

# Sources of Lifetime Inequality

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## Abstract

Is lifetime inequality mainly due to differences across people established early in life or to differences in luck experienced over the working lifetime? We answer this question within a model that features idiosyncratic shocks to human capital, estimated directly from data, as well as heterogeneity in ability to learn, initial human capital, and initial wealth – features which are chosen to match properties of earnings dynamics. We find that as of age 20, differences in initial conditions account for more of the variation in lifetime utility, lifetime earnings and lifetime wealth than do differences in shocks received over the lifetime. Among initial conditions, variation in initial human capital is substantially more important than variation in learning ability or initial wealth for determining how an agent fares in life.

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# 1 Introduction

To what degree is lifetime inequality due to differences across people established early in life as opposed to differences in luck experienced over the working lifetime? Among the individual differences established early in life, which ones are the most important?

A convincing answer to these questions is of fundamental importance. First, and most simply, an answer serves to contrast the potential importance of the myriad policies directed at modifying or at providing insurance for initial conditions (e.g. public education) against those directed at shocks over the lifetime (e.g. unemployment insurance). Second, a discussion of lifetime inequality cannot go too far before discussing which type of initial condition is the most critical for determining how one fares in life. Third, a useful framework for answering these questions should also be central in the analysis of a wide range of policies considered in macroeconomics, public finance and labor economics.

We view lifetime inequality through the lens of a risky human capital model. Agents differ in terms of three initial conditions: initial human capital, learning ability and financial wealth. Initial human capital can be viewed as controlling the intercept of an agent's mean earnings profile, whereas learning ability acts to rotate this profile. Human capital and labor earnings are risky as human capital is subject to uninsured, idiosyncratic shocks each period.

We ask the model to account for key features of the dynamics of the earnings distribution. To this end, we document how mean earnings and measures of earnings dispersion and skewness evolve for cohorts of U.S. males. We find that mean earnings are hump shaped and that earnings dispersion and skewness increase with age over most of the working lifetime.<sup>1</sup>

Our model produces a hump-shaped mean earnings profile by a standard human capital channel. Early in life earnings are low as agents allocate time to accumulating human capital. Earnings rise as human capital accumulates and as a greater fraction of time is devoted to market work. Earnings fall later in life because human capital depreciates and little time is put into producing new human capital.

Two forces within the model account for the increase in earnings dispersion. One force is that agents differ in learning ability. Agents with higher learning ability have steeper mean

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<sup>1</sup>Mincer (1974) documents related patterns in U.S. cross-section data. Deaton and Paxson (1994), Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2005) and Huggett, Ventura and Yaron (2006) examine cohort patterns in U.S. repeated cross section or panel data.

earnings profiles than low ability agents, other things equal.<sup>2</sup> The other force is that agents differ in idiosyncratic human capital shocks received over the lifetime.

To identify the contribution of each of these forces, we exploit the fact that the model implies that late in life little or no new human capital is produced. As a result, moments of the change in wage rates for these agents are almost entirely determined by shocks, rather than by shocks and the endogenous response of investment in human capital to shocks and initial conditions. We estimate the shock process using precisely these moments. Given an estimate of the shock process and other model parameters, we choose the initial distribution of financial wealth, human capital and learning ability across agents to best match the earnings facts described above.<sup>3</sup> We find that learning ability differences produce an important part of the rise in earnings dispersion over the lifetime, given our estimate of the magnitude of human capital risk.

We use our estimates of shocks and initial conditions to quantify the importance of different proximate sources of lifetime inequality. We find that initial conditions established by age 20 are more important than are shocks received over the remaining lifetime as a source of variation in realized lifetime utility, lifetime earnings and lifetime wealth.<sup>4</sup> In the benchmark model, variation in initial conditions accounts for 61 to 62 percent of the variation in lifetime earnings and 65 to 67 percent of the variation in lifetime utility. There are two values for each statistic because we analyze two separate empirical descriptions of earnings distribution dynamics.

Among initial conditions, we find that, as of age 20, variation in initial human capital is substantially more important than variation in either learning ability or initial wealth for how an agent fares in life. This analysis is conducted for an agent with the median value of each initial condition. We find that a one standard deviation increase in initial wealth increases expected lifetime wealth by 3 to 4 percent. In contrast, a one standard deviation increase in learning ability or initial human capital increases expected lifetime wealth by 6 to 14 percent and 32 to 38 percent, respectively. An increase in human capital lifts an

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<sup>2</sup>This mechanism is supported by the literature on the shape of the mean age-earnings profiles by years of education. It is also supported by the work of Lillard and Weiss (1979), Baker (1997) and Guvenen (2007). They estimate a statistical model of earnings and find important permanent differences in individual earnings growth rates.

<sup>3</sup>Since a measure of financial wealth is observable, we set the tri-variate initial distribution to be consistent with features of the distribution of wealth among young households.

<sup>4</sup>Lifetime earnings equals the present value of earnings, whereas lifetime wealth equals lifetime earnings plus initial wealth.

agent's mean earnings profile, whereas an increase in learning ability rotates this profile counterclockwise.

We also analyze how an agent in the model values these changes in initial conditions. Specifically, we calculate the permanent percentage change in consumption which is equivalent in expected utility terms to these changes in initial conditions. We find that these permanent changes in consumption are roughly in line with how a change in an initial condition impacts, in percentage terms, expected lifetime wealth.

A leading view of lifetime inequality is based on the standard, incomplete-markets model in which labor earnings over the lifetime is exogenous. Storesletten et. al. (2004) analyze lifetime inequality from the perspective of such a model. Similar models have been widely used in the macroeconomic literature on economic inequality.<sup>5</sup> Storesletten et. al. (2004) estimate an earnings process from U.S. panel data to match features of earnings over the lifetime. Within their model, slightly less than half of the variation in realized lifetime utility is due to differences in initial conditions as of age 20. On the other hand, and in the context of a career-choice model, Keane and Wolpin (1997) find a more important role for initial conditions. They find that unobserved heterogeneity realized at age 16 accounts for about 90 percent of the variance in lifetime utility.

Given the popularity of the incomplete-markets model with exogenous earnings in macroeconomic research, and its natural connection to our framework, we note three difficulties with this model. First, the importance of idiosyncratic earnings risk may be overstated because all of the rise in earnings dispersion with age is attributed to shocks. In our model initial conditions account for some of the rise in dispersion. Second, although the exogenous earnings model produces the rise in U.S. within cohort consumption dispersion over the period 1980-90, the rise in consumption dispersion is substantially smaller in U.S. data over a longer time period. Our model produces less of a rise in consumption dispersion because part of the rise in earnings dispersion is due to initial conditions. Third, the incomplete-markets model is not useful for some purposes. Specifically, since earnings are exogenous, the model can not shed light on how policy may affect inequality in lifetime earnings or may affect welfare through earnings. Models with exogenous wage rates (e.g. Heathcote et. al. (2006)) face this criticism, but to a lesser extent, since most earnings variation is attributed to wage variation. It therefore seems worthwhile to pursue a more fundamental approach that endog-

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<sup>5</sup>See Huggett (1996), Castañeda, Diaz-Jimenez and Rios-Rull (2003), Krueger and Perri (2006), Heathcote, Storesletten and Violante (2006) and Guvenen (2007) among many others.

enizes wage rate differences via human capital theory. In our view, a successful quantitative model of this type would bridge an important gap between the macroeconomic literature with incomplete markets and the human capital literature and would offer an important alternative workhorse model for quantitative work and policy analysis.

The paper is organized as follows. Section 2 presents the model. Section 3 documents earnings distribution facts and estimates properties of shocks. Section 4 sets model parameters. Sections 5 and 6 analyze the model and answer the two lifetime inequality questions. Section 7 concludes.

## 2 The Model

An agent maximizes expected lifetime utility, taking initial financial wealth  $k_1(1+r)$ , initial human capital  $h_1$  and learning ability  $a$  as given.<sup>6</sup>

$$\max_{\{c_j, l_j, k_{j+1}\}} E\left[\sum_{j=1}^J \beta^{j-1} u(c_j)\right]$$

subject to

$$c_j + k_{j+1} = k_j(1+r) + e_j, \forall j \text{ and } k_{J+1} \geq 0$$

$$e_j = R_j h_j L_j \text{ if } j < J_R, \text{ and } e_j = 0 \text{ otherwise.}$$

$$h_{j+1} = \exp(z_{j+1})H(h_j, l_j, a), \forall j \text{ and } L_j + l_j = 1, \forall j$$

In this decision problem the fundamental source of risk over the working lifetime comes from idiosyncratic shocks to an agent's human capital. Thus, consumption  $c_j(x_1, z^j)$  at age  $j$  is risky as it depends on the  $j$ -period history of human capital shocks  $z^j = (z_1, \dots, z_j)$  in addition to initial conditions  $x_1 = (h_1, k_1, a)$ . An agent faces a period budget constraint in which consumption  $c_j$  plus financial asset holding  $k_{j+1}$  equal earnings  $e_j$  plus the value of assets brought into the period  $k_j(1+r)$ . Financial assets pay a risk-free, real return  $r$ .

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<sup>6</sup>The model generalizes Ben-Porath (1967) to allow for risky human capital and extends Levhari and Weiss (1974) and Eaton and Rosen (1980) to a multi-period setting. Krebs (2004) analyzes a multi-period model of human capital with idiosyncratic risk. Our work differs in its focus on lifetime inequality among other differences.

Earnings  $e_j$  before a retirement age  $J_R$  equal the product of a human capital rental rate  $R_j$ , an agent's human capital  $h_j$  and the fraction  $L_j$  of available time put into market work. Earnings are zero at and after the retirement age  $J_R$ . An agent's future human capital  $h_{j+1}$  is an increasing function of an idiosyncratic shock  $z_{j+1}$ , current human capital  $h_j$ , time devoted to human capital production  $l_j$  and an agent's learning ability  $a$ . Learning ability is fixed over an agent's lifetime and is exogenous.

We now embed this decision problem within a general equilibrium framework. We assume there is an aggregate production function  $F(K_t, L_t A_t)$  with constant returns in aggregate capital and labor  $(K_t, L_t)$  and with labor augmenting technical change  $A_{t+1} = A_t(1 + g)$ . Aggregate variables are sums of the relevant individual decisions. Thus,  $C_t \equiv \sum_j \mu_j \int E[c_{jt}(x_1, z^j)] d\psi$ ,  $K_t \equiv \sum_j \mu_j \int E[k_{jt}(x_1, z^j)] d\psi$  and  $L_t \equiv \sum_j \mu_j \int E[h_{jt}(x_1, z^j) L_{jt}(x_1, z^j)] d\psi$ . In defining aggregates,  $\psi$  is a time-invariant distribution over initial conditions  $x_1$  and weights  $\mu_j$  describe the fraction of age  $j$  agents in the population. Weights are time invariant and are determined by a constant population growth rate  $n$  as follows:  $\mu_{j+1} = \mu_j / (1 + n)$

*Definition:* An equilibrium is a collection of agent decisions  $\{(c_{jt}, l_{jt}, L_{jt}, h_{jt}, k_{jt}) \mid j = 1, \dots, J\}_{t=1}^{\infty}$ , factor prices  $\{R_t, r_t\}_{t=1}^{\infty}$  and a distribution  $\psi$  over initial conditions such that

- (1) Decisions are optimal, given factor prices.
- (2) Competitive Factor Prices:  $R_t = F_L(K_t, L_t A_t)$  and  $r_t = F_K(K_t, L_t A_t) - \delta$
- (3) Resource Feasibility:  $C_t + K_{t+1}(1 + n) = F(K_t, L_t A_t) + K_t(1 - \delta), \forall t$

We focus on equilibria with balanced growth. In such an equilibrium all aggregates and factor prices grow at constant rates. More specifically, the rental rate on labor services  $R_{t+1} = R_t(1 + g)$  grows at the rate of labor augmenting technological change and the interest rate  $r_t$  is time invariant. The existence of a balanced-growth equilibrium relies on the technology assumptions already stated and on homothetic preferences. The distribution of initial human capital and learning ability is not critical for balanced growth but initial assets are important. The benchmark model sets initial assets to zero. The optimal consumption, asset holdings and earnings of age  $j$  agents at different time periods  $t$  but with the same shocks and initial conditions are simply scaled up by the level of the rental rate, whereas the time allocation and human capital levels of such age  $j$  agents at different time periods are time invariant.

We comment on three key features of the model. First, while the earnings of an agent are stochastic, the earnings distribution for a large cohort of agents evolves deterministically.

This occurs because the model has idiosyncratic but no aggregate risk.<sup>7</sup> Second, the model has two sources of growth in earnings dispersion within a cohort - agents have different learning abilities and different shock realizations. The next section characterizes empirically the rise in U.S. earnings dispersion over the life cycle. Third, the model implies that the nature of human capital shocks can be identified from wage rate data, independently from all other model parameters. This holds, as an approximation, because the model implies that the production of human capital goes to zero towards the end of the working lifetime. The next section develops the logic of this point.

### 3 Data and Empirical Analysis

In this section we use data to address two issues. First, we characterize how mean earnings and measures of earnings dispersion and skewness evolve with age for a cohort. Second, we estimate a human capital shock process from wage rate data.

#### 3.1 Age Profiles

We estimate age profiles for mean earnings and measures of earnings dispersion and skewness for ages 23 to 60. We use earnings data for males who are the head of the household from the Panel Study of Income Dynamics (PSID) 1969-2004 family files. To calculate earnings statistics at a specific age and year, we employ a centered 5-year age bin.<sup>8</sup> For males over age 30, we require that they work between 520 and 5820 hours per year and earn at least 1500 dollars (in 1968 prices) a year. For males age 30 and below, we require that they work between 260 and 5820 hours per year and earn at least 1000 dollars (in 1968 prices).

These selection criteria are motivated by several considerations. First, the PSID has many observations in the middle but relatively fewer at the beginning or end of the working life cycle. By focusing on ages 23-60, we have at least 100 observations in each age-year bin with which to calculate earnings statistics. Second, labor force participation falls near the traditional retirement age for reasons that are abstracted from in the model. This motivates the use of a terminal age that is earlier than the traditional retirement age. Third,

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<sup>7</sup>More specifically, the probability that an agent receives a  $j$ -period shock history  $z^j$  is also the fraction of the agents in a cohort that receive this shock history.

<sup>8</sup>To calculate statistics for the age 23 and the age 60 bin we use earnings for males age 21-25 and 58-62.

the hours and earnings restrictions are motivated by the fact that within the model the only alternative to time spent working is time spent learning. For males above 30, the minimum hours restriction amounts to a quarter of full-time work hours and the minimum earnings restriction is below the annual earnings level of a full-time worker working at the federal minimum wage.<sup>9</sup> For younger males, we lower both the minimum hours and earnings restrictions to capture students doing summer work or working part-time while in school.

We now document how mean earnings, two measures of earnings dispersion and a measure of earnings skewness evolve with age for cohorts. We consider two measures of dispersion: the variance of log earnings and the Gini coefficient of earnings. We measure skewness by the ratio of mean earnings to median earnings.

The methodology for extracting the age effects of these earnings statistics follows in two parts. First, we calculate the statistic of interest for males in age bin  $j$  at time  $t$  and label this  $stat_{j,t}$ . For example, for mean earnings we set  $stat_{j,t} = \ln(e_{jt})$ , where  $e_{jt}$  is real mean earnings of all males in the age bin centered at age  $j$  in year  $t$ .<sup>10</sup> Second, we posit a statistical model governing the evolution of the earnings statistic of interest as indicated below. Here the earnings statistic is viewed as being generated by several factors that we label cohort (c), age (j), and time (t) effects. We wish to estimate the age effects  $\beta_j^{stat}$ . We employ a statistical model as our economic model is not sufficiently rich to capture all aspects of time variation in the data.

$$stat_{j,t} = \alpha_c^{stat} + \beta_j^{stat} + \gamma_t^{stat} + \epsilon_{j,t}^{stat}$$

The linear relationship between time  $t$ , age  $j$ , and birth cohort  $c = t - j$  limits the applicability of this regression specification. Specifically, without further restrictions the regressors in this system are co-linear and these effects cannot be estimated. This identification problem is well known.<sup>11</sup> In effect any trend in the data can be arbitrarily reinterpreted as due to year (time) effects or alternatively as due to age or cohort effects.

Given this problem, we provide two alternative measures of the age effects. These correspond to the cohort effects view where we set  $\gamma_t^{stat} = 0, \forall t$  and the time effects view where

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<sup>9</sup>A full-time worker (working 40 hours per week and 52 weeks a year) who receives the federal minimum wage earns 3,328 dollars in 1968 prices.

<sup>10</sup>We use the Consumer Price Index to convert nominal earnings to real earnings.

<sup>11</sup>See Weiss and Lillard (1978) and Deaton and Paxson (1994) among others.



we set  $\alpha_c^{stat} = 0, \forall c$ . We use ordinary least squares to estimate the coefficients. For the cohort effects view, the regression has  $J \times T$  dependent variables regressed on  $J + T$  cohort dummies and  $J$  age dummies, where  $T$  and  $J$  denote the number of time periods in the panel and the number of distinct age groups. For the time effects view the regression has  $J \times T$  dependent variables regressed on  $T$  time dummies and  $J$  age dummies.<sup>12</sup>

In Figure 1(a) we graph the age effects of the levels of mean earnings implied by each regression. Figure 1 highlights the familiar hump-shaped profile of mean earnings for both the time and cohort effects views. Mean earnings almost doubles from the early 20's to the peak earnings age. Figure 1 is constructed using the coefficients  $exp(\beta_j)$  from the regression based upon mean earnings. The age effects in Figure 1(a) are first scaled so that mean earnings at age 38 in both views pass through the mean value across panel years at this age and are then both scaled so the time effects view is normalized to equal 100 at age 60.

Figure 1(b)-(d) graphs the age effects for measures of earnings dispersion and skewness setting time or cohort effects to zero. Our measures of dispersion are the variance of log earnings and the Gini coefficient, whereas skewness is measured by the ratio of mean earnings to median earnings. Each profile is normalized to run through the mean value of each statistic across panel years at age 38. Figures 1(b)-(d) show that measures of dispersion and skewness increase with age in both the time and cohort effects views. The cohort effect view implies a rise in the variance of log earnings of about 0.4 from age 23 to 60 whereas the time effects view implies a smaller rise of about 0.2. Thus, there is an important difference in the rise in dispersion coming from these two views. The same qualitative pattern holds for the Gini coefficient measure of dispersion. Figure 1(d) shows that the rise in earnings skewness with age is also greater for the cohort effects view than for the time effects view.

We will ask the economic model to match the time effects view of the evolution of the earnings distribution as well as the cohort effects view. We will then determine whether or not the answers to the two main questions concerning lifetime inequality are particularly sensitive to the differences in these two views.<sup>13</sup>

<sup>12</sup>A third approach, discussed in Huggett et. al. (2006), allows for age, cohort and time effects but with the restriction that time effects are mean zero and are orthogonal to a time trend. That is  $(1/T) \sum_{t=1}^T \gamma_t^{stat} = 0$  and  $(1/T) \sum_{t=1}^T t\gamma_t^{stat} = 0$ . Thus, trends over time are attributed to cohort and age effects rather than time effects. The results of this approach for means, dispersion and skewness are effectively the same as those for cohort effects.

<sup>13</sup>Heathcote et. al. (2005) present an argument for stressing the importance of time effects. Given the lack of a consensus in the literature, we are agnostic about which view should be stressed.

## 3.2 Human Capital Shocks

The model implies that an agent's wage rate, measured as market compensation per unit of work time, equals the product of the rental rate and an agent's human capital. The model also implies that late in the working life cycle human capital investments are approximately zero. This occurs as the number of working periods over which an agent can reap the returns to these investments falls as the agent approaches retirement. The upshot is that when there is no human capital investment over a period of time, then the change in an agent's wage rate is entirely determined by rental rates and the human capital shock process and not by any other model parameters.<sup>14</sup>

This logic is restated in the equations below. The first equation indicates that the wage  $w_{t+s}$  is determined by the rental rate  $R_{t+s}$ , shocks  $(z_{t+1}, \dots, z_{t+s})$  and human capital  $h_t$ , in the absence of human capital investment from period  $t$  to  $t+s$ . Here it is understood that  $h_{t+1} = \exp(z_{t+1})H(h_t, l_t, a) = \exp(z_{t+1})[h_t + f(h_t, l_t, a)]$  and that  $H(h, 0, a) = h$  in all periods with no investment. The second equation takes logs of the first equation, where a hat denotes the log of a variable.

$$w_{t+s} \equiv R_{t+s}h_{t+s} = R_{t+s}\exp(z_{t+s})H(h_{t+s-1}, 0, a) = R_{t+s} \prod_{j=1}^s \exp(z_{t+j})h_t$$

$$\hat{w}_{t+s} \equiv \ln w_{t+s} = \hat{R}_{t+s} + \sum_{j=1}^s z_{t+j} + \hat{h}_t$$

Let measured s-period log wage differences (denoted  $y_{t,s}$ ) be true differences plus measurement error differences  $\epsilon_{t+s} - \epsilon_t$ . This is the first equation below. Human capital shocks and measurement errors are assumed to be independent and identically distributed over time and people. Furthermore,  $z_t \sim N(\mu, \sigma^2)$  and  $Var(\epsilon_t) = \sigma_\epsilon^2$ . These assumptions imply the three cross-sectional moment conditions below.

$$y_{t,s} \equiv \hat{w}_{t+s} - \hat{w}_t + \epsilon_{t+s} - \epsilon_t = \hat{R}_{t+s} - \hat{R}_t + \sum_{j=1}^s z_{t+j} + \epsilon_{t+s} - \epsilon_t$$

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<sup>14</sup>Heckman, Lochner and Taber (1998) use a similar line of reasoning to estimate rental rates across skill groups within a model which abstracts from idiosyncratic risk.

$$\begin{aligned}
E[y_{t,s}] &= \hat{R}_{t+s} - \hat{R}_t + s\mu \\
Var(y_{t,s}) &= s\sigma^2 + 2\sigma_\epsilon^2 \\
Cov(y_{t,s}, y_{t,r}) &= r\sigma^2 + \sigma_\epsilon^2 \text{ for } r < s
\end{aligned}$$

To make use of these moment restrictions, one needs to be able to measure the variable  $y_{t,s}$  and to have individuals for which the assumption of no time spent accumulating human capital is a reasonable approximation. The focus on older workers addresses both issues. Wage data for younger workers are potentially problematic for both issues. Specifically, on the first issue it may be difficult to accurately measure the wage rates emphasized in the model when measured time at work is a mix of work time and learning time.

We calculate wages in PSID data as total male labor earnings divided by total hours for male head of household. We follow males for four years and thus calculate three log wage differences (i.e.  $y_{t,s}$  for  $s = 1, 2, 3$ ).<sup>15</sup> In utilizing the wage data we impose the same selection restriction as in the construction of the age-earnings profiles but also exclude observations for which earnings growth is either above (below) 20 (1/20) to trim potential extreme measurement errors. In estimation we use cross-sectional variances and all cross-sectional covariances aggregated across panel years. For each year we generate the sample analog to the moments:  $\mu_{t,s} \equiv \frac{1}{N_t} \sum_{i=1}^{N_t} y_{t,s}^i$  and  $\frac{1}{N_t} \sum_{i=1}^{N_t} (y_{t,s}^i - \mu_{t,s})^2$  and  $\frac{1}{N_t} \sum_{i=1}^{N_t} (y_{t,s}^i - \mu_{t,s})(y_{t,r}^i - \mu_{t,r})$ . We stack the moments across the panel years and use a 2-step General Method of Moments estimation with an identity matrix as the initial weighting matrix.

Table 1 provides the estimation results. The top panel of Table 1 considers the sample of males age 55-65 and two comparison samples. The first of these comparison samples considers males age 50-60, whereas the second pools all males age 23-60 together. The point estimate for the age 55-65 sample is  $\sigma = .106$  so that a one standard deviation shock moves wages by about 11 percent. We find that for the age 50-60 and the age 23-60 sample the point estimates are  $\sigma = .114$  and  $\sigma = .153$  respectively. Thus, the shock estimate is slightly larger for the 50-60 age group and is substantially larger for the 23-60 age group.

The remainder of Table 1 examines sensitivity in two directions. First, the middle panel of Table 1 examines sensitivity to alternative minimum annual earnings levels stated in 1968 dollars. We find that the point estimate of  $\sigma$  increases for males age 55-65 as the minimum earnings level in the sample is lowered. As a reference point, we note that a full-time worker

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<sup>15</sup>The PSID data is not available for the years 1997, 1999, 2001, and 2003.

(working 40 hours per week and 52 weeks) who receives the federal minimum wage earns 3,328 dollars in 1968 prices. Second, the bottom panel of Table 1 considers estimates based on different subperiods. The point estimates for the 1969-1981 period are lower across the board compared to the 1982-2004 period. It is well known that cross-sectional measures of earnings and wage inequality increased over the period 1982-2004.

### 3.3 Discussion

The benchmark model examined in the paper imposes that shocks to human capital arriving each period have a standard deviation set to  $\sigma = .106$  - the point estimate from the first row of Table 1. Although the empirical strategy for estimating this parameter is consistent with the structure of the theory developed, one might ask whether such shocks are consistent with several additional dimensions of the data. We now discuss these in turn:

**Dimension 1:** Are the shocks consistent with wage data?

Heathcote, Storesletten and Violante (2006) estimate that the persistent component of male wage shocks in their PSID sample averages  $\sigma_{HSV} = .136$  across years. Their estimate is based on wage rate data for male workers age 20-59. Our estimate from Table 1 is  $\sigma = .106$ . Thus, one might question whether our shocks are too small. However, under the theory developed here, it is not surprising that an estimate based on data for an older age group is smaller than for younger age groups or for pooled age groups. The reason is that the variance of log wage differences for younger age groups is in theory determined both by shocks and by endogenous responses to shocks and initial conditions. More specifically, measured log wage differences have an extra human capital production term, stated below, when human capital production is non-zero. Thus,  $Var(y_{t,s})$  may rise faster with the gap  $s$  between periods for younger age groups or pooled age groups than for older age groups.<sup>16</sup>

$$w_{t+s} = R_{t+s} \exp(z_{t+s}) H(h_{t+s-1}, l_{t+s-1}, a) = R_{t+s} \exp(z_{t+s}) \left(1 + \frac{f(h_{t+s-1}, l_{t+s-1}, a)}{h_{t+s-1}}\right) h_{t+s-1}$$

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<sup>16</sup>Unlike the estimates of wage shocks in the literature, our estimates in the top panel of Table 1 do not adjust log wage differences for differences related to age, education or other individual characteristics.

$$y_{t,s} = \hat{R}_{t+s} - \hat{R}_t + \sum_{j=1}^s z_{t+j} + \sum_{j=1}^s \ln\left(1 + \frac{f(h_{t+j-1}, l_{t+j-1}, a)}{h_{t+j-1}}\right) + \epsilon_{t+s} - \epsilon_t$$

**Dimension 2:** Are the shocks consistent with earnings data?

A large literature estimates statistical models of earnings. They are used as an exogenous input into incomplete-market models to analyze wealth inequality, consumption smoothing and many other issues. A typical model posits that log earnings equal a deterministic component influenced by individual characteristics plus a stochastic component. The stochastic part of earnings has a permanent component and a persistent component. A common view is that the persistent component is especially important for policy directed at shocks over the lifetime and for an assessment of lifetime inequality.

We show that if one uses earnings data produced by our model and applies standard estimation procedures, one finds that the autoregressive structure of the persistent shocks and the innovation variance are roughly in line with estimates from the literature. This is interesting as the driving shocks within our model are serially uncorrelated. These results are provided in section 5 after all model parameters are specified and after some quantitative features of the model are documented.

**Dimension 3:** Are the shocks consistent with consumption data?

A common view (e.g. Storesletten, Telmer and Yaron (2004)) is that a useful model of inequality over the life cycle should be broadly consistent with the rise in both earnings and consumption dispersion over the life cycle. The rise in consumption dispersion will reflect in part the impact of uninsured shocks. We set model parameters to match the rise in earnings dispersion, given the shock estimates from Table 1. We do not target the rise in consumption dispersion. Nevertheless, we show in section 5 that the resulting model produces a rise in consumption dispersion that lies between estimates of this rise from U.S. data.

## 4 Setting Model Parameters

The strategy for setting model parameters is in three steps. First, we estimate the key parameter governing human capital shocks directly. This was done in the previous section. Second, we choose parameters governing the utility function, the interest rate and the ag-

gregate production function. These choices are based on a number of considerations but they are not selected to match the earnings distribution facts. Third, we set the parameters governing the distribution of initial conditions and the parameter governing the elasticity of the human capital production function so that the model best matches the age profiles of the evolution of the male earnings distribution documented in the previous section. In choosing the initial distribution and elasticity, we take all model parameters, other than the discount factor, as given.

Model parameter values relevant for solving an agent's decision problem are summarized in Table 2. We set the model period to be a year which is the time period used to measure earnings data. Agents live  $J = 56$  model periods or from a real-life age of 20 to 75. We set a retirement age at  $J_R = 42$  or at a real-life age of 61. At the retirement period an agent can no longer engage in market work. The real interest rate in the model is set to  $r = 0.042$ . This is the average of the annual real return to stock and long-term bonds over the period 1946-2001 from Siegel (2002, Table 1-1 and 1-2).

The utility function is of the constant relative risk aversion class. The parameter  $\rho$  governing risk aversion and intertemporal substitution is set to  $\rho = 2$ . This value lies in the middle of the range of estimates based upon micro-level data which are surveyed by Browning, Hansen and Heckman (1999, Table 3.1). This parameter value is the value used in Storesletten et. al. (2004) in their analysis of lifetime inequality.

We set the value  $g$  governing the growth in the rental rate of human capital in the model equal to the average growth rate of mean male real earnings in US cross-section data. We calculate that in the PSID over the period 1968-2001 the mean arithmetic growth rate of mean male earnings equals 0.19 percent. The benchmark model, with homothetic preferences, implies that the earnings distribution of different cohorts is proportional to the initial level of this rental rate, other things equal. Thus, in a balanced-growth equilibrium the average cross-sectional earnings in the model grows at rate  $g$ .

We set the standard deviation  $\sigma$  of the log human capital shocks to  $\sigma = 0.106$  based on the estimate from Table 1. We set  $\mu$ , governing the mean log human capital shock, so that the model matches the average rate of decline of mean earnings for the cohorts of older workers in U.S. data documented in Figure 1. The ratio of mean earnings between adjacent model periods equals  $(1 + g)e^{\mu + \sigma^2/2}$  when agents make no human capital investments. Thus,  $\mu$  is set, given the value  $g$  and  $\sigma$ , so that mean earnings in the model late in life fall at

the rates documented in Figure 1. The quantity  $E[\exp(z)] = e^{\mu+\sigma^2/2}$  can be viewed as one minus the mean rate of depreciation of human capital. The values in Table 2 imply a mean depreciation rate of just over 1 (2) percent per year for the case of cohort (time) effects.

We assume that  $H(h, l, a) = h + a(hl)^\alpha$  which is the functional form analyzed in Ben-Porath (1967). We assume that the distribution of initial conditions  $\psi$  is lognormal so that  $\ln(x) \sim N(\mu_x, \Sigma)$ , where  $x = (h_1, a)$ . In the benchmark model initial assets are zero for all agents. Later in the paper we explore a tri-variate distribution, where the initial asset distribution matches features of net-wealth holdings for young households in the PSID. We set the elasticity parameter  $\alpha$  and the parameters governing the initial distribution so the age-profiles (means, log variance, and skewness) implied by the model best match their U.S. data counterparts. The Appendix describes the distance metric between data and model statistics that we minimize. For the case of time effects the elasticity parameter is  $\alpha = .70$  and initial conditions are described by  $(\mu_a, \mu_h, \sigma_a^2, \sigma_h^2, \rho_{ha}) = (-1.08, 4.63, .023, .186, .805)$ . For the case of cohort effects  $\alpha = .625$  and initial conditions are described by  $(\mu_a, \mu_h, \sigma_a^2, \sigma_h^2, \rho_{ha}) = (-.938, 4.48, .089, .164, .719)$ .<sup>17</sup>

We have examined the fit of the model at prespecified values of the parameter  $\alpha$ , while choosing the parameters of the initial distribution to best match the earnings distribution facts. The distance between model and data statistics displays a U-shaped pattern in the parameter  $\alpha$ , where the bottom of the U is the value in Table 2. For values of  $\alpha$  above the values in Table 2 the model produces too large of a rise in dispersion and skewness compared to the patterns in Figure 1. The values of  $\alpha$  that minimize the distance between model and data moments are  $\alpha = 0.70$  for the time effects case and  $\alpha = 0.625$  for the cohort effects case. The parameter  $\alpha$  has been estimated in the human capital literature. The estimates, surveyed by Browning et. al. (1999, Table 2.3- 2.4), lie in the range 0.5 to just over 0.9. These estimates are based upon models that abstract from idiosyncratic risk.

We focus on balanced-growth equilibria. Thus, we need to specify an aggregate production function  $F$ , a depreciation rate on physical capital  $\delta$  and a population growth rate  $n$ . The production function is Cobb-Douglas  $Y = F(K, LA) = K^\gamma(LA)^{1-\gamma}$ , where  $1 - \gamma = .678$  is labor's share of output. The population growth rate is  $n = .012$ . The depreciation rate is  $\delta = .067$ . The implied equilibrium capital-output ratio is  $K/Y = 2.947$ .<sup>18</sup> We note that

<sup>17</sup>It is important to note that the model does not trivially fit the age profiles. After estimating the process for shocks, there are 5 parameters governing initial conditions and 1 parameter governing human capital production to fit 3 age profiles,  $3 \times 38$  moments.

<sup>18</sup>The population growth rate is the 1959-2007 geometric average from the (EROP) Economic Report of

the discount factor  $\beta$  is set, given all other model parameters, to produce an equilibrium capital-output ratio of  $K/Y = 2.947$ . The values of  $\beta$  are listed in Table 2.<sup>19</sup>

## 5 Properties of the Benchmark Model

In this section, we report on the ability of the benchmark model to reproduce the earnings facts documented in section 3. We also provide a number of other properties of the benchmark model to help understand how the model works and to address issues raised at the end of section 3.

### 5.1 Dynamics of the Earnings Distribution

The age profiles of mean earnings, earnings dispersion and skewness produced by the benchmark model are displayed in Figure 2 for the time effects view and in Figure 3 for the cohort effects view of earnings distribution dynamics. The procedure for setting model parameters so that model age profiles for mean earnings, the variance of log earnings and earnings skewness best match age profiles from U.S. data was discussed in the last section.

For both the time and cohort views, the model generates the hump-shaped earnings profile for a cohort by a standard human capital argument. Early in the working life cycle, individuals devote more time to human capital production than at later ages. These time allocation decisions lead to a net accumulation of human capital in the early part of the working life cycle. Thus, mean earnings increase with age as human capital and mean time worked increase with age.

Towards the end of the working life-cycle, mean human capital levels fall. This happens as the mean multiplicative shock to human capital is smaller than one (i.e.  $E[\exp(z)] = e^{\mu+\sigma^2/2} < 1$ ). This corresponds to the notion that on average human capital depreciates.

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the President (2008, Table B35). Labor's share is the 1959-2006 average based upon EROP (2008, Table B26 and B28) and Bureau of Economic Analysis (2008, Table 1.1.2) and is calculated as the compensation of employees divided by national income plus depreciation less proprietor's income and indirect taxes. Following the methodology described in Cooley and Prescott (1995), we calculate that the average US capital-output ratio over the period 1959-2000 is  $K/Y = 2.947$  when the capital measure includes fixed private capital, durables, inventories and land. The data for capital and land are from Bureau of Economic Analysis (Fixed Asset Account Tables) and Bureau of Labor Statistics (Multifactor Productivity Program Data).

<sup>19</sup>Methods for computing balanced-growth equilibria are described in the Appendix.



The implication is that average earnings in Figure 2 and 3 fall late in life because growth in the rental rate of human capital is not enough to offset the mean fall in human capital.

Figure 4 graphs the age profiles of the mean fraction of time allocated to human capital production and the mean human capital levels that underlie the earnings dynamics in the model. Figure 4(a) shows that approximately 25 percent of available time is directed at human capital production early in life but less than 5 percent of available time is used after age 55. This result is consistent with a key assumption we employed to identify human capital shocks: human capital production is negligible towards the end of the working lifetime.

Figure 4(b) shows that the mean human capital profile is hump shaped and that it is flatter than the earnings profile. A relatively flat mean human capital profile and a declining time allocation profile to human capital production is how the model accounts for a hump-shaped earnings profile. This point is quite important. The fact that the mean human capital profile is flatter than the earnings profile means that initial mean human capital is quite high. This a key reason behind why we find in the next section that human capital differences are such an important source of individual differences at age 20 compared to ability differences. We note that the mean human capital profile for the cohort effects case increases more over the lifetime than the profile for time effects. The lower value of mean initial human capital under the cohort effects view is part of the reason behind the slightly smaller role for human capital differences and the larger role for ability differences under cohort effects compared to time effects that we find later in the paper.

Two forces account for the rise in earnings dispersion. First, since individual human capital is repeatedly hit by shocks, these shocks are a source of increasing dispersion in human capital and earnings as a cohort ages. Second, differences in learning ability across agents produce mean earnings profiles with different slopes. This follows since within an age group, agents with high learning ability choose to produce more human capital and devote more time to human capital production than their low ability counterparts. Huggett et. al. (2006, Proposition 1) establish that this holds in the absence of human capital risk. This mechanism implies that earnings of high ability individuals are relatively low early in life and relatively high late in life compared to agents with lower learning ability, holding initial human capital equal.

## 5.2 Earnings Dispersion: Risk Versus Ability Differences

We now try to understand the quantitative importance of risk and ability differences for producing the increase in earnings dispersion in the benchmark model. We do so by either eliminating ability differences or eliminating shocks. The analysis holds factor prices constant as risk or ability differences are varied.

We eliminate ability differences by changing the initial distribution so that all agents have the same learning ability, which we set equal to the median ability. In the process of changing learning ability, we do not alter any agent's initial human capital. Figure 5(a) and 5(c) show that eliminating ability differences flattens the rise in the variance of log earnings with age. Even more striking, earnings dispersion actually falls over part of the working lifetime. This latter result is due to two opposing forces. First, human capital risk leads ex-ante identical agents to differ ex-post in human capital and earnings. Second, the model has a force which leads to decreasing dispersion in human capital and earnings with age. This force has received almost no attention in work which interprets patterns of earnings dispersion over the lifetime. Without risk and without ability differences, all agents within an age group produce the same amount of new human capital regardless of the current level of human capital – see Huggett et. al. (2006, Proposition 1). This holds for any value of the elasticity parameter  $\alpha \in (0, 1)$  of the human capital production function. This implies that both the distribution of human capital and earnings are Lorenz ordered by age. Thus, measures of earnings or human capital dispersion that respect the Lorenz order (e.g. the Gini coefficient) decrease for a cohort as the cohort ages.

Figure 5(a) and 5(c) show that earnings dispersion increases at the very end of the working lifetime. This occurs as human capital production at the end of life goes to zero because the time allocated to production goes to zero. This means that the opposing force leading to convergence is gradually eliminated with age.

To highlight the role of human capital risk, we eliminate idiosyncratic risk altogether by setting  $\sigma = 0$ . We adjust the mean log shock  $\mu$  to keep the mean shock level constant but maintain all other initial conditions. Removing idiosyncratic risk leads to a counter-clockwise rotation of the mean earnings profile and a U-shaped earnings dispersion profile. Figure 5(b) and 5(d) display these results.

When idiosyncratic risk is eliminated, human capital accumulation becomes more attractive for risk-averse agents. Thus, all else equal, agents spend a greater fraction of time

accumulating human capital early in life. The result is a counter-clockwise movement in the mean earnings profile.<sup>20</sup> In terms of dispersion in labor earnings, eliminating risk significantly changes the time allocation decisions of agents with relatively high learning ability. Absent risk, these agents allocate an even larger fraction of time into human capital accumulation. This leads to a U-shaped dispersion profile since early in life these agents have lower earnings and later in life have higher earnings than agent's with lower learning ability, other things equal.

### 5.3 Properties of the Initial Distribution

Table 3 summarizes properties of the distribution of initial conditions for the time and cohorts effects views of earnings distribution data. These views differ notably in that the level of earnings dispersion early in life is greater in the time effects view and in that the rise in earnings dispersion over the lifetime is smaller in the time effects view. These views also differ in that the mean earnings profile has a smaller rise in mean earnings over the lifetime under the time effects view.

We highlight three important differences in initial conditions that help account for the differences in the time and cohort effects views. These are that the mean initial human capital is larger, the coefficient of variation in human capital is larger and the coefficient of variation in learning ability is lower under the time effects view compared to the cohort effects view. Intuitively, the model produces a higher initial level of earnings dispersion with more initial human capital dispersion. Similarly, holding the shock variance unchanged, the model produces a smaller rise in earnings dispersion with a smaller amount of ability dispersion. Recall that shocks and ability differences are the two forces within the model that can generate a rise in earnings dispersion with age.

One interesting property in Table 3 is that human capital and learning ability are positively correlated at age 20 in the initial distributions which best account for the earnings facts. To develop some intuition for this result, consider two agents who differ in learning ability but have the same initial human capital. The mean earnings profile for the agent with higher learning ability is rotated counter clockwise from his lower ability counterpart due to the extra time spent in learning early in life. Thus, if there were a zero correlation

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<sup>20</sup>Levhari and Weiss (1974) examine this issue in a two-period model. They show that time input into human capital production is smaller with human capital risk than without when agents are risk averse.

in learning ability and human capital at age 20, then the model would produce a U-shaped earnings dispersion profile. The rise in dispersion later in life would be due in part to high ability agents overtaking the earnings of low learning ability agents. Given that Figure 1 does not support a strong U-shaped dispersion profile over ages 23-60 in U.S. data, the model accounts for this fact by making learning ability and human capital positively correlated at age 20. Thus, if high learning ability agents also have high initial human capital, then this produces an upward shift of an agent's mean earnings profile to eliminate the strong U-shaped dispersion profile that otherwise would occur.

## 5.4 Inferring Earnings Shocks Using Model Data

We now examine what an empirical researcher might conclude about the nature of earnings risk in the benchmark model from the vantage point of a standard statistical model of earnings. A common view in the literature is that log earnings is the sum of a predictable component capturing age effects among other things and an idiosyncratic component with mean zero. The latter is the sum of a fixed effect, a persistent shock and a purely temporary component  $(\alpha^i, z_{j,t}^i, \epsilon_{j,t}^i)$ , respectively, where  $i$  indexes an agent and  $j$  and  $t$  index age and time. That is log earnings are assumed to follow,

$$\begin{aligned} \log e_{j,t}^i &= g(\theta_t, X_{j,t}^i) + \alpha^i + z_{j,t}^i + \epsilon_{j,t}^i \\ z_{j,t}^i &= \rho z_{j-1,t-1}^i + \eta_{j,t}^i, \quad z_{1,t}^i = 0 \end{aligned}$$

where  $\rho$  is the autocorrelation of the persistent component and  $\sigma_\alpha$ ,  $\sigma_\eta$ , and  $\sigma_\epsilon$  are the standard deviation of the fixed effects, and innovations to the persistent and transitory component respectively. The variables  $(\alpha^i, \eta_{j,t}^i, \epsilon_{j,t}^i)$  are uncorrelated.

This type of a model has been extensively examined in the literature (e.g., Hubbard, Skinner and Zeldes (1995), Storesletten, Telmer and Yaron (2004) and Guvenen (2007)). Table 4 presents what an empirical researcher would conclude about the nature of earnings data drawn from our benchmark human capital model. Specifically, we simulate the model 500 times with each simulation having 200 observations per age group. We add zero mean and normally distributed measurement error to the log of model earnings. The standard deviation of the measurement error is set to 0.15 following the estimate in Guvenen and Smith (2008). For each simulated panel we use variances and autocovariances to estimate the

parameters.<sup>21</sup> Table 4 shows the point estimates and the standard errors of the parameters of the earnings dynamics when the data is simulated from the cohort and time effects view. The standard errors are simply the standard deviation of the point estimates across the 500 samples.

Table 4 shows that when we follow agents for only two periods (1 covariance term per person) the persistent component has an autoregressive coefficient of .983, .959 with an innovation variance of .018, .021 for the cohort and time effect views respectively. These findings are not far from the properties that Storesletten, Telmer and Yaron (2004) and Guvenen (2007) find in US data especially after accounting for standard errors. The former (latter) paper provides an autoregressive coefficient estimate of 0.984 (0.988) and an innovation variance of .022 (.015) respectively. Table 4 also shows that these results are quite robust to changing the number of years we follow each agent (from 2 to 6). This extends the number of autocovariances used in the estimation. Finally, a researcher would conclude that there are substantial permanent differences in the fixed effect component of earnings as its variance is .179, .218 respectively for the cohort and time effects view.

We also analyze the autocorrelation structure of first differences of log earnings. It is well known in the labor literature that these earnings growth rates display negative first-order autocorrelation and close to zero higher-order autocorrelations (e.g., MaCurdy (1981) and Abowd and Card (1989)). Using the simulated earnings data from our model, we find that the first-order, autocorrelation coefficient is  $-.393, -.397$  for the time and cohort effects view. As a reference, Abowd and Card (1989) estimates range from -0.23 to -0.44 across sample years. Higher order autocorrelations of model data are found to be statistically indistinguishable from zero. In contrast, Huggett, Ventura and Yaron (2006) find that the Ben-Porath model, which abstracts from shocks, generates autocorrelation coefficients for earnings growth rates that are large and positive. Hence, this dimension of the data is useful in highlighting the important role that human capital shocks play within our framework.

In sum, the analysis in this subsection clearly indicates that our benchmark human capital model, with human capital shocks inferred from wages of older workers, is broadly consistent with the dynamics of earnings and earnings growth rates as documented in the literature.

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<sup>21</sup>First, we estimate the deterministic component to calculate estimated residuals. This amounts to calculating the sample mean of log earnings at each age. Second, calculate sample variances and covariances of the residuals. Third, the estimate is the parameter vector minimizing the equally-weighted squared distance between sample and model moments. All variance and covariance restrictions of the model are used in estimation.

## 5.5 Consumption Implications

A common view is that a useful model for analyzing lifetime inequality within an incomplete-markets framework should also be broadly consistent in terms of its implications for consumption inequality. We therefore compare the model’s implications for the rise in consumption dispersion over the lifetime with the patterns found in U.S. data. A number of studies analyze the variance of log adult-equivalent consumption in U.S. data by regressing the variance of log adult-equivalent consumption for households in different age groups on age and time dummies or alternatively on age and cohort dummies. The coefficients on age dummies are then used to highlight how consumption dispersion varies for a cohort with age.

Figure 6 plots the variance of log adult-equivalent consumption in U.S. data from three such studies and from the model economy. Deaton and Paxson (1994) analyze U.S. Consumer Expenditure Survey (CEX) data from 1980 to 1990. Heathcote et. al. (2005), Slesnick and Ulker (2005), Primiceri and van Rens (2006) and Aguiar and Hurst (2008) examine this issue using CEX data over a longer time period. All of these later studies find that the rise in dispersion with age is smaller than the rise in Deaton and Paxson (1994). Aguiar and Hurst (2008) find that the rise is somewhat larger than that found by Heathcote et. al. (2005a), Slesnick and Ulker (2005) and Primiceri and van Rens (2006). Aguiar and Hurst (2008) note that the increase in the variance is about 12 log points when consumption is measured as total nondurable expenditures but about 5 log points when consumption is measured as core nondurable expenditures.

The exogenous earnings model analyzed in Storesletten et. al. (2004) produces the rise in consumption dispersion documented in Deaton and Paxson (1994). This is the case when their exogenous earnings model has a social insurance system. Without social insurance, their model produces a rise in dispersion greater than the rise in Deaton and Paxson (1994). We analyze the consumption implications both within the benchmark model and when we add a social insurance system to the benchmark model.<sup>22</sup> The model with a social insurance system is analyzed in general equilibrium and produces the factor prices described in Table 2. The elasticity parameter of the human capital production function is set to the value in

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<sup>22</sup>The social insurance system includes a social security system and an income tax. Social security has a proportional earnings tax of 10.6 percent, the old-age and survivors insurance tax rate in the U.S., and a common retirement benefit paid to all agents after the retirement age set equal 40 percent of mean earnings in the last period of the working lifetime. The income tax in the model captures the pattern of effective average federal tax rates as documented in Congressional Budget Office (2004, Table 3A and 4A). Details for implementing this income tax function follows closely Huggett and Parra (2006).

Table 2 but initial conditions are selected to best match the earnings facts.

Figure 6 shows that the rise in consumption dispersion within the model is always less than the rise in Deaton and Paxson (1994) and always more than the rise in Primiceri and van Rens (2006). This holds for both for the model with and without social insurance and for the case of both time and cohort effects. The rise in the model from age 25 to 60 is approximately 8 log points for the benchmark model with social insurance in both the time and cohort effects cases. This 8 log point rise lies between the estimates of the rise for core nondurable expenditures and total nondurable expenditures found by Aguiar and Hurst (2008).

Our model with social insurance produces less of a rise in consumption dispersion than the model with social insurance analyzed by Storesletten et. al. (2004). A key reason for this is that part of the rise in earnings dispersion in our model is due to initial conditions. These differences are foreseen and therefore built into consumption differences early in life. This mechanism relies on agents knowing their initial conditions. Cunha, Heckman and Navarro (2005), Guvenen (2007) and Smith and Guvenen (2008) analyze the information individuals have about future earnings as revealed by economic choices. They conclude that much is known early in life about future earnings prospects.

## 6 Lifetime Inequality

We now use the model analyzed in the previous section to answer the two lifetime inequality questions posed in the introduction.

### 6.1 Initial Conditions Versus Shocks

We decompose the variance in lifetime inequality into variation due to initial conditions versus variation due to shocks. This is done for lifetime utility and lifetime earnings in the benchmark model.<sup>23</sup> Such a decomposition makes use of the fact that a random variable can be written as the sum of its conditional mean plus the variation from its conditional mean.

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<sup>23</sup>Lifetime utility and lifetime earnings along a shock history  $z^J$  from initial condition  $x_1 = (h_1, k_1, a)$  are defined as  $U(x_1, z^J) = \sum_{j=1}^J \beta^{j-1} u(c_j(x_1, z^j))$  and  $E(x_1, z^J) = \sum_{j=1}^J e_j(x_1, z^j)/(1+r)^{j-1}$ . The decomposition of lifetime utility and earnings are invariant to an affine transformation of the period utility function.

As these two components are orthogonal, the total variance equals the sum of the variance in the conditional mean plus the variance around the conditional mean.

Table 5 presents lifetime inequality within the benchmark model. Lifetime inequality is analyzed as of the start of the working life cycle, which we set to a real-life age of 20 following Storesletten et. al. (2004). We find 62 percent of the variation in lifetime earnings and 67 percent of the variation in lifetime utility is due to initial conditions for the time effects view of earnings distribution data. In the cohort effects view, we find that 61 percent of the variation in lifetime earnings and 65 percent of the variation in lifetime utility is due to initial conditions. Thus, the majority of the variation in either lifetime utility or lifetime earnings is due to initial conditions for both views of the evolution of the earnings distribution. Moreover, the overall importance of initial conditions is nearly the same in both views.

Figure 7 describes lifetime inequality as the elasticity parameter  $\alpha$  of the human capital production function is varied over the interval  $[\.5, .9]$ . This interval includes the values,  $\alpha = .70$  and  $\alpha = .625$ , that best match the earnings profiles for the time and cohort effect views. Figure 7 shows that the fraction of the variance in lifetime earnings that is due to initial conditions tends to fall as the elasticity parameter increases. These findings are related to some results in Kuruscu (2006). He analyzes the importance of training in producing differences in lifetime earnings in models without idiosyncratic risk. An interpretation of one aspect of his work is that as the elasticity parameter increases, then the marginal benefit and marginal cost curves from producing extra units of human capital move closer together.<sup>24</sup> Lifetime earnings for agent's with quite different learning abilities but the same initial human capital will not differ strongly when this holds.

### 6.1.1 Allowing Initial Wealth Heterogeneity

The benchmark model abstracts from initial wealth differences. This is a potentially important source of lifetime wealth and lifetime utility differences. We now determine how the decomposition of lifetime inequality changes when we account for variation in initial wealth found in U.S. data. To examine this issue, we use PSID net-wealth data for households with

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<sup>24</sup>In the Ben-Porath model, the marginal benefit of extra units of human capital is constant at any age, whereas the marginal cost of producing an additional unit is  $MC_j(q; a) = \frac{R_j}{\alpha a} (\frac{q}{a})^{\frac{1}{\alpha}-1}$  when total production is  $q$ .



a male head age 20 to 25.<sup>25</sup> For each male household head age 20 to 25 in a given year, we calculate net wealth as a ratio to mean earnings of males in this age group. We then pool these ratios across years.

We maintain the multi-variate log-normal structure for describing initial conditions. However, we do allow for negative wealth holding. Specifically, we approximate the empirical pooled wealth distribution with a lognormal distribution which is shifted a distance  $\delta$ . We choose  $\delta$  so that 95 percent of the distribution has a wealth to mean earnings ratio above  $-\delta$ . The distribution of the wealth to mean earnings ratio in the model is given by  $e^x - \delta$ , where  $x$  is distributed  $N(\mu_1, \sigma_1^2)$ . The parameters  $(\mu_1, \sigma_1^2)$  are set equal to the sample mean and sample variance of the log of the sum of the wealth to earnings ratio plus  $\delta$  for ratios above  $-\delta$ . The median, mean and standard deviation of the wealth to mean earnings ratio in the model is then  $(0.377, 0.778, 1.340)$ .<sup>26</sup> This implies that there is a substantial amount of initial wealth dispersion within the model. Specifically, a one standard deviation change in initial wealth is 1.34 times mean yearly earnings for young agents.

The distribution of initial wealth, human capital and learning ability is selected to best match the earnings facts documented earlier. The distribution is a tri-variate lognormal, where the parameters describing the mean and variance of shifted log wealth are those calculated above in U.S. data. Thus, wealth in the model is skewed and mean wealth is more than double median wealth.<sup>27</sup>

Table 6 analyzes lifetime inequality with initial wealth differences. We find that initial conditions now account for 67 to 71 percent of the variation in lifetime utility, 62 to 63 percent of the variation in lifetime earnings and 64 to 66 percent of the variation in lifetime wealth. Lifetime wealth equals lifetime earnings plus initial wealth. Thus, when we account for initial wealth differences the importance of initial conditions for lifetime wealth or lifetime utility increases slightly over our findings for the benchmark model.

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<sup>25</sup>The data is from the PSID wealth supplement for 1984, 1989, 1994, 1999, 2001 and 2003. The sample size is 1176 when pooled across these years.

<sup>26</sup>In the PSID sample we calculate that  $(\mu_1, \sigma_1^2, \delta) = (-0.277, 0.849, 0.381)$  and that the median, mean and standard deviation of the wealth to mean earnings ratio is  $(0.313, 0.776, 1.432)$ .

<sup>27</sup>This is a partial equilibrium exercise. We fix factor prices and all model parameters, except those describing the initial distribution, at the values specified in Table 2. A balanced-growth equilibrium with non-zero initial wealth would require some mechanism for producing the initial wealth distribution.

### 6.1.2 Sensitivity: Social Insurance, Shocks and Moments

We examine the sensitivity of lifetime inequality in the benchmark model to three changes. First, we add a social insurance system to the model. The social insurance system is the one analyzed in section 5.5 and includes both social security and federal income taxation. We have already seen that adding such a tax-transfer program dampens the rise in consumption dispersion over the lifetime. To the degree that such a system differentially provides insurance over initial conditions as opposed to shocks over the working lifetime, one might expect the decomposition of lifetime inequality to change. The direction of change is not obvious. For example, income taxation is not so clearly targeted at initial conditions or at shocks. A high income tax payment in a period may result from either high initial human capital or favorable shocks over the lifetime.

We find that the decomposition of lifetime inequality changes very little when this social insurance system is added to the benchmark model. Previously, Table 5 documented that the fraction of the variance in lifetime utility due to initial conditions was (.667, .646) for the time and cohort effects cases. With social insurance, the corresponding fractions are (.625, .621). Thus, there is a modest reduction in the fraction of variance coming from initial conditions with social insurance. We suspect that a more realistic treatment of the social security retirement benefit payment would raise these fractions.

Second, we analyze lifetime inequality when the shock variance is moved up or down by our estimate of the standard error of the shock variance from Table 1. Thus, we examine a low shock case  $\sigma = .091$  and a high shock case  $\sigma = .121$  by changing the point estimate (i.e.  $\sigma = .106$ ) up or down by the estimated standard error. In each case the parameter controlling the mean shock is adjusted to match the fall in mean earnings at the end of the working lifetime.

Intuitively, when the driving shocks are smaller, then initial conditions must play a larger role in accounting for the increase in earnings dispersion over the lifetime and, thus, account for more of the variance in lifetime earnings and utility. This is precisely what occurs. In the low shock case initial conditions account for fractions (.722, .710) of the variance in lifetime earnings and for fractions (.748, .723) of lifetime utility for the time and cohort effects cases. The results for the high shock case are (.495, .495) for lifetime earnings and (.571, .545) for lifetime utility. Thus, the importance of initial conditions in the benchmark model is fairly

sensitive to the precision of the estimate of  $\sigma$ .<sup>28</sup>

Third, we change the data moments that we select initial conditions to match. We now set initial conditions to match both the mean earnings profile and the age profiles of the 90-10 and 75-25 quantile ratios of the earnings distributions. These ratios will not emphasize earnings observations in the extreme tails of the distribution. With these new moments we find that initial conditions account for fractions (.549, .578) of the variance in lifetime earnings and for fractions (.609, .621) of lifetime utility for the time and cohort effects cases. Thus, by not emphasizing the tails the importance of initial conditions falls by few percentage points.

## 6.2 How Important are Different Initial Conditions?

The analysis so far has not addressed how important variation in one type of initial condition is compared to variation in other types. We analyze the importance of different initial conditions at age 20 by asking an agent how much compensation is equivalent to starting at age 20 with a one standard deviation change in any initial condition. We express this compensation, which we call an equivalent variation, in terms of the percentage change in consumption in all periods that would leave an agent with the same expected lifetime utility as an agent with a one standard deviation change in the relevant initial condition. The baseline initial condition is set equal to the mean log values of initial human capital and learning ability and equal to the mean of the shifted log initial wealth to earnings ratio. The changes in initial conditions are also in standard deviations of log variables.

The analysis of equivalent variations is presented in the upper panel of Table 7. We find that a one standard deviation movement in log human capital is substantially more important than a one standard deviation movement in either log learning ability or log initial wealth. A one standard deviation increase in initial human capital is equivalent to a 33 – 37 percent increase in consumption. In contrast, a one standard deviation increase in learning ability or initial wealth is equivalent to an 6 – 11 percent and 5 – 6 percent increase in consumption, respectively. Thus, we find that an increase in human capital leads to the largest impact and an increase in learning ability or initial financial wealth have substantially smaller impacts.

We remind the reader that the distribution of initial conditions in the time and cohort

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<sup>28</sup>These results fix the elasticity coefficient of the human capital production function to their values in Table 2 while choosing initial conditions to best match the earnings distribution facts.

effects views differ. Specifically, in the cohort effects view the coefficient of variation in learning ability is much higher and the coefficient of variation in human capital is lower compared to their values in the time effects view. In addition, mean initial human capital is lower under cohort effects than under time effects. Despite this, it turns out that as of age 20 a one standard deviation increase in human capital is much more important than a one standard deviation increase in learning ability in both views of the earnings distribution dynamics.

We also analyze the importance of different initial conditions by determining how changes in initial conditions affect an agent's budget constraint. More specifically, we determine the percent by which an agent's expected lifetime wealth changes in response to a one standard deviation change in an initial condition. The lower panel of Table 7 presents the results of this analysis. In interpreting these results, it is useful to keep two points in mind. First, an increase in human capital acts as a vertical shift of the expected earnings profile, whereas an increase in learning ability rotates this profile counter clockwise. Second, the impact of additional initial financial wealth is both through a direct impact on lifetime resources as well as the indirect impact through earnings arising from different time allocation decisions.

Broadly speaking, the lower panel of Table 7 shows that the impact on expected lifetime wealth of changes in initial conditions is roughly in line with the calculation of equivalent variations. Thus, a one standard deviation change in initial human capital has the greatest impact and the corresponding changes in learning ability and initial wealth have substantially smaller impacts on expected lifetime wealth.

## 7 Concluding Remarks

This paper analyzes the proximate sources of lifetime inequality. We find that differences in initial conditions as of a real-life age of 20 account for more of the variation in realized lifetime utility, lifetime earnings and lifetime wealth than do shocks over the working lifetime. Among initial conditions evaluated at age 20, a one standard deviation change in human capital is substantially more important as of age 20 than either a one standard deviation change in learning ability or initial wealth for how an agent fares over the remaining lifetime.

The conclusions stated above come from a specific model and reflect the choice of a specific age to evaluate lifetime inequality. Below we discuss three issues that are useful for

providing perspective on these choices and the conclusions that depend upon them:

**Issue 1:** We address lifetime inequality as of age 20. This choice brings up several issues. First, analyzing lifetime inequality at later ages would likely produce an even greater importance for “initial conditions” established at later ages within our model. Second, analyzing lifetime inequality at even earlier ages might lead one to conjecture that the relative importance of learning ability compared to initial human capital might increase. This might hold if learning ability is crystallized before age 20 and produces some of the human capital differences as of age 20. We think that this is an interesting conjecture. However, pushing back the age at which lifetime inequality is evaluated will raise the issue of the importance of one’s family more directly than is pursued here. The importance of one’s family, or more broadly one’s environment, up to age 20 is not modeled in our work but is implicitly captured through their impact on initial conditions as of age 20: human capital, learning ability and initial wealth.

**Issue 2:** One can ask what key features of the data lead us to conclude that variation in human capital must be so important as of age 20 compared to learning ability or to initial wealth, given our model. Four features of the data are key: (i) the magnitude of the persistent component of wage variation at the end of the working lifetime, (ii) the steepness of the mean earnings profile, (iii) the amount of earnings dispersion early in the working lifetime together with the nature of the rise in dispersion at later ages and (iv) the distribution of financial wealth among young households.

These features of the data shape our answer. First, absent persistent wage variation among older workers, learning ability differences would have to account for all of the rise in earnings dispersion over the lifetime. Thus, a greater magnitude of such wage variation implies a smaller role for ability differences in accounting for rising earnings dispersion over the lifetime. Second, the shape of the mean human capital profile must be flatter than the shape of the mean earnings profile implying that mean human capital is quite high early in life. This limits the importance of learning ability differences as of age 20. Third, the dispersion in earnings early in life is large and this needs to be accounted for largely by differences in human capital. The lack of a strong U-shaped earnings dispersion profile dictates large differences in human capital early in life rather than solely large differences in learning ability and dictates a positive correlation between human capital and learning ability at age 20. Fourth, given our estimate of shocks, a larger rise in earnings dispersion over the lifetime dictates a larger role for learning ability differences. Thus, learning ability

differences play a larger role under the cohort effects view of the dynamics of the earnings distribution. Fifth, we find that a one standard deviation increase in initial financial wealth moves wealth by 1.43 times the mean earnings of young males. Such a change in initial wealth leads to only a small direct impact on expected lifetime wealth, given a working lifetime of at least 40 years.

**Issue 3:** Our model does not capture all shocks impacting individuals after age 20. Instead, the model has a single source of shocks impacting human capital. This shock structure successfully captures the permanent and persistent idiosyncratic earnings shocks highlighted by statistical models of earnings. It does not capture purely temporary idiosyncratic earnings shocks, any aggregate shocks or interactions between aggregate and idiosyncratic shocks. We doubt that adding a source of temporary earnings shocks to our model will substantially change our conclusions about lifetime inequality. Theoretical work by Yaari (1976) and simulations of related models suggest that shocks that impact earnings in a purely temporary way are reasonably well insured. It remains to be determined if a richer modeling of multiple sources of idiosyncratic shocks or the interaction of idiosyncratic and aggregate shocks will substantially change our conclusions. We conjecture that moments of the wages of older workers will continue to be valuable as part of a procedure to identify shocks in multiple-shock models nesting our model.

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# A Appendix: Computation

## A.1 Algorithm

We compute a balanced-growth equilibrium to the benchmark model. The algorithm finds a discount factor  $\beta$  so that the computed equilibrium has the factor prices specified in Table 2, given all other model parameters.

Algorithm:

1. Set  $(R_j, r)$  as specified in Table 2. Find  $(K_1/L_1, A_1)$  satisfying equilibrium condition 2 (i.e.  $R_1 = F_L(K_1, L_1 A_1)$  and  $r = F_K(K_1, L_1 A_1) - \delta$ ).
2. Compute solutions  $(c_j, h_j, L_j, k_j)$  to the agent's problem for the cohort of agents born at time 1, given a guess of the agent's discount factor  $\beta$ .
3. Compute the implied capital-labor ratio  $K'_1/L'_1$ :  

$$K'_1 \equiv \sum_j \mu_j \int E\left[\frac{k_j(x_1, z^j)}{(1+g)^{j-1}}\right] d\psi$$
 and  $L'_1 \equiv \sum_j \mu_j \int E[h_j(x_1, z^j)L_j(x_1, z^j)] d\psi$ .
4. If  $K'_1/L'_1 = K_1/L_1$ , then stop. Otherwise, update  $\beta$  and repeat steps 2-3.

## A.2 Solving the Agent's Problem

We compute solutions to the agent's problem using dynamic programming. The dynamic programming problem is given below, where the state is  $x = (h, k, a)$ . The model implies that the period borrowing limits should depend upon age, human capital, learning ability and the distribution of shocks. We impose ability-specific limits  $\underline{k}(a)$  and relax these limits until they are not binding. We also directly penalize choices leading to negative consumption later in life. This is a device for effectively imposing the endogenous limits implied by the model.

$$V_j(x) = \max_{(c, k', L)} u(c) + \beta E[V_{j+1}(h', k', a)]$$

$$\text{subject to } c + k' \leq R_j h L + k(1+r), \quad h' = \exp(z')H(h, l, a), \quad l + L = 1, \quad k' \geq \underline{k}(a)$$

We compute solutions by backwards recursion. We use a rectangular grid on the state variables  $(h, k)$  which is learning-ability specific. For each gridpoint and age  $j$ , we numerically solve the maximization problem on the right-hand-side of the Bellman's equation. To evaluate the objective,

we employ a bi-linear interpolation of  $V_{j+1}$  across gridpoints. To compute expectations, we follow Tauchen (1986) and discretize the shock into 5 equally-spaced values over the interval  $[\mu-2\sigma, \mu+2\sigma]$ . Proceeding in this way gives a computed value function  $V_j(x)$  and decision rules  $(c_j(x), k_j(x), L_j(x))$  at gridpoints.

### A.3 Computing Capital-Labor Ratios

To calculate implied capital-labor ratios, we put a grid on initial conditions  $(h_1, k_1, a)$ . We draw gridpoint  $x_1 = (h_1, k_1, a)$  with a probability proportional to the density of the distribution  $\psi$  at  $x_1$ . For any draw of an initial condition, we also draw a lifetime shock history  $z^J$ . We calculate realizations  $(k_j(x_1, z^J), h_j(x_1, z^J), L_j(x_1, z^J))$ , using computed decision rules. Capital-labor ratios are computed from age group sample averages using 20,000 draws of initial conditions and lifetime histories. These draws are fixed both across iterations in the algorithm that computes an equilibrium and across the search over model parameters which we discuss next. To calculate aggregate capital we divide the mean wealth held by a cohort over the life cycle by  $(1+g)^{j-1}$  to capture the mean wealth held by age  $j$  agents in cross section.

### A.4 Selecting Model Parameters

We set the parameter  $\alpha$  and the parameters governing the distribution  $\psi$  to minimize the squared distance of log model moments from log data moments. To do so, we compute balanced-growth equilibria for given  $(\alpha, \psi)$  and simulate to find the corresponding model moments. The objective of the minimization problem is

$$\sum_{j=1}^{J_R-1} [(\log(m_{1j}/d_{1j}))^2 + (\log(m_{2j}/d_{2j}))^2 + (\log(m_{3j}/d_{3j}))^2],$$

where  $(m_{1j}, m_{2j}, m_{3j})$  denote mean earnings, var (log earnings) and earnings skewness at age  $j$  in the model and where  $(d_{1j}, d_{2j}, d_{3j})$  are the corresponding data moments. The simplex minimization routine AMOEBA, from Press et. al. (1992), is used to solve this minimization problem.

Table 1: Estimation of Human Capital Shocks

Min-Age	Max-Age	Period	$e_{min}$	$e_{max}$	N	$\sigma$	S.E.( $\sigma$ )	$\sigma_\epsilon$	S.E.( $\sigma_\epsilon$ )
55	65	1969-2004	2,000	1.8 M	103	0.106	0.015	0.142	0.007
50	60	1969-2004	2,000	1.8 M	199	0.114	0.011	0.145	0.005
23	60	1969-2004	2,000	1.8 M	1385	0.153	0.004	0.167	0.003
55	65	1969-2004	3,000	1.8 M	98	0.100	0.015	0.135	0.007
55	65	1969-2004	2,000	1.8 M	103	0.106	0.015	0.142	0.007
55	65	1969-2004	1,500	1.8 M	106	0.112	0.015	0.142	0.007
55	65	1969-2004	1,000	1.8 M	110	0.123	0.015	0.141	0.008
55	65	1969-1981	2,000	1.8 M	104	0.095	0.019	0.135	0.008
50	60	1969-1981	2,000	1.8 M	210	0.098	0.014	0.142	0.006
23	60	1969-1981	2,000	1.8 M	1270	0.137	0.007	0.169	0.004
55	65	1982-2004	2,000	1.8 M	102	0.107	0.026	0.171	0.012
50	60	1982-2004	2,000	1.8 M	193	0.137	0.015	0.147	0.009
23	60	1982-2004	2,000	1.8 M	1492	0.164	0.005	0.171	0.003

Note: The entries provide the estimates for  $\sigma$  and  $\sigma_\epsilon$  for various samples. The first two columns provide the minimum and maximum respective age in the sample. The third column specifies which PSID years are included. The columns labeled  $e_{min}$  and  $e_{max}$  refer to the minimum and maximum earnings levels in 1968 dollars. The column labeled  $N$  refers to the median number of observation across panel years. Columns labeled  $S.E.$  refer to standard errors.

Table 2: Benchmark Parameter Values

Definition	Symbol	Value
Model Periods	$J$	$J = 56$
Retirement Period	$J_R$	$J_R = 42$
Interest Rate	$r$	$r = 0.042$
Period Utility Function	$u(c) = \frac{c^{(1-\rho)}}{(1-\rho)}$	$\rho = 2$
Rental Rate	$R_j = (1 + g)^{j-1}$	$g = 0.0019$
Human Capital Shocks	$z \sim N(\mu, \sigma^2)$	$\sigma = 0.106$ $\mu = -.0284$ time effects $\mu = -.0184$ cohort effects
Human Capital Technology	$h' = \exp(z')H(h, l, a)$	$H(h, l, a) = h + a(hl)^\alpha$ $\alpha = 0.70$ time effects $\alpha = 0.625$ cohort effects
Distribution of Initial Conditions	$\psi$	$\psi = LN(\mu_x, \Sigma)$ discussed in text
Discount Factor	$\beta$	$\beta = .949$ time effects $\beta = .958$ cohort effects

Table 3: Properties of Initial Distributions: Benchmark Model

Statistic	Time Effects View	Cohort Effects View
Mean Learning Ability ( $a$ )	0.344	0.407
Coefficient of Variation ( $a$ )	0.155	0.306
Mean Initial Human Capital ( $h_1$ )	112.9	95.4
Coefficient of Variation ( $h_1$ )	0.454	0.404
Correlation ( $a, h_1$ )	0.742	0.717

Note: Entries show the moments of the distribution of initial conditions that best match the profiles of mean earnings, earnings dispersion and skewness for the parameters in Table 2.

Table 4: A Statistical Model of Earnings: Model Data

Source of Data	$\rho$	$\sigma_\alpha^2$	$\sigma_\eta^2$	$\sigma_\epsilon^2$
Cohort Effects				
cov=1	.983	.179	.0182	.0173
	(.008)	(.020)	(.0043)	(.0081)
cov=5	.984	.179	.0182	.0146
	(.008)	(.021)	(.0046)	(.0101)
Time Effects				
cov=1	.959	.218	.0207	.0176
	(.013)	(.024)	(.0058)	(.0062)
cov=5	.962	.221	.0193	.0162
	(.011)	(.019)	(.0046)	(.0076)

Note: Results are based on drawing 500 samples, each sample has 200 agents at each age cohort. cov=# indicates the number of covariance terms per agent used in estimation. Measurement errors distributed  $N(0, .15^2)$ , are added to each value of log earnings. Estimation on model data uses all variance and covariance restrictions.

Table 5: Sources of Lifetime Inequality: Benchmark Model

Statistic	Time Effects View	Cohort Effects View
Fraction of Variance in Lifetime Utility		
Due to Initial Conditions	.667	.646
Fraction of Variance in Lifetime Earnings		
Due to Initial Conditions	.619	.611

Note: Entries show the fraction of the variance of lifetime utility and lifetime earnings accounted for by initial conditions (initial human capital and learning ability).

Table 6: Sources of Lifetime Inequality: Model with Initial Wealth Differences

Statistic	Time Effects View	Cohort Effects View
Fraction of Variance in Lifetime Utility		
Due to Initial Conditions	.705	.674
Fraction of Variance in Lifetime Earnings		
Due to Initial Conditions	.633	.615
Fraction of Variance in Lifetime Wealth		
Due to Initial Conditions	.655	.636

Note: Entries show the fraction of the variance of lifetime utility, lifetime earnings and lifetime wealth accounted for by initial conditions (initial human capital, learning ability and initial wealth). Wealth differences are measured directly from PSID data as explained in the text.

Table 7: Importance of Changes in Initial Conditions: Model with Initial Wealth

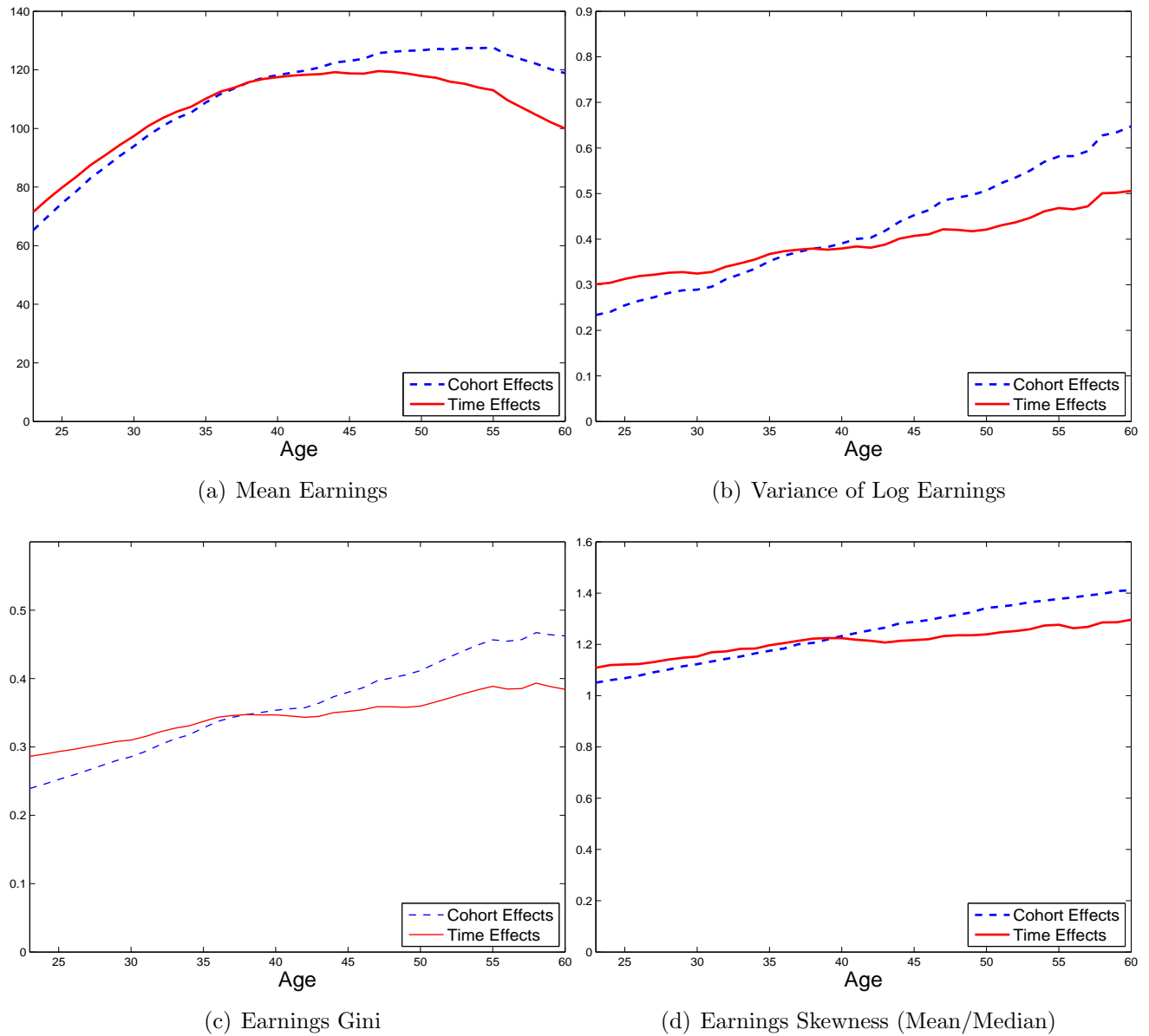
Equivalent Variations In Lifetime Utility			
Variable	Change in Variable	Time Effects	Cohort Effects
Human Capital	+ 1 st. deviation	37.2	32.8
	- 1 st. deviation	-26.9	-23.4
Learning Ability	+ 1 st. deviation	5.9	11.0
	- 1 st. deviation	-3.1	-5.4
Initial Wealth	+ 1 st. deviation	6.1	5.1
	- 1 st. deviation	-1.2	-2.0

Percentage Change in Expected Lifetime Wealth			
Variable	Change in Variable	Time Effects	Cohort Effects
Human Capital	+ 1 st. deviation	38.0	32.3
	- 1 st. deviation	-26.5	-21.4
Learning Ability	+ 1 st. deviation	5.8	13.6
	- 1 st. deviation	-5.6	-6.4
Initial Wealth	+ 1 st. deviation	2.7	3.9
	- 1 st. deviation	-2.8	-4.4

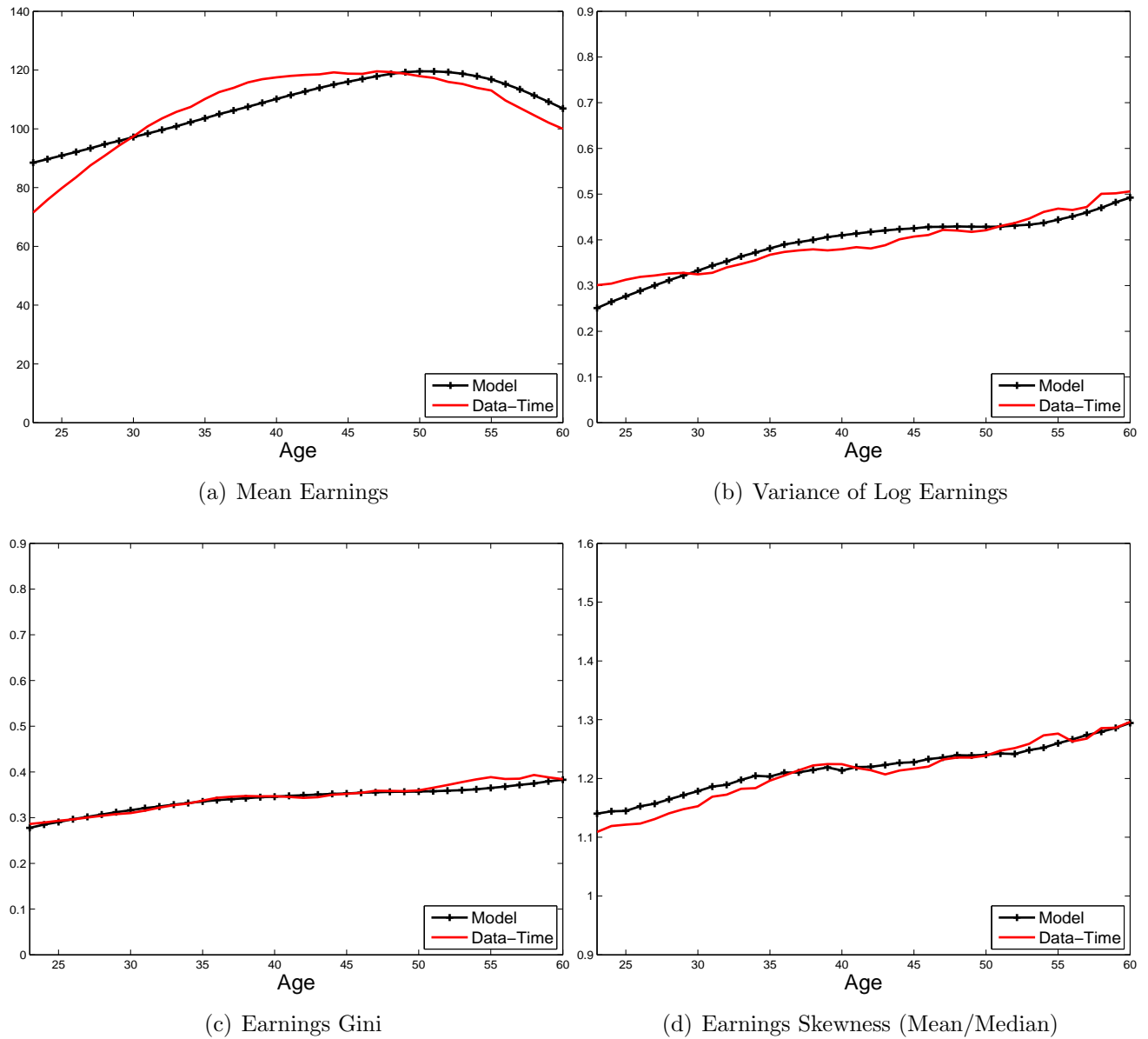
Note: The top panel states equivalent variations, whereas the bottom panel states the percentage change in the expected lifetime wealth associated with changes in each initial condition. The baseline initial condition is set equal to the mean log values of initial human capital, learning ability and wealth. Changes in initial conditions are also in log units.





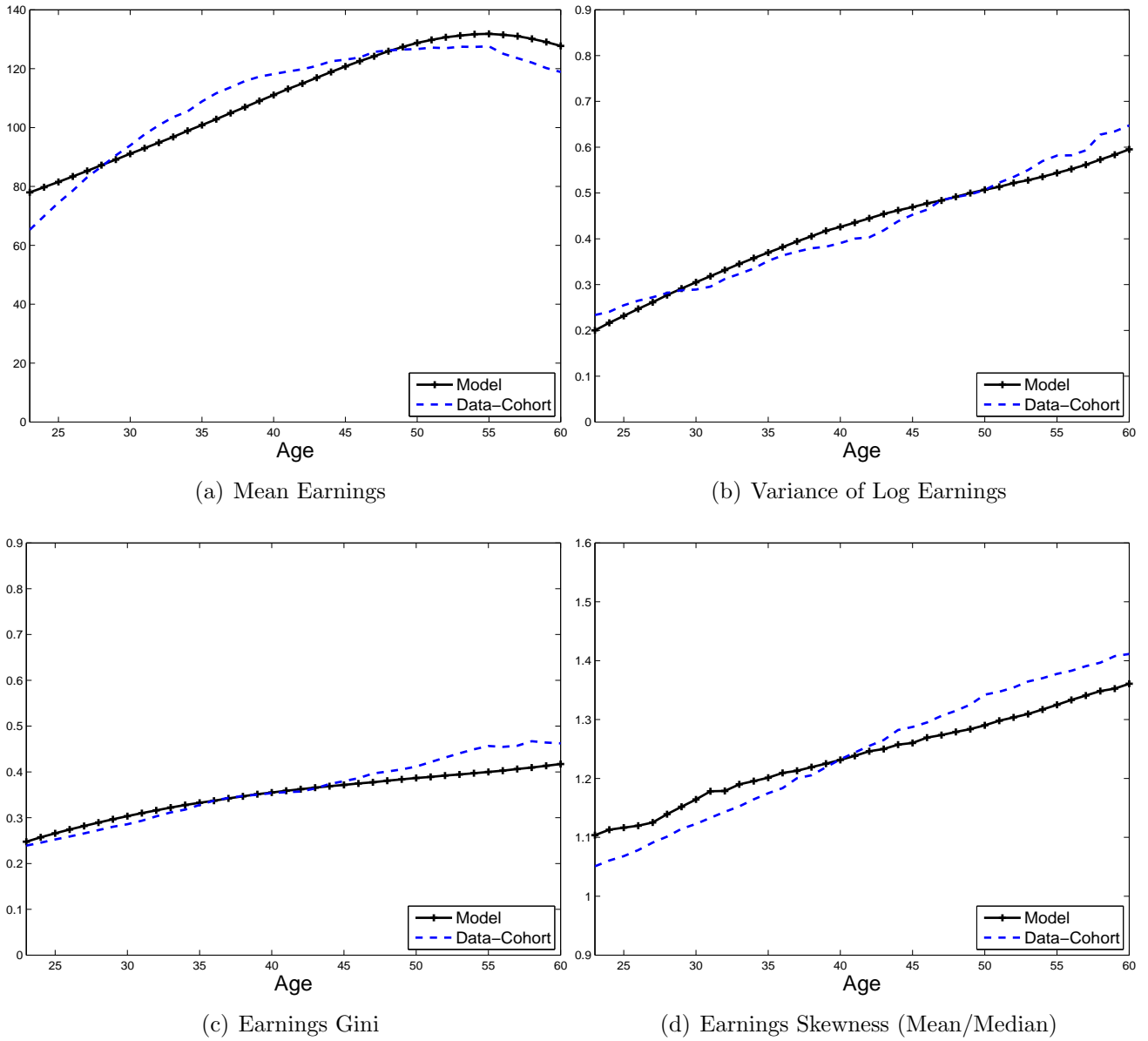
**Figure 1.** Mean, Dispersion and Skewness of Earnings by Age

Note: Figure 1 plots the age effects in mean earnings, earning dispersion measured by the variance of log earnings and the Gini coefficient, and in a measure of skewness after controlling for either time or cohort effects based on data from the PSID, 1969-2004.



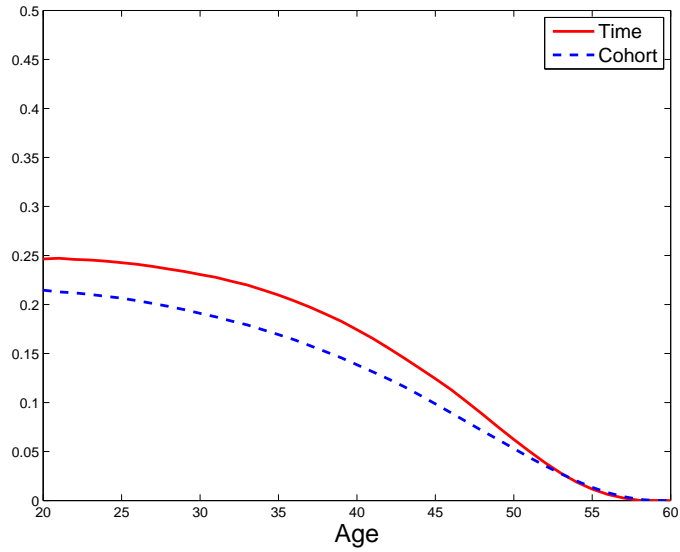
**Figure 2.** Mean, Dispersion and Skewness of Earnings by Age: Time Effects

Note: Figure 2 plots the age effects in mean earnings, earnings dispersion measured by the variance of log and the Gini coefficient, and in a measure of skewness after controlling for time effects based on data from the PSID, 1969-2004. The figure compares the model implication against the data estimates.

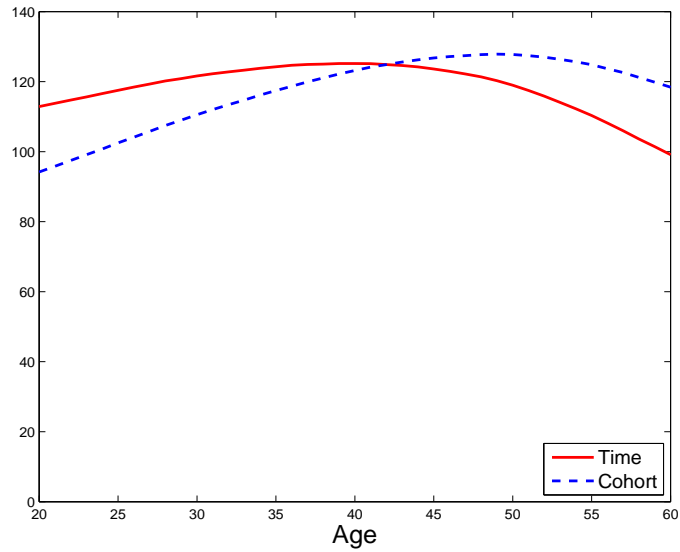


**Figure 3.** Mean, Dispersion and Skewness of Earnings by Age: Cohort Effects

Note: Figure 3 plots the age effects in mean earnings, earnings dispersion measured by the variance of log and the Gini coefficient, and in a measure of skewness after controlling for cohort effects based on data from the PSID, 1969-2004.



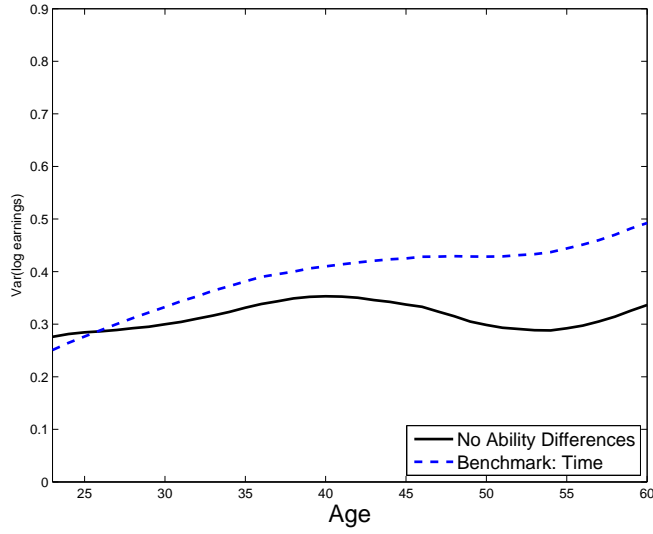
(a) Mean Time in Human Capital



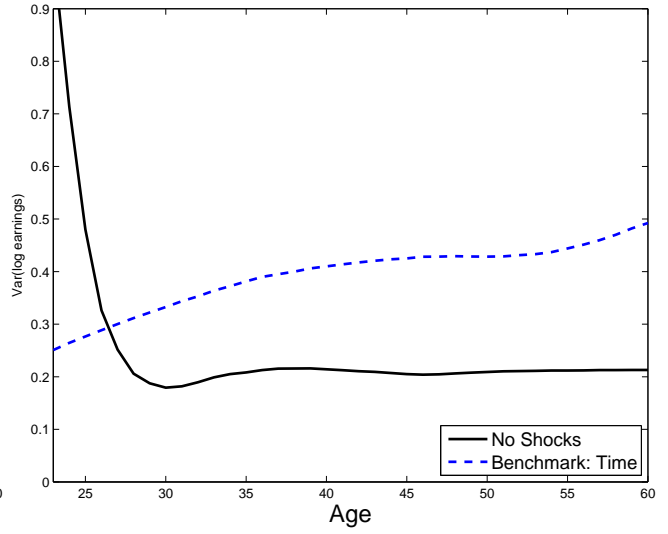
(b) Mean Human Capital Profile

**Figure 4.** Properties of Human Capital by Age

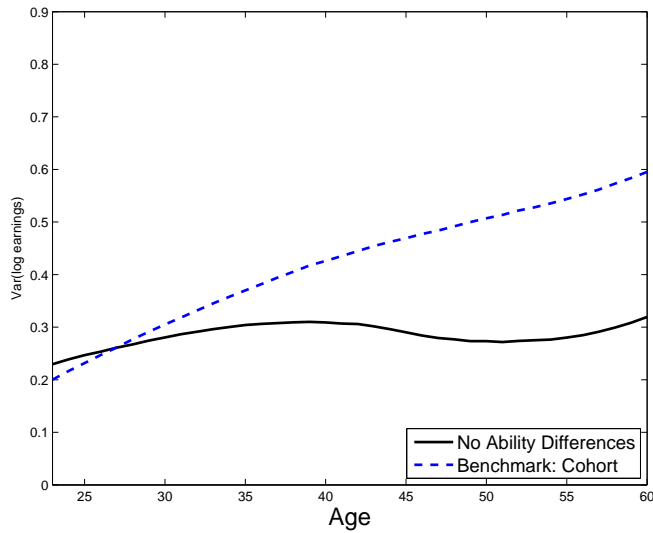
Note: Figure 4 plots properties of human capital by age. Figure 4a plots the mean time spent on human capital. Figure 4b plots the overall mean level of human capital profile by age. The figures plot the human capital properties for both the model that fits the cohort and time effects view.



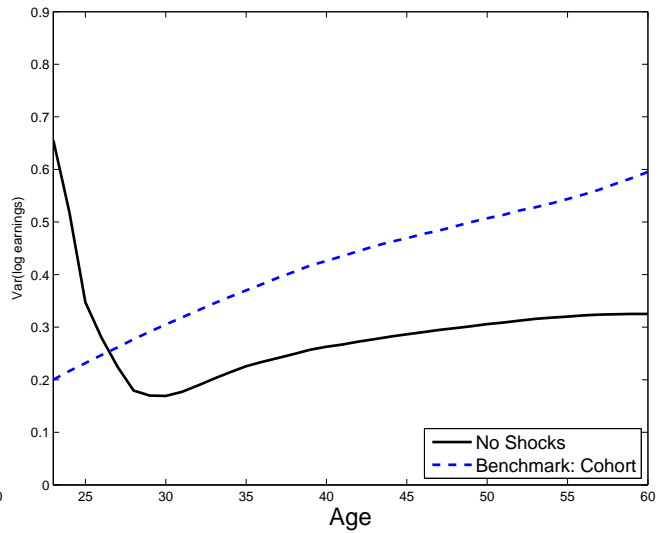
(a) Eliminating Ability Differences



(b) Eliminating Shocks



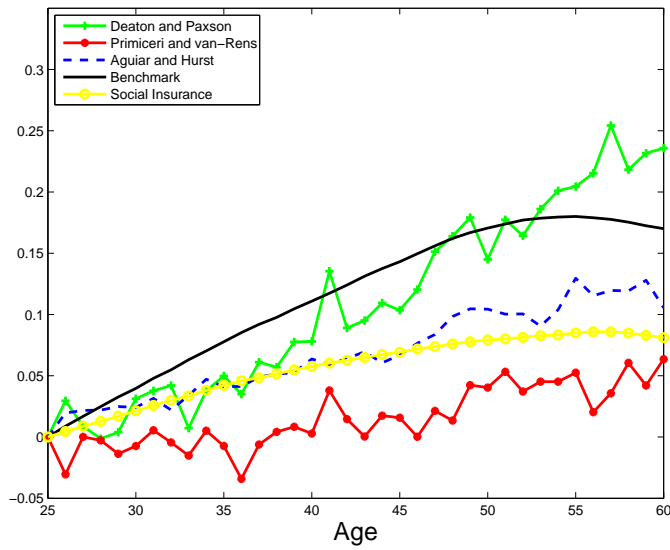
(c) Eliminating Ability Differences



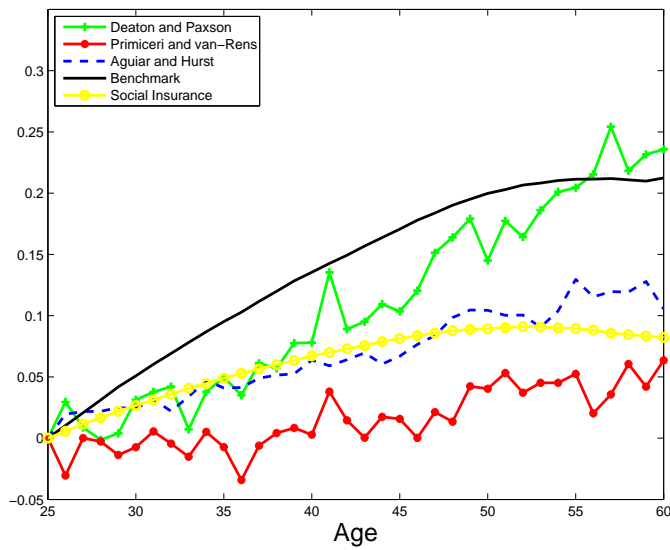
(d) Eliminating Shocks

**Figure 5.** Dispersion of Earnings by Age

Note: Figures 5a and 5c compare the earnings dispersion measured by the variance of log earnings for the benchmark model and one in which all agents share similar ability levels. Figures 5b and 5d compare the earnings dispersion measured by the variance of log earnings for the benchmark model and one in which there are no idiosyncratic shocks to human capital.



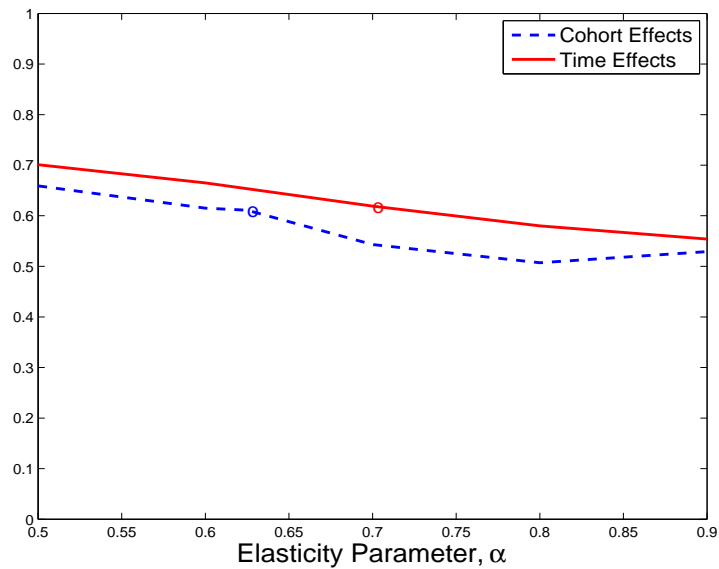
(a) Rise in Consumption Dispersion: Time Effects



(b) Rise in Consumption Dispersion: Cohort Effects

**Figure 6.** Rise in Consumption Dispersion: Model and Data.

Note: Figure 6 plots the rise in the variance of log-consumption for ages 25-60 both in U.S. data and the model. The data is based on Deaton and Paxson (1994), Primiceri and van Rens (2006), Aguiar and Hurst (2008). The model results are for the benchmark model and a version of the model with a social insurance system (SI)– see text for details.



**Figure 7.** Lifetime Inequality and the Elasticity Parameter  $\alpha$

Note: Figure 7 displays the fraction of the variance in lifetime earnings due to initial conditions as the elasticity parameter  $\alpha$  varies. In each line, the values corresponding to the best estimate of  $\alpha$  are highlighted with a circle.