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TECHNICAL CHANGE AND THE DECENTRALIZATION PENALTY

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We consider the organizer of a firm who compares a decentralized arrangement where divisions are granted total autonomy with a centralized arrangement where perfect monitoring and policing guarantee that all divisions make the choices the organizer wants them to make. We ask: when does improvement in the divisions' technology strengthen the case for decentralization and when does it weaken it? In a simple one-division model with complete information and linear contracts we obtain conditions under which the welfare loss due to decentralizing rises (falls) when technology improves.

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1. Introduction

Does the case for decentralizing a firm get stronger or weaker when the production technology used by one or more of its divisions improves? Consider the Organizer of the firm, who seeks a good balance between the cost of the divisions' efforts and the revenue which those efforts yield. One way to achieve a good balance may be intrusive but perfect monitoring and policing, which fully reveals the chosen efforts and guarantees that they are those the Organizer prefers.

Perfect monitoring/policing may be very costly. A better mode of organizing might be "decentralization", where the divisions are totally autonomous, though their choices may be influenced by appropriate rewards and penalties. In the decentralized mode that we shall study there is a Principal who treats each division as an Agent. Each Agent freely chooses her effort and bears the effort's cost. The Principal observes the realized revenue and rewards the Agents. Each Agent's reward is a function of revenue, and her net earnings are her reward minus the cost of her chosen effort. The reward functions the Principal chooses are acceptable to the Agents and are preferred by the Principal to other possible reward functions that are also acceptable to the Agents. The Principal pockets the residual revenue which is left over after the rewards have been paid. When an Agent's technology improves, the cost of a given effort drops.

The Organizer compares the decentralized Principal/Agents mode with perfect monitoring/policing. Many production technologies rapidly improve, but at the same time the costs of perfect monitoring may rapidly drop as well, because of dramatic advances in monitoring techniques. So the relative merit of the two modes requires regular reassessment. We shall let the Organizer take a "welfare" point of view in comparing the two modes. The Organizer's focus is the firm's surplus: the revenue earned by the divisions' efforts minus the cost of those efforts. Perfect monitoring/policing guarantees maximal surplus. The Decentralization Penalty is the welfare loss due to decentralizing. It is the gap between maximal surplus and the surplus achieved in the decentralized Principal/Agents mode.

Our central question is whether the Decentralization Penalty grows or shrinks when a technical advance lowers Agents' effort costs. If the Penalty substantially grows, then perfect monitoring may now be worth what it costs. (We will not explicitly model the cost of monitoring). If the Penalty shrinks, then perfect monitoring becomes less attractive even if monitoring techniques have advanced. Our central question is tricky for the following reason: when the Agents' technology improves, maximal surplus rises (under weak assumptions). Maximal surplus is a "moving target". Decentralized surplus also rises, under reasonable assumptions. But that does NOT mean, in general, that as technology improves, the rising decentralized surplus gets *closer* to the moving surplus target. Our question appears to be very rarely asked in the abundant Principal/Agent literature. The cost of an Agent's effort appears in many papers and so does the welfare loss due to Agents' second-best choices. But the effect of cost improvement on welfare loss seems to be widely neglected.

2. The model

We shall study a highly simplified model. There is a single effort variable x, chosen from a set $\Sigma \subseteq I\!R^{+}$ of possible positive efforts. The set Σ may be finite or it may be a continuum. There is no uncertainty about the consequences of a given effort. The effort x generates a positive *revenue* $R(x)$, where R is strictly increasing. The effort x costs $t \cdot C(x)$, where C is positive and strictly increasing. A drop in t occurs when technology improves (or there is a fall in the price of the inputs which effort requires). For a given t, we consider the *surplus at the effort* x, denote $\tilde{W}(x,t)$. Thus

$$\tilde{W}(x,t) = R(x) - t \cdot C(x).$$

In the centralized mode perfect monitoring/policing guarantees that effort is "first-best": it maximizes surplus. In the decentralized mode there is no direct monitoring. Instead there is a self-interested Principal and a single self-interested Agent who freely chooses $x \in \Sigma$ and bears the cost $t \cdot C(x)$. The functions R and C, and the technology parameter t, are known to both parties. The Principal observes the revenue $R(x)$. Since R is strictly increasing, that observation also reveals the Agent's chosen x. The Principal rewards the Agent, using a reward which is a function of the observed revenue. We study an extremely simple reward scheme, namely linear revenue sharing. The Principal pays the Agent a *share* $r \in (0,1]$ of the revenue. So if the Agent chooses the effort x, she earns $rR(x) - t \cdot C(x)$ and the net amount received by the Principal is the residual $(1-r) \cdot R(x)$. We will assume that for every (r,t) there is an effort $x \in \Sigma$ such that the Agent's gain $rR(t) - tC(x)$ is nonnegative, and this is sufficient for the Agent to be willing to participate. The Agent chooses to exert the effort $\hat{x}(r,t)$, the smallest maximizer of $rR(x) - t \cdot C(x)$ on the set Σ. We denote *the surplus when the share is* r by $W(r,t)$ (the tilde is deleted). So

$$W(r,t) \equiv \tilde{W}(\hat{x}(r,t),t) = R(\hat{x}(r,t)) - t \cdot C(\hat{x}(r,t)).$$

Note that if $r = 1$, then the Agent's effort choice $\hat{x}(1,t)$ is surplus-maximizing. Thus

$$W(1,t) = \tilde{W}(\hat{x}(1,t),t) \textit{ is the largest possible surplus.}$$

In the centralized mode, perfect monitoring/policing insures that $W(1,t)$ is achieved.

In our study of the Decentralization Penalty we consider two cases. In the *exogenous* case, the reward share is determined outside the model. It might, for example, be the result of previous bargaining between Principal and Agent, or it might be prescribed by law. In the *endogenous case*, the Principal considers all the shares in the open interval $(0,1)$ and chooses a share which maximizes $(1-r) \cdot R(\hat{x}(r,t))$, the residual when the Agent uses the best-effort function $\hat{x}$ in responding to a given share. We let $r^*(t)$ denote the maximizer which the Principal chooses. So in the endogenous case the Agent's effort is $\hat{x}(r^*(t),t)$ and surplus is $\tilde{W}(\hat{x}(r^*(t),t),t) = W(r^*(t),t)$.

3. The main results.

The "moving target" remark that we made above suggests that the effect of a drop in t on the Decentralization Penalty is subtle. On the other hand, it is hard to imagine a model simpler than ours. So one might hope that in our simple model there are simple conditions on Σ, R, C under which the Penalty rises (falls) when t drops. It turns out, however, that even in our model there is a striking diversity of results. There are simple examples where the Penalty rises and simple examples where it falls. That is the case in both the exogenous and endogenous settings.

There are, however, basic results that do not directly concern the Penalty and hold for all examples, whether the effort set is finite or a continuum, and whether or not the functions R, C are differentiable. Exogenous-case basic results are given in Theorem 1 and endogenous-case basic results in Theorem 2. The main findings of Theorem 1 are that the Agent never works less hard when the share rises and when technology improves (t drops); that surplus cannot fall when t drops and maximal ("first-best") surplus must rise; that a drop in t is never bad news for the Principal, must be good news for the Agent, and is never bad news from the welfare point of view. A final finding is that a rise in the share r is never bad from the welfare point of view and and is good if and only if the Agent's effort changes. Thus if r is determined through Principal/Agent bargaining, then it is in the "social" interest to strengthen the bargaining power of the Agent (who prefers larger values of r).

There are fewer basic results for the more difficult endogenous case, where the share is $r^*(t)$, a maximizer of $(1-r) \cdot R(\hat{x}(r,t))$ on the interval $(0,1)$. Theorem 2 finds that when t drops, there cannot be a fall in the ratio $\dfrac{r^*(t)}{t}$ or in the Agent's effort $\hat{x}(r^*(t),t)$. A drop in t, moreover, can never be bad news for the Principal and must be good news from the welfare point of view.

In the examples which follow we obtain very diverse results about the Decentralization Penalty. To bring some order to this diversity, we divide examples (Σ, R, C) into classes. To do so, we consider the effect of a drop in t on the example's best effort $\hat{x}$ and on the example's endogenous-case best share r^*. A higher share stimulates the Agent to work harder, but the strength of the stimulus depends on t. Consider any pair of shares (r_L, r_H), where $0 < r_L < r_H < 1$, and suppose that t drops. In some examples the effort increase $\hat{x}(r_H,t)-\hat{x}(r_L,t)$ rises and in other examples it falls. In some examples, moreover, the Principal's best share r^* rises when t drops, and in other examples it falls. So we have four classes of examples.

For each class we examine the *effort gap* — the amount by which decentralized effort falls short of the "first-best" effort. The effort gap is $\hat{x}(1,t) - \hat{x}(r,t)$ when r is exogenous and it is $\hat{x}(1,t) - \hat{x}(r^*(t),t)$ in the endogenous case. The effect of a drop in t on the effort gap is again tricky — just as it was for the Decentralization Penalty, or "surplus gap". Under broad conditions, both terms of the gap rise when t drops, but the gap itself may rise or fall.

The effect of a drop in t on the effort gap is interesting in itself. There are classes of examples, moreover, in which the effort gap *tracks* the surplus gap (the Decentralization Penalty): when t drops, the two gaps move in the same direction. Imagine that first-best effort $\hat{x}(1,t)$ has been studied for many triples (R,C,t). Then for an impending new technology t, first-best effort is already known but the welfare effects of decentralizing remain to be discovered. If we indeed have tracking, then it suffices to observe the Agent's work to see whether, with the new technology, her effort has moved closer to first-best effort or further away from it. In the former case we know — if we indeed have tracking — that the new technology has shrunk the Decentralization Penalty, so it has made perfect monitoring/policing less attractive. In the latter case it has increased the Penalty.

Our first result about tracking is Theorem 4, which requires R and C to be thrice differentiable. The theorem concerns the exogenous case. It considers the effectiveness of a share increase in stimulating higher effort and classifies examples according to the change in effectiveness when t drops, i.e., the sign of the cross partial $\hat{x}_{rt}(r,t)$. It finds that we indeed have tracking, provided that the following monotonicity condition holds: either $\hat{x}_{rt}(r,t) > 0$ at all (r,t) or $\hat{x}_{rt}(r,t) < 0$ at all (r,t). The theorem has a Corollary which directly addresses our central question for the exogenous case. It finds conditions on the signs of C'', R'', C''', R''' under which the Decentralization Penalty rises when technology improves (t drops) and conditions under which the Penalty falls.

As one would expect, the tracking question is more difficult in the endogenous case. Theorem 5 again classifies examples with regard to the effect of technical improvement on effectiveness (the sign of $\hat{x}_{rt}(r,t)$) but also classifies them with regard to the effect of technical improvement on the Principal's endogenous-case "generosity", i.e., the sign of the derivative $r^{*'}(t)$. It finds that we have tracking if either of the following conditions hold: (i) for every possible (r,t), $\hat{x}_{rt}(r,t) < 0$ and $r^{*'}(t) \geq 0$; (ii) for every possible (r,t), $\hat{x}_{rt}(r,t) > 0$ and $r^{*'}(t) < 0$. There is now no Corollary, analogous to Theorem 4's Corollary, in which the effect of technical improvement on the Penalty is related to the signs of the second and third derivatives of R and C.

Theorem 6 shows that we cannot have an example where marginal revenue is declining ($R'' < 0$), and a drop in t (weakly) increases both effectiveness and the Principal's generosity (i.e., at every possible (r,t) we have both $\hat{x}_{rt}(r,t) \geq 0$ and $r^{*'}(t) \geq 0$).

Finally, Theorem 7 addresses bargaining between Principal and Agent over the share r. For a given t, consider the curve which shows the Principal's gain $(1-r)\cdot R(\hat{x}(r,t))$ as a function of $r \in (0,1)$. Suppose the curve is single-peaked, i.e., the gain rises to a peak at the principal's preferred share $r^*(t)$ and then falls (perhaps after an interval where it is flat). In the interval $(0, r^*(t))$, where the gain curve rises, both parties favor higher r. (That follows from results in Theorem 1). The negotiation set — where one party prefers higher r and the other lower r — is the interval $(r^*(t), 1)$. The final result in Theorem 1 showed that welfare increases when the exogenous share increases. So — informally speaking — it is in the "social" interest for the Agent's bargaining strength to be high. Moreover, if r^* is increasing (decreasing) in t, then the negotiation interval $(r^*(t), 1)$ shrinks (widens) when technology improves. To conclude that this shrinkage (widening) raises or lowers the share on which the bargainers finally agree would require a precise model of the bargaining procedure. In any case, Theorem 7 provides conditions on R and C under which the Principal's gain curve is indeed single-peaked.

Plan of the remainder of the paper. In Section 5 we examine six examples where the set of possible efforts and the set of possible values of t are not finite and calculus methods can be used. The examples will illustrate the theorems we informally sketched above. The examples, together with the preceding sketch of the main results, provide an extensive preview of the theorems. It then remains to state each of them formally. In Section 6 we formally state the exogenous-case Theorem 1 and the endogenous-case Theorem 2 (which do not require differentiability) and we comment on the proof techniques. Section 7 states two exogenous-case theorems which require differentiability. The second of these is Theorem 4 which concerns tracking and has a Corollary that relates directly to our central question — when does a drop in t increase (decrease) the Decentralization Penalty? Section 8 presents two endogenous-case theorems which again require differentiability: Theorem 5, which again concerns tracking, and Theorem 6, which concerns the case where marginal revenue is decreasing. Section 9 presents Theorem 7, about bargaining and the shape of the Principal's gain curve. Section 10 sketches several of the many possible extensions and modifications of our model. An Appendix contains portions of the proofs. The complete proofs are given in Liang, Marschak, and Wei (2017), abbreviated henceforth as LMW.

4. Related literature

A great many Principal/Agent papers, starting with the earliest ones, use a framework that allows the Agent's effort to have a cost. The Agent has a utility function on her actions and rewards. In many papers Agent utility for the action a and the reward y takes the form $V(y) - g(a)$. Among the early papers where this occurs are Holmstrom (1979), (1982) and Grossman and Hart (1983). The action a might be effort and $g(a)$ could be its cost. Welfare loss also appears very early in the literature. Ross (1973), for example, finds conditions under which the solution to the Principal's problem maximizes welfare (as measured by the sum of Agent's utility and Principal's utility) and notes that these conditions are very strong. But Principal/Agent papers whose main concern is the relation between effort cost and welfare loss are scarce.

The Principal/Agent papers closest to ours are Balmaceda, Balseiro, Correa, and Stier-Moses (2016) and Nasri, Bastin, and Marcotte (2015). They study an Agent who has m possible efforts. Each effort has a cost, which the Agent pays. There are n possible revenues. For a given effort, the probability of each of the possible revenues is common knowledge, but the revenue actually realized only becomes known after the Agent's effort choice has been made. The Principal announces a vector of n nonnegative wages. For each of the n possible revenues, the vector specifies a wage received by the Agent when that revenue is realized. Both Principal and Agent are risk-neutral. Surplus for a given effort equals expected revenue minus the effort's cost. The socially preferred effort maximizes surplus. In the decentralized (Principal/Agent) mode, on the other hand, surplus is not maximal. Instead it is the surplus when the effort is the one the Principal chooses to induce. The papers study a fraction. Its numerator is maximal surplus and its denominator is "worst-case" Principal/Agent surplus. (When the Principal is indifferent between several efforts, the denominator of the fraction selects the one that is socially worst). The fraction is a measure of the welfare loss due to decentralizing. It is shown, under standard assumptions on the probabilities and on the possible (revenue,effort-cost) pairs, that the ratio cannot exceed m, the number of efforts. That upper bound does not depend on the effort costs, so the papers are silent on the effect of a drop in those costs on welfare loss.

Note that welfare loss is also defined as a fraction in a larger literature, initially developed by computer scientists. Typically the object of study is a game. The fraction studied is often called "the price of anarchy". Its numerator is the payoff sum in the "socially worst" equilibrium of the game. Its denominator — attainable when the players cooperate — is the largest possible payoff sum.[1] In our setting it is natural to use the surplus gap rather than a ratio in defining the Penalty (welfare loss) due to decentralizing. Perfect monitoring (centralization) would eliminate the gap, but in reality it would be expensive. If its cost exceeds the gap then decentralization is the preferred mode.

If we allow more than one Agent, then parts of the large literature on the design of organizations become relevant. The designer has a goal, say surplus (profit) maximization, and can choose between a structure where a single member commands the choices made by all the others, and a structure where everyone is autonomous. The latter structure might be modeled as a game. A rather small piece of the design literature studies the communication and computation costs of each structure and the trade-off between those costs and some measure of gross performance (e.g., gross expected surplus, before the costs are subtracted). The problem is far more complex than the one we consider here and the results remain scarce and specialized. [2]

[1] A variety of social situations are studied from this point of view. One of them concerns optimal versus "selfish" routing in transportation (Roughgarden (2005)). Others are found in Nissan, Roughgarden, Tardos, and Vazirani (eds.) (2007). Many of these studies develop bounds on the price of anarchy. Several of them (e.g., Babaioff, Feldman, and Nissan (2009)) consider a Principal/Agent setting.

[2] Surveys of the design literature with communication and computation costs are found in Garicano and Prat

Finally, it seems appropriate to mention a paper co-authored by Leo Hurwicz, whom this volume honors (Hurwicz and Shapiro, 1978). Here the Principal is a landlord and the Agent is a sharecropper who chooses how hard to work. The landlord knows neither the sharecropper's utility function nor her production function, but does observe the revenue that the sharecropper's labor has achieved. The landlord rewards the sharecropper with a share of the revenue. It is shown that a fifty/fifty split is preferred by the Principal. If the reward function is required to be linear, then that split is also socially optimal, where "optimal" means that the largest possible social "regret" is minimized. One could study the welfare effect of lowering the Agent's cost for every effort, but the paper does not do so

5. Examples.

In each example we specify a triple (Σ, R, C) and we also specify a set Γ of possible pairs (r, t). The set Γ has the property that for each of its pairs (r, t): (i) $0 < r < 1$; and (ii) there exists a positive effort $\hat{x}(r, t)$ which maximizes $R(x) - tC(x)$ on Σ. It will be convenient to use the symbol $\tilde{\Gamma}$ for the set of possible values of t. Thus

$$\tilde{\Gamma} \equiv \{t \ : \ (r, t) \in \Gamma \text{ for some } r\}.$$

In discussing our example we will use the terms *exogenous tracking* and *(exogenous) "opposite directions"*. Here is the definition:

Definition 1

An example (R, C, Γ, Σ), with R and C thrice differentiable, has the *exogenous tracking (exogenous "opposite directions")* property if

$$\frac{d}{dt}\left[\hat{x}(1, t) - \hat{x}(r, t)\right] \cdot \frac{d}{dt}\left[W(1, t) - W(r, t)\right] > 0 \ (< 0) \text{ at all } (r, t) \in \Gamma.$$

Each of our examples will be an *interior* example. In such an example first-order conditions suffice to identify both $\hat{x}(r, t)$ and the Principal's endogenous-case share $r^*(t)$.

Definition 2

An example (Σ, R, C, Γ) is Interior if

- $\Sigma \subseteq I\!R^+$, and $\Gamma \subseteq I\!R^{2^+}$, are open sets.
- R, C are thrice differentiable on Σ and $R' > 0, C' > 0$.
- There exists a twice differentiable function $\hat{x} : \ (0, 1] \times \tilde{\Gamma} \to \Sigma$ such that for $r \in (0, 1]$, $\hat{x}(r, t)$ satisfies the first-order condition $0 = rR'(x) - tC''(x)$ and is the unique maximizer of $rR(x) - tC(x)$ on Σ.

(2013) and Marschak (2006). A model in which revenue is shared by a group of game-playing Agents is studied in Courtney and Marschak (2009). Each player chooses effort and bears its cost. Equilibria of the game are compared with the welfare-maximizing efforts. The paper finds conditions under which the welfare loss drops (rises) when effort costs shift down.

- For every $t \in \tilde{\Gamma}$, there exists a share $r^*(t) \in (0,1)$ which satisfies the first-order condition $0 = \frac{d}{dr}\left[(1-r)\cdot R(\hat{x}(r,t))\right]$ and is the unique maximizer of $(1-r)\cdot R(\hat{x}(r,t))$ on $(0,1)$.

In discussing an interior example we use the terms *endogenous tracking* and *endogenous "opposite directions"*. The definitions are analogous to Definition 1.

Definition 3

An interior example (R, C, Γ, Σ) has the *endogenous tracking (endogenous "opposite directions")* property if

$$\frac{d}{dt}\left[\hat{x}(1,t)-\hat{x}(r^*(t),t)\right]\cdot\frac{d}{dt}\left[W(1,t)-W(r^*(t),t)\right] > 0 \; (< 0) \text{ at all } t \in \tilde{\Gamma} = \{t \;:\; (r,t) \in \Gamma \text{ for some } r\}.$$

Example 1: A classic monopoly

For brevity we shall call this the Classic example. The firm is a monopolist and the effort x is product quantity. Price is $A - Bx$, where $A > 0, B > 0$, so revenue is $R(x) = Ax - Bx^2$. Cost is $t \cdot C(x) = tx$. Marginal revenue becomes negative at $x = \frac{A}{2B}$. To keep price and marginal revenue positive, our set of possible efforts will be

$$\Sigma = \left(0, \frac{A}{2B}\right).$$

In the decentralized mode the monopolist acts as a Principal, lets an Agent choose quantity, and announces a share $r \in (0,1)$. We consider the following set Γ of possible pairs (r,t):

$$\Gamma \equiv \{(r,t) \;:\; 0 < r < 1;\; 0 < t < Ar\}.$$

Thus the set of possible values of t is $\tilde{\Gamma} = (0, A)$. If $(r,t) \in \Gamma$, the Agent's best quantity is

$$\hat{x}(r,t) = \frac{A}{2B} - \frac{t}{2Br}.$$

That belongs to Σ and is the unique maximizer of the Agent's net gain $R(x) - t \cdot C(x)$.

Note that our Γ in this example is the interior of a triangle. In a diagram with r on the horizontal axis and t on the vertical axis the triangle has vertices at the points $(0,0), (1,0)$ and $(1, A)$. In other examples Γ might be a rectangle, as in Example 3 below. In still other examples one of the boundaries of Γ might have curvature.

We now differentiate $\hat{x}$ and obtain some exogenous-case statements. The subsequent theorems will generalize them.

- $\hat{x}_r(r,t) = \frac{t}{2Br^2}$, which is positive. For a given t, increasing the share evokes more effort. It is easily shown, in Part **(a)** of Theorem 1, that in any example, finite or nonfinite, increasing the share never evokes less effort.

- $\hat{x}_t(r,t) = -\frac{1}{2Br}$, which is negative. When r is fixed and technology improves, the Agent works harder. Part **(b)** of Theorem 1 says that the Agent never works less when t drops.

- $\hat{x}_{rt}(r,t) = \frac{1}{2Br^2} > 0$. So technology improvement (a drop in t) *diminishes* the effectiveness of a small rise in the share as a stimulus to higher effort. We use effectiveness (the sign of $\hat{x}_{rt}(r,t)$) in classifying examples. The classification will be especially important in the endogenous case.

- When the Agent uses the best effort $\hat{x}(r,t)$, he receives $r \cdot R(\hat{x}(r,t) - t \cdot \hat{x}(r,t)$. The derivative of that expression with respect to t is negative.[3] So technology improvement is good news for the Agent. Part **(f)** of Theorem 1 uses a simple argument to show that this is always true in the exogenous case.

- We find that surplus is

$$W(r,t) = R(\hat{x}(r,t)) - t \cdot C(\hat{x}(r,t)) = \frac{1}{4B^2r^2} \cdot [(Ar - t) \cdot (BAr + Bt - 2Brt)].$$

The derivative with respect to t of the expression in square brackets is

$$-2BAr^2 - 2Bt + 4Brt.$$

Our requirement that $t < Ar$ implies that this is negative.[4] So for a fixed $r < 1$, decentralized exogenous-case surplus rises when technology improves (t drops). Part **(g)** of Theorem 1 says that this always holds.

- For all $t \in \tilde{\Gamma}$, we have $W_t(1,t) < 0$. Maximal surplus rises when technology improves (t drops). In Part (**d**) of Theorem 1, a trivial argument shows that this always holds.

- $W_{rt}(r,t) = \dfrac{(1-r) \cdot t}{Br^3} > 0$. So $W_{rt}(r,t)$ and $\hat{x}_{rt}(r,t)$ have the same sign. Theorem 3 shows, using a very simple argument, that whenever $\hat{x}_{rt}(r,t) > 0$ (< 0), we also have $\dfrac{d}{dt}\ [\hat{x}(1,t) - \hat{x}(r,t)] > 0$ (< 0). An analogous argument shows that whenever $\hat{W}_{rt}(r,t) > 0$ (< 0) we also have $\dfrac{d}{dt}\ [\hat{W}(1,t) - W(r,t)] > 0$ (< 0). So, in our example, the exogenous Decentralization Penalty (surplus gap) $W(1,t) - W(r,t)$ and the exogenous effort gap $\hat{x}(1,t) - \hat{x}(r,t)$ move in the same direction when technology improves, i.e., the exogenous surplus gap *tracks* the exogenous effort gap. Theorem 4 shows that this must be so as long as R and C are thrice differentiable.

We now turn to the endogenous case. In our example we can verify that the Principal's gain $(1-r) \cdot (R(\hat{x}(r,t))$ is positive for all $r \in (0,1)$ and is concave on $(0,1)$. That implies — as Theorem 7 shows — that there is a share in $(0,1)$, denoted $r^*(t)$, which solves the first-order condition

$$0 = \frac{d}{dr}\ [(1-r) \cdot R(\hat{x}(r,t)] = -R(\hat{x}(r,t)) + (1-r) \cdot R'(\hat{x}(r,t)) \cdot \hat{x}_r(r,t)$$

and maximizes the Principal's gain on the set $(0,1)$. In our example the Principal's first-order condition turns out to be the cubic equation

$$0 = A^2r^3 + rt^2 - 2t^2.$$

[3] The derivative is $\hat{x}_t(r,t) \cdot [rR'(\hat{x}(r,t)) - t \cdot C'(\hat{x}(r,t)] - C(\hat{x}(r,t))$. That is negative, since $0 < r < 1$ and $\hat{x}(r,t)$ satisfies the first-order condition $0 = rR' - tC'$.

[4] The derivative is negative if $Ar^2 > t \cdot (2r - 1)$. That is the case at $r = 0$ and at $r = 1$ (since $t < A$). At all $r \in (0,1)$ our requirement $t < Ar$ implies that $2Ar$, the derivative of the left side of the inequality with respect to r exceeds $2t$, the derivative of the right side. So at all $(r,t) \in \Gamma$ the inequality holds.

When we graph the implicit function $r^*(t)$, we find that for the case $A = 2, B = 3$, r^* is increasing in t. The share-choosing Principal becomes less generous when technology improves. But without any graphing, Theorem 6 tells that r^* cannot be *decreasing* in t if $\hat{x}_{rt}(r,t) > 0$ (as in our example), and in addition $R'' < 0$, which holds in our example, since $R'' = -2B < 0$.

Next consider the endogenous effort $\hat{x}(r^*(t),t)$. If we graph this for our example we find that it rises when technology improves (t drops). Part **(b)** of Theorem 2 shows that this must happen, for the endogenous case, in every example, finite or nonfinite.

We now turn to the tracking question. Figure 1 shows both the Penalty (surplus gap) and the effort gap $\hat{x}(1,t) - \hat{x}(r^*(t),t)$. Figure 1 shows that when t increases each gap first rises and then falls and for each t the gaps move in the same direction, so we indeed have tracking.

FIGURE 1 HERE

But it is *not* the case that endogenous tracking in our example is implied by the fact that we have $\hat{x}_{rt}(r,t) > 0$ and $r^{*'} < 0$. The next example has the same inequalities but it does not exhibit endogenous tracking.

Example 2: A "Cubic-revenue" example.

In this example: $R(x) = x^3 - x^2$, $C(x) = x$ and the set of possible (r,t) pairs is the triangle $\Gamma = \{(r,t) \ : \ r \in (0,1); t \leq r\}$, so the set of possible values of t is $\tilde{\Gamma} = (0,1)$. We find[5] that — just as in the Classic Monopoly example — we have $\hat{x}_{rt}(r,t) > 0$ for all (r,t) in Γ and $r^{*'}(t) < 0$ for all t in $\tilde{\Gamma}$. But when we graph the two endogenous-case gaps, we find that for t in the interval $(.48, .63)$, the effort gap rises but the surplus gap falls.

Example 3: A "Price-taker" example.

The Principal takes the price of one as given and the cost function is quadratic. The set of possible efforts is $\Sigma = I\!R^+$; $R(x) = x$; $C(x) = \frac{1}{2}(x-1)^2$. The set of possible pairs (r,t) is the rectangle $\Gamma = \{(r,t) : \ 0 < r < 1; 0 < t < 1\}$. The Agent's exogenous-case effort choice is $\hat{x}(r,t) = \frac{r}{t} + 1$. So $\hat{x}_{rt}(r,t) = \frac{-1}{t^2} < 0$. That contrasts with Examples 1 and 2. Surplus-maximizing (first-best) effort is $\frac{1}{t} + 1$. The exogenous effort gap is $\frac{1-r}{t}$, which has a negative derivative with respect to t. Exogenous surplus is $R(\hat{x}(r,t)) - t \cdot C(\hat{x}(r,t)) = \frac{r}{t} + 1 - \frac{r^2}{2t}$ and maximal surplus is $1 + \frac{1}{2t}$. Hence the exogenous surplus gap (the Penalty) is $\frac{1}{2t} - \frac{r}{t} + \frac{r^2}{2t}$. Its derivative with respect to t is negative, just like the derivative of effort gap. So — as Theorem 4 tells us — the exogenous surplus gap tracks the exogenous effort gap.

We now turn to the endogenous case. The unique solution to the Principal's first order condition $0 = \frac{d}{dr}[R(\hat{x}(r,t)) - t \cdot C(\hat{x}(r,t))]$ is $r^*(t) = \frac{1-t}{2}$. So we have $r^{*'}(t) < 0$ at every possible t. (Recall that $t < 1$). That contrasts sharply with Example 1 (Classic monopoly). Is a drop in t good news from the welfare point of view in the endogenous case? That cannot be directly answered in Example 1, where there is no closed form for welfare. But in the present example it is easily answered. We have

$$\frac{d}{dt} \, [R(\hat{x}(r^*(t),t)) - t \cdot C(\hat{x}(r^*(t),t))] = [\hat{x}_r \cdot r^{*'} + \hat{x}_t] \cdot (R' - tC') - C.$$

[5] The details of this calculation, as well as a graph of effort gap and surplus gap, are given in LMW (2017). Details for the remaining examples are given there as well.

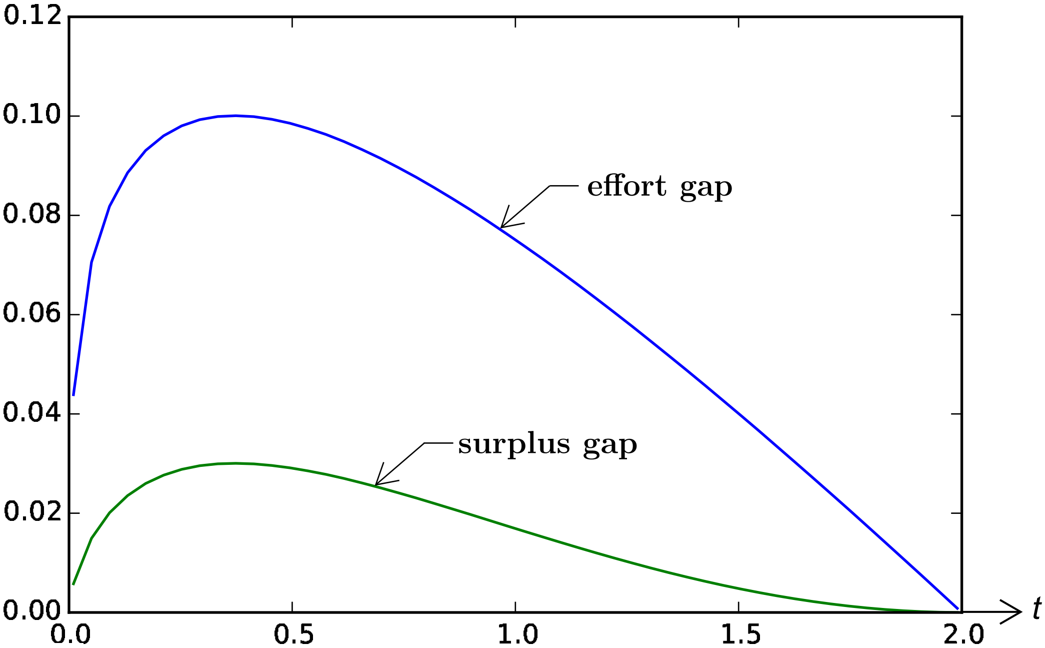

Figure 1: The two gaps: surplus gap $(W(1,t) - W(r^*(t),t))$ and effort gap $(\hat{x}(1,t) - \hat{x}(r^*(t),t))$ for the Classic case, with $A = 2, B = 3$

We have $R' - tC' > 0$ (because of the first-order condition $rR' - tC' = 0$, where $0 < r < 1$). Since $\hat{x}_t < 0$ and $r^{*'} \leq 0$, we conclude that the derivative is negative, so we indeed have "good news" from the welfare point of view. Part (**d**) of Theorem 2, moreover, shows that this *must* be the case, with or without differentiability.

The endogenous Penalty (surplus gap) is $\frac{1}{4} + \frac{1}{8t} + \frac{t}{8}$. Its derivative with respect to t is $\frac{1}{8t^2} \cdot (t^2 - 1)$, which is negative, since $t < 1$. The Penalty always rises when technology improves. Note the contrast with Example 1 (Classic Monopoly), where the Penalty *drops* when technology improves, once t has dropped below a critical value. The endogenous effort gap is $\frac{1+t}{t} - \frac{r^*(t)+t}{t} = \frac{1}{2t} + \frac{1}{2}$. That also has a negative derivative. So the endogenous effort gap tracks the endogenous surplus gap. But that is NOT implied, as we shall see, by the fact that $\hat{x}_{rt} < 0$ and $r^{*'}(t) < 0$, just as the endogenous tracking that we found in Example 1 was not implied by the fact that $\hat{x}_{rt} > 0$ and $r^{*'}(t) > 0$. *We will find a sharply different pattern in our final two examples (5 and 6); there we have endogenous tracking and that DOES follow from the signs of $\hat{x}_{rt}$ and $r^{*'}(t)$.*

Example 4: A "Cubic-cost" example.

In this example

$$R(x) = \frac{1}{2}x^2$$

and

$$C(x) = \frac{1}{3}x^3 + \frac{a}{2}x^2 - \epsilon x,$$

where $\epsilon > 0$ and $a > 0$. The numbers a, ϵ and the set Σ of possible efforts will be chosen as we proceed. The triple (a, ϵ, Σ) will have the property that $C(x) > 0$ for all $x \in \Sigma$.

The Agent's first-order condition for given r, t is

$$rx = t \cdot (x^2 + ax - \epsilon).$$

This is solved by

$$\hat{x}(r,t) = \frac{\sqrt{\left(a - \frac{r}{t}\right)^2 + 4\epsilon} - \left(a - \frac{r}{t}\right)}{2} > 0.$$

Our set Γ of possible (r, t) pairs will be

$$\Gamma = \left\{ (r,t)\ ;\ t \in \left(\frac{1}{a}, \frac{2}{\sqrt{a^2 + 4\epsilon}} \right); 0 < r < 1 \right\}.$$

Now assume that

- $t \geq \frac{1}{a}$
- $\epsilon < \frac{3}{4}a^2$.

Then $\dfrac{1}{a} < \dfrac{2}{\sqrt{a^2 + 4\epsilon}}$, so Γ is not empty. Moreover $a - r/t \geq 0$ for all $r \in (0, 1)$. Under these assumptions we can show[6] that $\hat{x}_{rt}(r,t) < 0$ at all (r,t) in Γ.

[6]Once again, the details are in LMW.

Turning to the endogenous case, we find that the Principal's chosen share $r^*(t)$ must satisfy

$$(+) \qquad r^*(t) = 1 - \frac{t\sqrt{\left(a - \frac{r^*(t)}{t}\right)^2 + 4\epsilon}}{2}.$$

That allows us to show that for every t in our set $\tilde{\Gamma} = \left(\frac{1}{a}, \frac{2}{\sqrt{a^2+4\epsilon}}\right)$ of possible values of t: there is a unique $r^*(t)$ satisfying (+) and, moreover, $r^{*'}(t) < 0$.

Now consider the case where $a = 1$ and $\epsilon = 0.6$. That meets our requirement $\epsilon < \frac{3}{4}a^2$. Define our set of possible efforts to be

$$\Sigma = (1, \infty].$$

Then $\tilde{\Gamma} = (1, 1.084)$ and $C(x) > 0$ for every $x \in \Sigma$. If we graph the surplus and effort gaps we find that the surplus gap rises in that interval but the effort gap falls. Instead of tracking we have "opposite directions". In the Price-taker example we also had $\hat{x}_{rt}(r,t) < 0$ and $r^{*'}(t) < 0$, but there we had tracking.

Example 5: A "Rising-marginals" example.

In this example marginal revenue rises but marginal cost rises faster, so there is an interior effort maximizing the Agent's gain. The set of possible efforts is $\Sigma = I\!R^+; R(x) = x^a$ and $C(x) = x^b$, where $0 < a < b$. The set of possible pairs (r,t) is $\Gamma = \{(r,t) \ : \ 0 < r < 1; t > 0\}$. We find that $\hat{x}(r,t) = \left(\frac{tb}{ra}\right)^{\frac{1}{a-b}}$ and $\hat{x}_{rt}(r,t) = -\frac{1}{(a-b)^2} \cdot t^{1/(a-b)-1} \cdot \left(\frac{b}{a}\right)^{1/(a-b)} \cdot r^{1/(b-a)-1}$, which is negative.

Turning to the endogenous case, we find that $r^*(t) = \frac{a}{b}$. The Principal' chosen share is independent of t. Even though we have an explicit expression for r^*, computing the derivative of endogenous effort gap (Penalty) with respect to t and the derivative of endogenous surplus gap with respect to t is cumbersome. It turns out that both are negative. So the endogenous surplus gap tracks the endogenous effort gap. Theorem 5 will show that this follows from the fact that we have both $\hat{x}_{rt}(r,t) < 0$ and $r^{*'}(t) \leq 0$.

Example 6: An "Exploding-marginals" example.

There remains one class, in our four-way classification of interior examples, which we have not yet illustrated. This is the class where we have both $\hat{x}_{rt}(r,t) > 0$ and $r^{*'}(t) < 0$. Theorem 6 will show us that in such an example we cannot have $R'' \leq 0$. So our search for an example is narrowed to the case $R'' > 0$. Moreover, preliminary exercises show that a modestly increasing marginal revenue (e.g., $R'' = 1$) is not enough. Marginal revenue has to rise rapidly and marginal cost has to rise even faster. In the following example both marginals "explode".

We have:

- $\Sigma = (0, 1)$.
- $\Gamma = \{(r,t) \ : \ 0 < r < 1; \frac{r}{t} \in (e, e^e)\}$ (e is the base of the natural logarithms).

- $R(x) = e^{x^2}$.
- $C(x) = \int_0^x \left[2e^{e^p} \cdot e^p \cdot p\right] \, dp$.

Since $\hat{x}_{rt}(r,t) > 0$ and $r^{*'}(t) < 0$, Part (**b**) of Theorem 5 tells us that in this example we have endogenous tracking.

A summary of the six interior examples and their relation to our theorems is provided in the table which follows.

$\boxed{\text{TABLE HERE}}$

THE EFFECT OF IMPROVED TECHNOLOGY (A DROP IN t) IN FOUR GROUPS OF INTERIOR EXAMPLES

	WHEN t DROPS, EFFECTIVENESS OF A SHARE INCREASE <u>*FALLS.*</u> HENCE SO DOES THE EXOGENOUS EFFORT GAP (SEE THEOREM 3). $\boxed{\hat{x}_{rt} > 0 \textbf{ and hence } \frac{d}{dt}[\hat{x}(1,t) - \hat{x}(r,t)] > 0}$	WHEN t DROPS, EFFECTIVENESS OF A SHARE INCREASE <u>*RISES*</u>. HENCE SO DOES THE EXOGENOUS EFFORT GAP (SEE THEOREM 3). $\boxed{\hat{x}_{rt} < 0 \textbf{ and hence } \frac{d}{dt}[\hat{x}(1,t) - \hat{x}(r,t)] < 0}$
WHEN t DROPS, PRINCIPAL BECOMES <u>*LESS*</u> GENEROUS OR GENEROSITY STAYS THE SAME. $\boxed{r^{*'} \geq 0}$	**1** *SEE "CLASSIC" AND "CUBIC-REVENUE" EXAMPLES.* WE HAVE ENDOGENOUS TRACKING IN THE CLASSIC EXAMPLE BUT IN THE CUBIC-REVENUE EXAMPLE WE HAVE "OPPOSITE DIRECTIONS" (IF THE SET OF POSSIBLE VALUES OF t IS PROPERLY CHOSEN).	**2** *SEE "RISING MARGINALS" EXAMPLE.* EVERY EXAMPLE THAT LIES IN THIS BOX HAS THE TRACKING PROPERTY. *(See Theorem 5, Part (a)).*
WHEN t DROPS, PRINCIPAL BECOMES <u>*MORE*</u> GENEROUS. $\boxed{r^{*'} < 0}$	**3** *SEE "EXPLODING MARGINALS" EXAMPLE.* EVERY EXAMPLE THAT LIES IN THIS BOX HAS THE ENDOGENOUS TRACKING PROPERTY. *(See Theorem 5, Part (b)). AN EXAMPLE WITH* $R'' < 0$ <u>*CANNOT BE IN THIS BOX.*</u> *(See Theorem 6).*	**4** *SEE THE "PRICE-TAKER" EXAMPLE, WHERE WE HAVE ENDOGENOUS TRACKING AND THE "CUBIC-COST" EXAMPLE, WHERE WE HAVE "OPPOSITE DIRECTIONS".*

6. Basic results that do not require differentiability.

Theorem 1

Let R and C be strictly increasing on Σ. Then:

(a) $\hat{x}(r_H,t) \geq \hat{x}(r_L,t)$ and $R(\hat{x}(r_H,t)) - tC(\hat{x}(r_H,t)) > R(\hat{x}(r_L,t)) - tC(\hat{x}(r_L,t))$ whenever $(r_L,t) \in \Gamma, (r_H,t) \in \Gamma$, and $0 < r_L < r_H < 1$.

(b) $\hat{x}(r,t_L) \geq \hat{x}(r,t_H)$ whenever $(r,t_L) \in \Gamma, (r,t_H) \in \Gamma$, and $0 < t_L < t_H$.

(c) $\hat{x}(1,t_L) \geq \hat{x}(1,t_H)$ whenever $t_L, t_H \in \tilde{\Gamma}$, and $0 < t_L < t_H$.

(d) $W(1,t_L) > W(1,t_H)$ whenever $t_L, t_H \in \tilde{\Gamma}$ and $0 < t_L < t_H$.

(e) $(1-r) \cdot R(\hat{x}(r,t_L)) \geq (1-r) \cdot R(\hat{x}(r,t_H))$ whenever $(r,t_L) \in \Gamma, (r,t_H) \in \Gamma$, and $0 < t_L < t_H$.

(f) $rR(\hat{x}(r,t_L)) - t_L C(\hat{x}(r,t_L)) > rR(\hat{x}(r,t_H)) - t_H C(\hat{x}(r,t_H))$ whenever $(r,t_L) \in \Gamma, (r,t_H) \in \Gamma$, and $0 < t_L < t_H$.

(g) $W(r,t_L) > W(r,t_H)$ whenever $(r,t_L) \in \Gamma, (r,t_H) \in \Gamma$, and $0 < t_L < t_H$.

(h) $W(r_H,t) \geq W(r_L,t)$ whenever $(r_H,t) \in \Gamma, (r_L,t) \in \Gamma$, and $0 < r_L < r_H < 1s$. The inequality is strict if and only if $\hat{x}(r_H,t) \neq \hat{x}(r_L,t)$.

In proving Parts **(a)**,**(b)**,**(c)**,**(d)** we use a basic proposition from monotone comparative statics. It concerns a function of two variables with strictly increasing differences, and describes the direction in which a maximizer moves when one of the variables increases.[7] For the other parts, we use the simple observation that when t drops or r rises the Agent could continue to muse the same effort as before the change. To show the pattern of the first argument, the Appendix provides the proof of part (**a**)). To show the second argument it gives the proofs of (**h**)and (**g**).[8]

Theorem 2 concerns the endogenous case.

Theorem 2

Let R and C be strictly increasing on Σ. Let $r^*(t)$ denote a maximizer of $(1-r) \cdot R(\hat{x}(r,t))$ on the interval $(0,1)$. Then

(a) $\frac{r^*(t_L)}{t_L} \geq \frac{r^*(t_H)}{t_H}$ whenever $t_L, t_H \in \tilde{\Gamma}$ and $0 < t_L < t_H$.

(b) $\hat{x}(r^*(t_L),t_L) \geq \hat{x}(r^*(t_H),t_H)$ whenever $t_L, t_H \in \tilde{\Gamma}$ and $0 < t_L < t_H$.

(c) $(1-r^*(t_L)) \cdot R(\hat{x}(r^*(t_L),t_L) \geq (1-r^*(t_H)) \cdot R(\hat{x}(r^*(t_H),t_H)$ whenever $t_L, t_H \in \tilde{\Gamma}$ and $0 < t_L < t_H$.

(d) $W(r^*(t_L),t_L) > W(r^*(t_H),t_H)$ whenever $t_L \in \tilde{\Gamma}, t_H \in \tilde{\Gamma}$, and $0 < t_L < t_H$.

[7] See, for example, Sundaram (1996).

[8] As already noted, the proofs of the parts not shown are given in LMW.

The Appendix provides the proofs of Parts (**a**) and (**b**). It is interesting to note that while Part (c) of Theorem 2 tells us that in the endogenous case technical improvement can never be bad news for the Principal, the situation is different for the Agent. We can construct examples where a drop in t leads to smaller net gain for the Agent. Informally: *in the endogenous case, the Principal is never the enemy of technical progress but the Agent might be.*

7. Two exogenous-case theorems which require differentiability.

Theorem 3

Let Γ be an open set in $I\!R^{2^+}$. Suppose that the functions R and C are thrice differentiable. Suppose that the following *monotonicity* condition is met:

we either have

$$\hat{x}_{rt} > 0 \text{ for all } (r,t) \in \Gamma$$

or

$$\hat{x}_{rt} < 0 \text{ for all } (r,t) \in \Gamma.$$

Suppose, in addition, $\hat{x}_t$ is continuous with respect to r at all points in $(0,1]$.

Then $\hat{x}_{rt}(r,t) > 0$ (< 0) at every $(r,t) \in \Gamma$ if and only if

$$\frac{d}{dt}\left[\hat{x}(1,t) - \hat{x}(r,t)\right] > \ (< 0) \text{ at every } (r,t) \in \Gamma.$$

Note that the pair $(r^*(t), t)$ belongs to Γ, so the theorem applies, in particular, to $\hat{x}_{rt}(r^*(t), t)$ and the endogenous effort gap $\hat{x}(1,t) - \hat{x}(r^*(t), t)$. The proof (in the Appendix) is very simple.

The next theorem concerns exogenous tracking in Interior examples.

Theorem 4

An interior example has the exogenous tracking property if the effort set is $\Sigma = (0, J)$, where $J > 0$, and the monotonicity condition of Theorem 2 holds (we either have $\hat{x}_{rt} > 0$ for all $(r,t) \in \Gamma$ or $\hat{x}_{rt} < 0$ for all $(r,t) \in \Gamma$).

Straightforward calculation yields the following Corollary.

Corollary

The following hold for an interior example in which the monotonicity condition of Theorem 2 is satisfied, the effort set is $\Sigma = (0, J)$ (where $J > 0$), and $\hat{x}_r(r,t) > 0, \hat{x}_t(r,t) < 0$ for all $(r,t) \in \Gamma$:

(i) the Decentralization Penalty (surplus gap) is decreasing in t (so the Penalty grows when technology improves) if at every effort $x \in (0, J)$ we have $R''(x) \geq 0, R'''(x) = C'''(x) = 0$.

(ii) the Decentralization Penalty (surplus gap) is increasing in t (so the Penalty shrinks when technology improves) if at every effort $x \in (0, J)$ we have $R''(x) < 0, C''(x) = 0, R'''(x) \leq 0$.

8. Endogenous-case results which require differentiability.

Theorem 5

Consider an interior example (Σ, Γ, R, C).

(**a**). Suppose the following holds:

for every $t \in \tilde{\Gamma}$ *we have* $r^{*'}(t) \geq 0$ *and for every* $(r,t) \in \Gamma$ *we have* $\hat{x}_{rt}(r,t) < 0$.

Then we have endogenous tracking.

(b). Suppose the following holds:

for every $t \in \tilde{\Gamma}$ *we have* $r^{*'}(t) < 0$ *and for every* $(r,t) \in \Gamma$ *we have* $\hat{x}_{rt}(r,t) > 0$.

Then we have endogenous tracking.

The next theorem does not directly concern the two gaps. But it implies that if marginal revenue is decreasing or constant ($R'' \leq 0$) in an interior example and the Principal has a unique best share, then the example cannot be in Box **3** of our table.

Theorem 6

Suppose that in the interior example (Σ, Γ, R, C) we have:

- $R''(x) \leq 0$ at every $x \in \Sigma$.
- $\hat{x}_{rt}(r,t) \geq 0, \hat{x}_t(r,t) < 0$ and $\hat{x}_r(r,t) > 0$ at every $(r,t) \in \Gamma$.
- $r^*(t)$ is the unique maximizer of $(1-r) \cdot R(\hat{x}(r,t))$ on $(0,1)$,

Then $r^{*'}(t) \geq 0$ for all $t \in \Gamma$.

It is difficult to give a clear intuition for Theorems 4 and 5. That is a little easier for Theorem 6, which says that if marginal revenue is decreasing, and effectiveness drops when technology improves, then when technology improves, the Principal does not become more generous ($r^{*'}(t) \geq 0$), i.e., we cannot be in Box **3**. Intuitively one might say: *when t drops, increasing the share above its previous level would damage the Principal, because the extra revenue due to extra effort has dropped (marginal revenue has declined) and at the same time the extra effort evoked by a share increase has dropped as well.*

9. Finding the Principal's best share for a given t: when is the Principal's gain a concave function of the share?

For a fixed t, consider the Principal's gain $(1-r) \cdot R(\hat{x}(r,t))$ as a function of $r \in (0,1)$. Our discussion in the "main results" section above argued that if we want to study bargaining between Principal and Agent over the share r, then it is very helpful if the graph of the gain curve is single-peaked. As long as the gain is positive at some $r \in (0,1)$, the curve is single-peaked if it is concave on $(0,1)$. The following theorem provides conditions under which the gain is indeed concave. The theorem has two parts. The first part does not require differentiability with respect to r, but the second part does. The second part says that we have concavity if marginal revenue drops ($R'' < 0$) and in addition the effectiveness of a share increase drops when the share increases ($\hat{x}_{rr} < 0$).

Theorem 7

(a) If, for a fixed t, $R(\hat{x}(r,t))$ is concave on $(0,1)$, then the Principal's gain $(1-r) \cdot R(\hat{x}(r,t)$ is also concave on $(0,1)$.

(b) Consider an interior example (Σ, Γ, R, C) where $\Sigma = (0, J)$, with $J > 0$. Then R is concave on $(0, J)$ if for all $x \in (0, J)$ we have $R''(x) < 0$, and for all $(r, t) \in \Gamma$ we have $\hat{x}_{rr}(r, t) < 0$. If $R''(x) < 0$, then a sufficient condition for $\hat{x}_{rr} < 0$ is

$$r \cdot R'''(x) - t \cdot C'''(x) \leq 0.$$

10. Concluding remarks.

Recall our central question: does technical improvement strengthen the case for full Agent autonomy or does it weaken it so much that perfect monitoring and policing has now become attractive? One might have reasonably hoped for a straightforward answer since our revenue-sharing Principal/Agent model is so simple. Specifically one might have hoped that a natural condition like rising marginal cost and falling marginal revenue unambiguously implies that the Decentralization Penalty rises (or falls) when technology improves. Instead we have found that there is no easy answer to our central question. On the other hand, we have found a rich array of other results. One of them is that in both the exogenous case and the endogenous case, an advance in technology increases welfare. Another is that an advance in technology causes the Agent to work harder. That is obvious in the exogenous case, since the Agent benefits from the advance even if she continues to use her previous effort. It is not obvious in the endogenous case.

Other interesting results for the challenging endogenous case concern the tracking question. If the effort gap always moves in the same direction as the surplus gap (the Penalty), then to see whether a technical advance has strengthened or weakened the case for autonomy, it suffices to observe (but not police) the Agent's effort before and after the advance and to compare it with first-best effort. We saw that two key properties of an example are the sign of $r^{*'}$ and the sign of $\hat{x}_{rt}$. We *must* have tracking if $r^{*'} \geq 0, \hat{x}_{rt} < 0$ or $r^{*'} < 0, \hat{x}_{rt} > 0$ — if a drop in t decreases generosity (or leaves it unchanged) and increases the effectiveness of a share increase in eliciting higher effort, or the drop increases generosity and decreases effectiveness. For the other combinations of the two signs, we may have tracking but we may also have "opposite directions".

Can we obtain an easier answer to our central question if we vary or complicate the model? There are many possible variations.

We could, in particular, turn to the framework of Balmaceda *et al* (2016), described in the Related Literature section above. Both Agent and Principal are risk-neutral. There are m possible efforts and n possible revenues. Consider the case $m = n = 2$. The efforts are x_L, x_H where $0 < x_L < x_H$. Letting the subscripts L, H again denote "low" and "high", the possible costs and revenues are C_L, C_H, R_L, R_H. The Agent's cost for the effort x_H (x_L) is tC_H (tC_L), where t is our technology parameter. For the effort x_L, the probability of R_H is p and $1 - p$ is the probability of R_L. For the effort x_H, the probabilities are $q, 1 - q$, where $q > p$. The Principal chooses a nonnegative wage pair before the Agent chooses effort. If — for a given t — the Principal wants to induce x_H, then he uses a pair (w_H^t, w_L^t), where the Agent is paid w_H^t (w_L^t) if revenue turns out to be R_H (R_L). When x_H is induced, the Agent's expected net gain is $qw_H^t + (1 - q) \cdot w_L^t - tC_H$, and the Principal's expected net gain is the remainder of the expected surplus, i.e., $[qR_H + (1 - q) \cdot R_L] - [qw_H^t + (1 - q) \cdot w_L^t]$. Another wage pair is used to induce x_L. The chosen wage pair, among those that induce a given effort, minimizes the average wage paid by the Principal, subject to an Individual Rationality (IR) constraint (the Agent's expected net gain is nonnegative) and an Incentive Compatibility constraint (the Agent's expected net gain for the given effort is not lower than her expected net gain for the other effort).

Do we again get one of the key results in our model: is it again true that the Agent never works less when technology improves? If the Principal chooses to induce x_H, then will he continue to do so if t drops? It turns out[9] that if a wage pair solves the Principal's induce-x_L problem, then the IR constraint for that pair must be binding. If the IR and IC constraints are both binding for a wage pair that solves the induce-x_H problem, and if the Principal prefers to induce x_H, then a drop in t cannot reverse that preference. Moreover the Decentralization Penalty is then zero for every t. If IR is slack in the induce-x_H solution, then it remains true that a drop in t cannot reverse the Principal's preference for x_H. But now a drop in t may raise or lower the Decentralization Penalty. A natural research path would consider all pairs (m, n) and would examine analogs of other results that we obtained in our model. One might then explore the same questions when we let the Agent be risk-averse. Does increasing risk aversion (when t is fixed) raise or lower the Penalty?

In another research path one could change the definition of "Decentralization Penalty", so that it becomes a fraction, as in Balmaceda *et al* (2016). The Penalty (in the endogenous case) would be $\frac{W(r^*(t),t)}{W(1,t)}$, rather than the surplus gap we have considered. Our central question becomes technically harder and again has no simple answer. Moreover there are examples where some of our results about the effect of a drop in t on the Penalty are now reversed.

A third research path would let t be a random variable with common-knowledge probabilities and would let one party have better information about the true t than the other. Technical improvement lowers the expected value of t. Does it increase or decrease the Decentralization Penalty?

It was natural to start with our stripped-down model, where we already saw the unexpected challenges posed by our central question. The question of the effect of improved technology on the merits of alternative modes of organizing is well motivated but has seldom been the focus of previous research. The variations and extensions that we have noted, and numerous others, merit further attention.

APPENDIX

Proofs of parts (a), (g), and (h) of Theorem 1

Proof of Part (a)

The function $r \cdot R(x) - tC(x)$, where t is fixed, displays strictly increasing differences in r, x if $r \cdot R(x)$ displays strictly increasing differences in r, x. But that is the case, since R is nondecreasing. Since, for fixed t, the effort $\hat{x}(r, t)$ maximizes $r \cdot R(x) - tC(x)$ on the effort set Σ, it is indeed the case that $\hat{x}(r_H, t) \geq \hat{x}(r_L, t)$, as (**a**) asserts. Part (**a**) also asserts that the Agent strictly prefers the higher share. That is the case since $\hat{x}(r_H, t)$ is a maximizer of $r_H \cdot R(x) - t \cdot C(x)$, so we have

$$r_H \cdot R(\hat{x}(r_H, t)) - t \cdot C(\hat{x}(r_H, t)) \geq r_H \cdot R(\hat{x}(r_L, t)) - t \cdot C(\hat{x}(r_L, t)) > r_H \cdot R(\hat{x}(r_L, t)) - t \cdot C(\hat{x}(r_L, t)).$$

[9]The details are provided in LMW.

Proof of Part (**g**)

Part **(g)** says:

$$W(r,t_L) > W(r,t_H) \text{ whenever } t_L, t_H \in \tilde{\Gamma} \text{ and } 0 < t_L < t_H.$$

The effort $\hat{x}(r,t_L)$ is a maximizer of $rR(x) - t_H \cdot C(x)$. Hence

$$r \cdot R(\hat{x}(r,t_L)) - t_L \cdot C(\hat{x}(r,t_L)) \geq r \cdot R(\hat{x}(r,t_H)) - t_L \cdot C(\hat{x}(r,t_H))$$

or

$$r \cdot [R(\hat{x}(r,t_L)) - R(\hat{x}(r,t_H))] \geq t_L \cdot [C(\hat{x}(r,t_L)) - C(\hat{x}(r,t_H))].$$

That implies — since $0 < r < 1$ — that

$$R(\hat{x}(r,t_L)) - R(\hat{x}(r,t_H)) > t_L \cdot [C(\hat{x}(r,t_L)) - C(\hat{x}(r,t_H))]$$

or

$$R(\hat{x}(r,t_L)) - t_L \cdot C(\hat{x}(r,t_L)) > R(\hat{x}(r,t_H)) - t_L \cdot C(\hat{x}(r,t_H))$$

and hence (since $t_H > t_L$)

$$R(\hat{x}(r,t_L)) - t_L \cdot C(\hat{x}(r,t_L)) > R(\hat{x}(r(t_H),t_H)) - t_H \cdot C(\hat{x}(r,t_H))$$

The term on the left of the inequality is $W(r,t_L)$ and the term on the right is $W(r,t_H)$. That completes the proof of Part **(g)**.

Proof of Part **(h)**:

When the Agent's share is r_H, he chooses an effort $\hat{x}(r_H,t)$ which satisfies

$$r_H R(\hat{x}(r_H,t)) - tC(\hat{x}(r_H,t)) \geq r_H R(\hat{x}(r_L,t)) - tC(\hat{x}(r_L,t)),$$

or equivalently

$$r_H \cdot [R(\hat{x}(r_H,t)) - R(\hat{x}(r_L,t))] \geq t \cdot [C(\hat{x}(r_H,t)) - C(\hat{x}(r_L,t))]. \tag{1}$$

Part **(a)** of Theorem 1 tells us that $\hat{x}(r_H,t) \geq \hat{x}(r_L,t)$. Since R is strictly increasing , that means that the left side of (1) is either positive or zero. First suppose that it is positive. Then, since $r_H < 1$, (1) implies that

$$R(\hat{x}(r_H,t)) - R(\hat{x}(r_L,t)) > t \cdot [C(\hat{x}(r_H,t)) - C(\hat{x}(r_L,t))], \tag{2}$$

or equivalently

$$R(\hat{x}(r_H,t)) - t \cdot C(\hat{x}(r_H,t)) > R(\hat{x}(r_L,t)) - t \cdot C(\hat{x}(r_L,t)), \tag{3}$$

i.e.,

$$W(r_H,t) > W(r_L,t). \tag{4}$$

If $\hat{x}(r_H,t) \neq \hat{x}(r_L,t)$, then, since R is strictly increasing, the left side of (1) is indeed positive, so (4) holds. If, on the other hand, $\hat{x}(r_H,t) = \hat{x}(r_L,t)$, then both sides of (1) equal zero and

(2),(3),(4) become equalities. So, as claimed, $W(r_H, t) \geq W(r_L, t)$ and the inequality is strict if and only if $\hat{x}(r_H, t) \neq \hat{x}(r_L, t)$.

Proofs of Parts (**a**) and (**b**) of Theorem 2

Proof of Part (a)

We note first that the Agent's chosen effort $\hat{x}(r, t)$ depends only on the ratio $\frac{r}{t}$, which we shall call ρ. The set of possible values of ρ is $\left(0, \frac{1}{t}\right]$. The Agent's effort is a value of x which maximizes $t \cdot (\rho R(x) - C(x))$ on the effort set Σ, and is therefore a maximizer of $\rho R(x) - C(x)$. We shall use a new symbol, namely $\phi(\rho)$ to denote the Agent's chosen effort when the ratio is ρ. So $\phi(\rho) = \hat{x}(r, t)$. The function $\rho R(x) - C(x)$ displays strictly increasing differences with respect to ρ, x. Hence the maximizer $\phi(\rho)$ is nondecreasing in ρ, so we have

$$(+) \qquad \phi(\rho_H) \geq \phi(\rho_L) \text{ whenever } 0 < \rho_L < \rho_H.$$

We can now reinterpret the Principal as the chooser of a ratio. For a given t, he chooses the ratio $\rho^*(t) = \frac{r^*(t)}{t}$, where

$$\rho^*(t) = \min\{\operatorname{argmax}_{\rho \in (0, 1/t)} M(\rho, -t) \},$$

and

$$M(\rho, -t) = (1 - t\rho) \cdot R(\phi(\rho)) = R(\phi(\rho)) - t \cdot \rho \cdot R(\phi(\rho)).$$

The function M has strictly increasing differences in $\rho, -t$ if the function $-t \cdot \rho \cdot R(\phi(\rho))$ has strictly increasing differences in $\rho, -t$. But that is the case, since R is nondecreasing, which implies (using (+)) that $R(\phi(\ \cdot\ ))$ is also nondecreasing. Since $\rho^*(t)$ is a maximizer of $M(\rho, -t)$, we conclude that

$$\frac{r^*(t_L)}{t_L} = \rho^*(t_L) \geq \rho^*(t_H) = \frac{r^*(t_H)}{t_H} \text{ whenever } 0 < t_L < t_H,$$

as Part (**a**) asserts.

Proof of Part (b)

We use the terminology just used in the proof of Part (**a**). Since $\phi\left(\frac{r^*(t)}{t}\right) = \hat{x}(r^*(t), t)$, we have, using (+) in the proof of part (**a**), $\hat{x}(r^*(t_L), t_L) \geq \hat{x}(r^*(t_H), t_H)$, as (**b**) asserts.

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