_{\omega \in \Omega_n^T} Q(\omega; \mathcal{F}_1^{\kappa, \pi})$ , completing the proof.  $\blacksquare$

**Proof of Proposition 3:** Suppose to the contrary that for almost all  $\omega \in \Omega^T$  the buyer takes the same action the seller would take if he controls the asset in state  $\omega$ . Then  $V(Q(\omega; \mathcal{F}_2^{\kappa, \pi})) = V(Q(\omega; \mathcal{F}_1^{\kappa, \pi}))$  for almost all  $\omega \in \Omega^T$ , which violates (5) and gives a contradiction.  $\blacksquare$

**Proof of Proposition 4:** Recall that if an agent has information given by the  $\sigma$ -algebra  $\mathcal{F}$  he knows the true realization of the fundamental. As such,

$$\int_{\Omega^T} (V(Q(\omega; \mathcal{F}_2^{\kappa, \pi}))) \mu(d\omega) \leq \int_{\Omega^T} (V(Q(\omega; \mathcal{F}))) \mu(d\omega),$$

i.e., the buyer's valuation of asset is less than the value of the asset to a perfectly informed agent. Moreover, since the seller's information becomes arbitrarily good,

$$\int_{\Omega^T} (V(Q(\omega; \mathcal{F}_1^{\kappa, \pi}))) \mu(d\omega) \rightarrow \int_{\Omega^T} (V(Q(\omega; \mathcal{F}))) \mu(d\omega).$$

It follows that the limit supremum of the lefthand side of (5) is weakly negative, giving the result.  $\blacksquare$

**Proof of Proposition 5:** We start with some preliminaries. Recall that  $L_i(s_i)$  denotes the likelihood ratio of signal  $s_i$ ; likewise, for any set  $S$  such that  $\eta_i^a(S) > 0$ , we let  $L_i(S)$  denote the likelihood ratio  $\eta_i^a(S)/\eta_i^b(S)$ . The asset value  $V$  is defined as a function of  $q$ , the probability the asset holder attaches to fundamental  $a$ . In the trade equilibria under consideration, for the seller the conditional probability  $q$  is of the form  $\Pr(a|s_1, s_2 \notin S_2^T)$ , while for the buyer it is of the form  $\Pr(a|s_1 \in S_1^T, s_2)$ . It is convenient to rewrite these probabilities as

$$\Pr(a|s_1, s_2 \notin S_2^T) = \frac{\frac{\Pr(a)}{\Pr(b)} L_1(s_1) L_2(\mathbb{R} \setminus S_2^T)}{\frac{\Pr(a)}{\Pr(b)} L_1(s_1) L_2(\mathbb{R} \setminus S_2^T) + 1}$$

$$\Pr(a|s_1 \in S_1^T, s_2) = \frac{\frac{\Pr(a)}{\Pr(b)} L_1(S_1^T) L_2(s_2)}{\frac{\Pr(a)}{\Pr(b)} L_1(S_1^T) L_2(s_2) + 1}.$$

Next, define a mapping from likelihood ratios to probabilities by

$$q(L) \equiv \frac{\frac{\Pr(a)}{\Pr(b)} L}{\frac{\Pr(a)}{\Pr(b)} L + 1} \text{ for any } L \in [0, \infty),$$

along with a transformation  $V^\ell$  of  $V$  that takes a likelihood ratio  $L$  as its argument, i.e.,  $V^\ell \equiv V \circ q$ . The function  $V$  is single-troughed (Lemma 1), and by hypothesis is non-monotone. As such, there exist probabilities  $q^*$  and  $q^{**} \geq q^*$  such that  $V$  is strictly decreasing over  $[0, q^*]$ , flat over  $[q^*, q^{**}]$ , and strictly increasing over  $[q^{**}, 1]$ . Since  $q$  is strictly increasing in  $L$ ,  $L^* = q^{-1}(q^*)$  and  $L^{**} = q^{-1}(q^{**})$  are well-defined, and  $V^\ell$  is strictly decreasing over  $[0, L^*]$ , flat over  $[L^*, L^{**}]$ , and strictly increasing over  $[L^{**}, \infty)$ .

We show there exists an equilibrium of the type described, i.e.,  $S_1^T \equiv [\underline{s}_1, \bar{s}_1]$  and  $S_2^T \equiv \mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)$ . If the seller sees signal  $s_1$  his payoff from offering to sell is

$$\Pr(s_2 \in S_2^T | s_1) p + \Pr(s_2 \notin S_2^T | s_1) V^\ell(L_1(s_1) L_2(\mathbb{R} \setminus S_2^T)),$$

while his payoff from not offering to sell is

$$\Pr(s_2 \in S_2^T | s_1) V^\ell(L_1(s_1) L_2(S_2^T)) + \Pr(s_2 \notin S_2^T | s_1) V^\ell(L_1(s_1) L_2(\mathbb{R} \setminus S_2^T)).$$

Thus it is a best response for the seller to offer to sell whenever  $s_1 \in S_1^T$  if and only if

$$\begin{aligned} V^\ell(L_1(s_1) L_2(S_2^T)) &\leq p \text{ for all } s_1 \in S_1^T \\ V^\ell(L_1(s_1) L_2(S_2^T)) &\geq p \text{ for all } s_1 \notin S_1^T. \end{aligned}$$

By continuity and the shape of  $V^\ell$ , these conditions are satisfied if and only if

$$V^\ell(L_1(\underline{s}_1) L_2(S_2^T)) = V^\ell(L_1(\bar{s}_1) L_2(S_2^T)) = p. \quad (8)$$

Likewise, in order for the buyer to offer to buy whenever  $s_2 \in S_2^T$ ,

$$\begin{aligned} V^\ell(L_1(S_1^T) L_2(s_2)) - \delta &\geq p \text{ for all } s_2 \in S_2^T \\ V^\ell(L_1(S_1^T) L_2(s_2)) - \delta &\leq p \text{ for all } s_2 \notin S_2^T, \end{aligned}$$

and these conditions are satisfied if and only if

$$V^\ell(L_1(S_1^T) L_2(\underline{s}_2)) - \delta = V^\ell(L_1(S_1^T) L_2(\bar{s}_2)) - \delta = p. \quad (9)$$

Thus a trade equilibrium exists if and only if there exist  $\underline{s}_1, \bar{s}_1 \neq \underline{s}_1, \underline{s}_2, \bar{s}_2 \neq \underline{s}_2$  such that (8) and (9) hold.

From the shape of  $V^\ell$ , there exists a unique quadruple  $\underline{L}_1, \bar{L}_1, \underline{L}_2, \bar{L}_2$  such that  $\underline{L}_i < L^* \leq L^{**} < \bar{L}_i$  for  $i = 1, 2$ , and

$$\begin{aligned} V^\ell(\underline{L}_1) &= V^\ell(\bar{L}_1) = p \\ V^\ell(\underline{L}_2) - \delta &= V^\ell(\bar{L}_2) - \delta = p. \end{aligned}$$

Consequently, a trade equilibrium of the type described exists if and only if there

exist  $\underline{s}_1, \bar{s}_1 \neq \underline{s}_1, \underline{s}_2, \bar{s}_2 \neq \underline{s}_2$  satisfying the following system of four equations:

$$L_1(\underline{s}_1) L_2(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)) = \underline{L}_1 \quad (10)$$

$$L_1(\bar{s}_1) L_2(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)) = \bar{L}_1 \quad (11)$$

$$L_1([\underline{s}_1, \bar{s}_1]) L_2(\underline{s}_2) = \underline{L}_2 \quad (12)$$

$$L_1([\underline{s}_1, \bar{s}_1]) L_2(\bar{s}_2) = \bar{L}_2. \quad (13)$$

To complete the proof we show that such a quadruple does exist. First note that (10) and (11) imply

$$\frac{L_1(\underline{s}_1)}{L_1(\bar{s}_1)} = \frac{\underline{L}_1}{\bar{L}_1} < 1 \quad (14)$$

and (12) and (13) imply

$$\frac{L_2(\underline{s}_2)}{L_2(\bar{s}_2)} = \frac{\underline{L}_2}{\bar{L}_2} < 1. \quad (15)$$

Fix  $\underline{s}_1$  and solve for  $\bar{s}_1(\underline{s}_1) > \underline{s}_1$  from (14). Similarly solve for  $\bar{s}_2(\underline{s}_2) > \underline{s}_2$  from (15).

Substituting for  $\bar{s}_1(\underline{s}_1)$  and  $\bar{s}_2(\underline{s}_2)$ , rewrite (10) and (12) as

$$L_1(\underline{s}_1) L_2(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2(\underline{s}_2))) = \underline{L}_1 \quad (16)$$

$$L_1([\underline{s}_1, \bar{s}_1(\underline{s}_1)]) L_2(\underline{s}_2) = \underline{L}_2. \quad (17)$$

Observe that  $\bar{s}_1(\underline{s}_1) \rightarrow \pm\infty$  as  $\underline{s}_1 \rightarrow \pm\infty$ . Consequently  $L_1([\underline{s}_1, \bar{s}_1(\underline{s}_1)]) \rightarrow \infty$  as  $\underline{s}_1 \rightarrow \infty$  and  $L_1([\underline{s}_1, \bar{s}_1(\underline{s}_1)]) \rightarrow 0$  as  $\underline{s}_1 \rightarrow -\infty$ . Thus from (17) define  $\underline{s}_2(\underline{s}_1)$ , and note that  $\underline{s}_2(\underline{s}_1) \rightarrow \mp\infty$  as  $\underline{s}_1 \rightarrow \pm\infty$ .

Also observe that  $\bar{s}_2(\underline{s}_2) \rightarrow \pm\infty$  as  $\underline{s}_2 \rightarrow \pm\infty$ , and so  $L_2(\mathbb{R} \setminus (\underline{s}_2, \bar{s}_2)) \rightarrow 1$  as  $\underline{s}_2 \rightarrow \pm\infty$ . So substituting in for  $\underline{s}_2(\underline{s}_1)$ , the lefthand side of (16) approaches 0 as  $\underline{s}_1 \rightarrow -\infty$  and grows without bound as  $\underline{s}_1 \rightarrow \infty$ . By continuity it follows that there exists some  $\underline{s}_1$  such that

$$L_1(\underline{s}_1) L_2(\mathbb{R} \setminus (\underline{s}_2(\underline{s}_1), \bar{s}_2(\underline{s}_2(\underline{s}_1)))) = \underline{L}_1,$$

completing the proof. ■

**Proof of Proposition 6:** The key to the proof is the following observation: for any set  $S_2 \subset \mathbb{R}$ , Jensen's inequality, the convexity and non-monotonicity of  $V$ , and unbounded MLRP together imply

$$\begin{aligned} & E [V (\Pr (a | \{s_1\} \times S_2)) | s_2 \in S_2] \\ &= \sum_{\theta=a,b} \Pr (\theta | s_2 \in S_2) \int_{-\infty}^{\infty} V (\Pr (a | \{s_1\} \times S_2)) f_1^\theta (s_1) ds_1 \\ &> \sum_{\theta=a,b} \Pr (\theta | s_2 \in S_2) V (\Pr (a | S_2)) = V (\Pr (a | S_2)). \end{aligned}$$

Likewise, for any  $S_1 \subset \mathbb{R}$ ,  $E [V (\Pr (a | S_1 \times \{s_2\})) | s_1 \in S_1] > V (\Pr (a | S_1))$ .

We show that the equilibrium established in Proposition 5 remains an equilibrium when the buyer and seller must pay  $k$  to acquire their signals. For an information acquisition cost of  $k = 0$ , the seller's equilibrium utility is

$$\begin{aligned} & p \Pr (s_1 \in S_1^T, s_2 \in S_2^T) \\ &+ E [V (\Pr (a | \{s_1\} \times S_2^T)) | s_1 \notin S_1^T, s_2 \in S_2^T] \Pr (s_1 \notin S_1^T, s_2 \in S_2^T) \\ &+ E [V (\Pr (a | \{s_1\} \times \mathbb{R} \setminus S_2^T))] \Pr (s_2 \notin S_2^T). \end{aligned} \tag{18}$$

Because the seller could instead always offer to sell, this quantity exceeds

$$p \Pr (s_2 \in S_2^T) + E [V (\Pr (a | \{s_1\} \times \mathbb{R} \setminus S_2^T)) | s_2 \notin S_2^T] \Pr (s_2 \notin S_2^T),$$

which by above is in turn strictly greater than

$$p \Pr (s_2 \in S_2^T) + V (\Pr (a | \mathbb{R} \setminus S_2^T)) \Pr (s_2 \notin S_2^T).$$

This last expression equals the seller's payoff under the deviation in which he does not buy his signal and always trades. Similarly, the seller's equilibrium utility (18) is greater than his payoff from never trading,

$$\begin{aligned} & E [V (\Pr (a | \{s_1\} \times S_2^T)) | s_2 \in S_2^T] \Pr (s_2 \in S_2^T) \\ &+ E [V (\Pr (a | \{s_1\} \times \mathbb{R} \setminus S_2^T)) | s_2 \notin S_2^T] \Pr (s_2 \notin S_2^T) \end{aligned}$$

which is in turn strictly greater than

$$V(\Pr(a|S_2^T)) \Pr(s_2 \in S_2^T) + V(\Pr(a|\mathbb{R} \setminus S_2^T)) \Pr(s_2 \notin S_2^T),$$

the value of the asset to the seller if he observes only the buyer's announcement of whether or not he is prepared to buy. So for all information acquisition costs  $k$  that are sufficiently low the seller chooses to buy his information.

For an information acquisition cost of  $k = 0$ , the buyer's equilibrium utility is

$$E[V(\Pr(a|S_1^T \times \{s_2\})) - p - \delta | s_1 \in S_1^T, s_2 \in S_2^T] \Pr(s_1 \in S_1^T, s_2 \in S_2^T).$$

Because the buyer could instead always offer to buy, this exceeds

$$E[V(\Pr(a|S_1^T \times \{s_2\})) - p - \delta | s_1 \in S_1^T] \Pr(S_1^T),$$

which in turn strictly exceeds

$$(V(\Pr(a|S_1^T)) - p - \delta) \Pr(S_1^T),$$

the buyer's payoff under the deviation in which he does not buy his signal and always trades. Finally, if the buyer deviates to not buying the signal and never trading, his payoff is simply zero, and which is strictly less than his equilibrium utility. Again, for all information acquisition costs  $k$  that are sufficiently low the buyer chooses to buy his information. ■

**Proof of Proposition 7:** We first show that trade is possible if and only if  $\mathcal{V}$  is non-monotone. As discussed in the main text, given prior results we need only show that  $\mathcal{V}$  is single-troughed, or equivalently,  $v/\bar{v}$  is single-peaked. The derivative of  $v/\bar{v}$  has the same sign as

$$v'(q) \bar{v}(q) - v(q) \bar{v}'(q). \tag{19}$$

The derivative of (19) in turn equals  $v''(q) \bar{v}(q) - v(q) \bar{v}''(q)$ . Recall that by assumption the optimal portfolio allocation of an agent without security 1 is unaffected

by the fundamental. Consequently,  $\bar{U}$  and hence  $\bar{v}$  are linear in  $q$ . So the derivative of (19) is simply  $v''(q)\bar{v}(q)$ , which is weakly negative since  $v$  is convex and  $\bar{v}$  is negative-valued. It follows that  $v/\bar{v}$  is either monotone, or else is increasing then decreasing. This completes the first half of the proof.

Second, we establish (6). Given that  $v/\bar{v}$  is single-peaked, it is non-monotone if and only if

$$\left. \frac{\partial}{\partial q} \left( \frac{v(q)}{\bar{v}(q)} \right) \right|_{q=0} > 0 > \left. \frac{\partial}{\partial q} \left( \frac{v(q)}{\bar{v}(q)} \right) \right|_{q=1}.$$

Consequently, trade is possible if and only if

$$v'(0)\bar{v}(0) - v(0)\bar{v}'(0) > 0 > v'(1)\bar{v}(1) - v(1)\bar{v}'(1). \quad (20)$$

Letting  $\psi(q)$  be the optimal market allocation of an investor who holds security 1 and attaches a probability  $q$  to fundamental  $a$ ,

$$v(q) = qE[-e^{-\gamma(r\psi(q)+R)}|a] + (1-q)E[-e^{-\gamma(r\psi(q)+R)}|b],$$

with a similar expression for  $\bar{v}(q)$ . Straightforward algebra and the envelope theorem together imply that  $v'(0)\bar{v}(0) - v(0)\bar{v}'(0)$  equals

$$E[-e^{-\gamma(r\psi_b+R)}|a]E[-e^{-\gamma r\bar{\psi}}|b] - E[-e^{-\gamma(r\psi_b+R)}|b]E[-e^{-\gamma r\bar{\psi}}|a],$$

while  $v'(1)\bar{v}(1) - v(1)\bar{v}'(1)$  equals

$$E[-e^{-\gamma(r\psi_a+R)}|a]E[-e^{-\gamma r\bar{\psi}}|b] - E[-e^{-\gamma(r\psi_a+R)}|b]E[-e^{-\gamma r\bar{\psi}}|a].$$

Substitution into (20) implies that trade is possible if and only if (6) holds. ■

**Details for the volatility-volume parameterization (see page 25):** In the CARA-normal framework, the utility of an agent holding security 1 and portfolio  $\psi$  in fundamental  $\theta$  is given by

$$-\exp\left(-\gamma\left(\mu_\theta\psi + \nu_\theta - \frac{\gamma}{2}\left(\psi'\Sigma\psi + \zeta_\theta^2 + 2\Sigma_{1\theta}\psi\right)\right)\right). \quad (21)$$

Since the correlation of security 1 with the other securities is independent of  $\theta$ , and positive, there exists some constant positive vector  $K$  such that  $\Sigma_{1\theta} = \zeta_\theta K$ . Substitution into (6) implies that trade is possible if and only if

$$(\zeta_a^2 + 2\zeta_a K \psi_b) - (\zeta_b^2 + 2\zeta_b K \psi_b) > 0 > (\zeta_a^2 + 2\zeta_a K \psi_a) - (\zeta_b^2 + 2\zeta_b K \psi_a),$$

or equivalently,

$$-2K\psi_b > \zeta_a + \zeta_b > -2K\psi_a.$$

Substituting in for  $\zeta_\theta$  and  $\psi_\theta$ , this condition is in turn equivalent to

$$-K\bar{\psi} + (\zeta + \varepsilon) K \Sigma_\theta^{-1} K' > \zeta > -K\bar{\psi} + (\zeta - \varepsilon) K \Sigma_\theta^{-1} K'.$$

■