INFORMATION IN TENDER OFFERS WITH A LARGE SHAREHOLDER[1]

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We study tender offers for a firm which is owned by one large shareholder who holds less than half of the total shares, and many small shareholders who each hold a unit share. Each shareholder is privately informed, yet uncertain, about the raider's ability to improve the value of the firm, whereas the raider is uninformed. In the benchmark model of complete information, the raider is unable to make a profit. As shown in Marquez and Yılmaz (2008), the same obtains when the raider is facing only privately informed small shareholders. We show, however, that the combination of private information on the side of shareholders and the presence of a large shareholder can facilitate profitable takeovers. More precisely, for any given information structure, the raider can make a profit if the large shareholder holds a sufficiently large stake in the company. In the unique equilibrium outcome, neither the probability of a successful takeover nor the equilibrium price offer depends on the large shareholder's information. Therefore, the large shareholder's information is not reflected in the price. When the equilibrium price offer is positive, the large shareholder tenders all of his shares regardless of his information. Finally, we show that the same type of equilibria arise when there are several large shareholders, as long as their total stake in the company is smaller than one-half.

KEYWORDS: takeovers, tender offers, lemons problem, large shareholder.

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1. Introduction

The threat of a potential takeover should discipline an incumbent management to serve in the best interest of the shareholders. If the incumbent management were to underperform, a more efficient raider would take over the company and introduce a better management. This reasoning fails, however, when the ownership of a company is widely dispersed. A raider will not initiate a value-increasing takeover of such a company because he cannot make any profits due to the free-riding incentives faced by the small shareholders (see Grossman and Hart (1980), Bagnoli and Lipman (1988), Harrington Jr and Prokop (1993)). The impact of one small shareholder on the probability of success of the takeover is negligible. Therefore, if the raider's price offer is lower than the post-takeover value of a share, then the small shareholders who anticipate that the takeover will succeed hold onto their shares, which collectively makes the takeover unsuccessful. Hence, successful takeovers are unprofitable. The free-riding problem is thus a fundamental source of inefficiency in the market for corporate control.

This insight extends to environments in which small shareholders have private yet imperfect information about the post-takeover value of the company, while the raider is uninformed (Marquez and Yılmaz (2008)). In fact, the raider's inability to make a profit is exacerbated due to the lemons problem arising from the asymmetry in information between the shareholders and the raider. For any price the raider offers, only shareholders who believe that the post-takeover value of the company will be low are willing to tender their shares.

We explore the roles of a minority large shareholder and of private information in takeover contests where the raider suffers from free-riding.[1] Our main result shows that under symmetric information and in the presence of a large shareholder who holds a minority stake in the company, the raider cannot make a profit. However, when there is asymmetric information, and hence the lemons problem, the raider may make strictly positive profits. In other words, the existence of a large shareholder together with asymmetric information can facilitate profitable takeovers, thus offering a new explanation for the occurrence of efficient takeovers.

Other mechanisms have been proposed to overcome the free-riding problem. Dilution (Grossman and Hart (1980)), squeeze-outs (Yarrow (1985), Amihud et al. (2004)), and debt financing (Müller and Panunzi (2004)) all reduce the post-takeover value of the shares that a minority shareholder holds, creating pressure on him to tender his shares. However, such

[1] Gadhoum et al. (2005) provides empirical support for the importance of considering large shareholders in takeovers. In our model, the large shareholder is a passive shareholder, i.e., he does not counter-bid the raider and try to undertake a takeover himself. Possible reasons for such a passive role for the shareholder are that he may be financially constrained, he may lack managerial skills, or he may be prohibited from such an action (for example, in the case of pension funds). See Burkart et al. (2006) for more detail about (*i*) the empirical evidence that the ownership structure we consider is a widely observed one, and (*ii*) support of considering a passive minority shareholder.

mechanisms also create a conflict between minority shareholder protection and efficiency. Since this conflict does not arise in our model, we interpret our result as suggesting that minority shareholder protection can occur with efficient takeovers. Another distinct solution proposed elsewhere is for the raider to secretly acquire a stake in the company before the takeover attempt (Shleifer and Vishny (1986), Chowdhry and Jegadeesh (1994)). Whether such acquisitions can take place depends on the depth of the market and the regulatory disclosure requirements. Finally, private benefits may facilitate takeovers (Grossman and Hart (1980), Bagnoli and Lipman (1988), Marquez and Yılmaz (2008)). In the latter models, unlike in ours, the raider does not make any profit on the shares he acquires from the shareholders.

Very little, however, is known about takeovers of companies in the presence of large shareholders. We study a model with incomplete information and one large shareholder.[2] With some probability, the state is low and the value added from the takeover is zero. With the complementary probability, the state is high and the value added is positive but only if the takeover is successful. The value of the company is unchanged if the takeover is unsuccessful. Each shareholder observes a private and imperfectly informative signal about the state. The raider, who has no information beyond the common prior about the state of the world, submits an unconditional offer for the equity shares of the firm by specifying a price per share. Each small shareholder then decides either to accept the offer and tender his share, or to reject the offer and keep his share, while the large shareholder decides how many shares to tender. The takeover succeeds only if the raider acquires at least half of all the shares.

We characterize the unique asymptotic equilibrium outcome of the tender game when the number of shares goes to infinity. If the large shareholder's stake is sufficiently high, then the equilibrium price is positive, the takeover succeeds with probability one, and the raider makes strictly positive profits. The small shareholders use a specific threshold strategy which leaves the large shareholder with only two options. Either he does not sell all of his shares and the takeover fails in the high state, or he sells everything and the takeover succeeds in the high state with a positive probability. Opting for the lesser of two evils, the large shareholder sells all his shares, and the raider acquires precisely half of the firm's shares in the high state of the world.

If the large shareholder is sufficiently large and sells all of his shares, then the raider needs to acquire a smaller fraction of the small shareholders' shares. The key to our result is that he can buy the latter from the more pessimistic shareholders by offering a low price. Asymmetric information therefore diminishes the free-riding incentives of some of the small shareholders.

[2] We also show that the results easily extend to the case of many large shareholders who together own less than the stake needed for a takeover.

The low price in turn gives the raider a profit from the large shareholders' shares, but a loss from the small shareholders' shares, due to the lemons problem. The larger the stake held by the large shareholder, the more the extent of the lemons problem is diminished, which results in a higher profit for the raider. The raider is unable to make a profit without the large shareholder due to the lemons problem; nor can he profit without asymmetric information due to severe free-riding. Finally, the large shareholder's equilibrium behavior is independent of his information; therefore, his information is reflected in neither the equilibrium price offer nor the probability of a successful takeover.

An important factor contributing to large shareholder formation is hedge fund activism. Some recent and ever-expanding research explores the consequences of hedge fund activism. Empirical evidence shows that the primary source of positive returns from hedge fund activism is through the takeover premium which such funds get from the acquisition of the target firm (Brav et al. (2008), Brav et al. (2010), Greenwood and Schor (2009)). Our results suggest that when a hedge fund becomes a minority large shareholder, this can help facilitate an efficient takeover, thereby serving as a powerful incentive to the incumbent management. Moreover, our finding that the large shareholder's information is not reflected in the likelihood of a successful takeover suggests that hedge funds need not have any firm-specific information in order to profit from takeovers.

Our model allows for several comparative statics that offer testable predictions on the relationship between ownership structure, takeover success, and the distribution of takeover gains. First, the larger the large shareholder's stake, the more likely the takeover will succeed. Second, as long as the large shareholder's share is less than a threshold fraction, the raider offers price zero, or does not attempt a takeover at all. However, once the large shareholder's stake exceeds that threshold, the raider's price offer has a discontinuous jump to a positive bid, which subsequently decreases with the large shareholder's share. Third, the raider's profit is zero as long as the large shareholder's share is less than the threshold level, and increases smoothly with the large shareholder's share, whenever it is above the threshold share. Finally, the small shareholders' payoffs are zero as long as the large shareholder's share is less than the threshold share; their payoffs have a discontinuous jump at the threshold share, and thereafter decreases as the large shareholder's share increases. A salient feature of our model is that the small shareholders' gains are nonmonotonic with respect to the large shareholder's initial stake.

In addition to the results described above, our paper offers a new methodology for studying takeovers with a continuum of shareholders. We introduce an equilibrium concept in the spirit of the rational expectations equilibrium. Our equilibrium concept differs from those commonly used in the literature on takeovers with atomistic shareholders (for example, Grossman and Hart (1980) and Shleifer and Vishny (1986)). While the standard models of takeovers with a continuum of shareholders assume that the probability of success is one when exactly half of the shares are sold, we leave this probability to be determined endogenously in equilibrium. We conclude that our equilibrium concept captures the behavior present in the model with a large, albeit finite, number of shares, by showing that the outcomes of perfect Bayes-Nash equilibria of the model that has finitely many shares converge to the unique equilibrium outcome of the continuum-shares model, as the number of shares goes to infinity. In other words, we provide a micro-foundation for the model using standard equilibrium concepts.

An important assumption in our paper is that the shareholders are privately informed agents. In this respect, the closest paper to ours is Marquez and Yılmaz (2008). In their model also, the shareholders are privately informed, while the raider holds no information. Both our models can be thought of as a reduced-form model in which the raider does have some information about the value of the takeover, yet this information is public. The crucial part of our model is that the shareholders have private information, in addition to the public information they share with the raider, and that they are asymmetrically informed. For example, employees in the firm, each of whom owns a very small number of shares, might know about the inner workings of the company, such as its corporate culture, which might affect the post-takeover value of the firm. Certainly the case in which the raider has some private information is also relevant. Indeed, our model serves as a building block for the analysis of such a signaling game, since our analysis applies to characterizing the shareholders' behavior after any price offer. Moreover, some previous studies show that in such games, the only equilibrium is the one in which the raiders' types are pooling, and hence the raider is unable to signal his information (Marquez and Yilmaz (2005), Burkart and Lee (2010)).

Shleifer and Vishny (1986) consider a model with a large shareholder and a continuum of small shareholders. In their model, the large shareholder is the raider, whereas in our model the large shareholder is passive. They show that the raider makes strictly positive profits, because he already owns a nontrivial share of the company and has strict incentives to facilitate the takeover in order to increase the value of the shares he owned from the start, even at the expense of a loss on the new shares he buys. Holmström and Nalebuff (1992) study a complete-information model in which the firm is owned by several large shareholders. They construct a particular type of equilibrium and show that the raider can extract a significant part of the surplus when the number of shares goes to infinity, while the number of shareholders and their relative position in the firm is held fixed.

Cornelli and Li (2002) analyze a setting with finitely many risk arbitrageurs participating in a tendering game. The risk arbitrageurs in their paper are similar to the large shareholders in our model. However, the takeover succeeds only when the arbitrageurs collectively hold at least half of the shares. In our model, the large shareholder does not have a controlling stake, yet takeovers can still be successful.

A more recent paper by Burkart et al. (2006) analyzes the effects of a minority shareholder in takeovers. They consider a complete-information environment with a large minority shareholder, in which a successful raider faces a moral hazard problem in that he may extract private benefits at the expense of the share value. In their model the large shareholder sells all the shares in equilibrium. Moreover, the large shareholder would like to sell more shares conditional on the takeover succeeding, because of the inefficiency of the post-takeover private-benefit extraction. In contrast, while the large shareholder does sell all of his shares in our model, he would have preferred to keep them in a scenario in which his behavior would have no effect on the success of the takeover. In the model of Burkart et al. (2006), the more shares held by the large shareholder, the more the raider's equilibrium price offer is, and the lower his profits are. We find the opposite in our model, where the price is nonincreasing and the profits nondecreasing functions of the large shareholder's stake.

The paper is organized as follows: In section 2, we first analyze the model with finitely many shareholders without uncertainty, i.e., the complete-information benchmark model. In section 3, we introduce the incomplete-information model with a continuum of shares, and develop the equilibrium concept for this model. In section 4, we solve the unique equilibrium of the incomplete information model, and present our main theorem. In later sections, we extend the model to allow for multiple large shareholders, or private benefits for the raider from a successful takeover. Before concluding, We flesh out the comparative statics and provide the incomplete-information model with finitely many shareholders with the convergence results.

2. Complete Information Model

A firm is owned by a large shareholder who holds xn shares, where $x \in (0, \frac{1}{2})$, and $(1-x)n$ small shareholders, each of whom holds a single share.[3] A raider wishes to take over the company, with intention to run it, which is currently run by an incumbent management. The raider needs to acquire at least $\frac{n}{2}$ shares to successfully take over the company.

In the complete-information model, the value of the firm under the incumbent is commonly known to be $V_0 = 0$. If the raider successfully takes over the firm, the value is increased to $V_1 = 1$, with a per-share value $v_1 = \frac{1}{n}$.

The raider makes an unconditional price offer p, which is followed by a subgame in which the shareholders simultaneously decide how many shares to sell. The raider acquires all of

[3]For convenience, we assume $xn \in \mathbb{N}$.

the shares that are tendered to him. We now develop the notation for the strategies and equilibrium in the subgames after the raider has made his price offer. A strategy for a small shareholder $\sigma_i \in [0,1]$ is the probability with which he tenders the share. Similarly, a strategy for the large shareholder σ_L is a probability distribution over $\{0, 1, ..., nx\}$. The probability that the large shareholder tenders k shares, where $k \in \{0, 1, ..., nx\}$, is denoted as $\sigma_L(k)$.[4]

A profile of pure strategies is an equilibrium if and only if exactly $n/2$ shares are tendered.[5] However, we will focus on the properties of symmetric equilibria of the tender subgames for any $p \in (0, 1/n)$.[6] Symmetric equilibria are equilibria in which all the small shareholders use the same strategy. We denote such an equilibrium strategy profile by the couple (ϕ, σ), where the first component is the probability that the small shareholder tenders his share (hence, $\phi \in [0,1]$), and the second component is the large shareholder's strategy.[7] An immediate observation is that, for the given range of prices, there does not exist a symmetric equilibrium in which small shareholders use pure strategies. Otherwise, either every small shareholder would sell his share or every small shareholder would keep it. In the former case, the takeover would succeed with certainty; hence, keeping a share would be more profitable than selling the share. In the latter case, the takeover would fail with certainty and therefore, selling rather than keeping the share would be the optimal action.

Suppose the raider offers a per-share price $p \in (0, 1/n)$, and in the tender subgame, the shareholders use a symmetric equilibrium strategy profile (ϕ, σ). In such an equilibrium the raider's payoff is:

$$U_R := v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} (i+k) - [(1-x)n\phi + E[k]]\, p.$$

The first term is the expected value of tendered shares conditional on the takeover succeeding. The second term is the price paid for the tendered shares, i.e., the expected number of tendered shares times the per-share price p. In expectation, small shareholders tender $(1-x)n\phi$ shares, and the large shareholder tenders $E[k]$ shares, where the expectation is taken with respect to the large shareholder's strategy σ.

As noted above, in any symmetric equilibrium the small shareholders use mixed strategies,

[4]Throughout the paper, we define the strategies for the shareholders in the subgames after the raider has made his price offer. If we had developed the strategies for the shareholders in the normal-form game, instead of the subgames after the raider's price offer, then a strategy would be a mapping from a set of prices to a mixed action.

[5]This argument follows the same reasoning offered by Bagnoli and Lipman (1988).

[6]We focus on the prices in $(0, 1/n)$ in order to explore whether the raider can make a profit. Clearly, he can facilitate a takeover by offering $p \geq 1/n$. But such a takeover will not be profitable.

[7]The existence of a symmetric equilibrium of any tender subgame is easily established using standard arguments.

$\phi \in (0,1)$; they must therefore be indifferent between tendering and not tendering their shares:

$$p = v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i} \phi^i (1-\phi)^{n(1-x)-1-i}.$$

If a small shareholder tenders his share, he receives price p. If he does not tender it, his payoff is the expected value of the share, conditional on the takeover succeeding, times the probability that the takeover succeeds, given that he is not tendering. This is represented by the right-hand side of the above equation.

The key difficulty that arises in our model is pinning down the large shareholder's behavior. His equilibrium behavior is more complicated than the small shareholders' behavior. Nevertheless, we can bound his equilibrium payoffs from below as follows: the large shareholder's payoff in any tender subgame is at least pnx, i.e., the payoff he obtains if he sells all his shares. This lower bound is sufficiently tight to show that the raider's profits approach zero when the number of shares approaches infinity.

We will say that $(p^n, \phi^n(\cdot), \sigma^n(\cdot))$, for $p^n \in [0, 1/n]$, is *an equilibrium of the tender game* with n shares if $(\phi^n(\tilde{p}^n), \sigma^n(\tilde{p}^n))$ is a symmetric equilibrium of the tender subgame after the price offer $\tilde{p}^n$, for every $\tilde{p}^n \in [0, 1/n]$; and if p^n is the raider's optimal price offer given $(\phi^n(\cdot), \sigma^n(\cdot))$.[8] In other words, an equilibrium of the tender game is a subgame perfect equilibrium in which, for every price offer, the shareholders play a symmetric equilibrium of the tender subgame.[9]

In the following theorem, we show that the raider's equilibrium profits converge to zero along any sequence of equilibria of the tender games as n approaches infinity.[10] This theorem allows us to conclude that the existence of a minority large shareholder is not sufficient to facilitate profitable takeovers.

THEOREM 1 *Let $\{p^n, \phi^n(\cdot), \phi^n(\cdot)\}_n$ be a sequence of equilibria of the tender games. The raider's equilibrium profit converges to zero as $n \to \infty$.*

PROOF: See Appendix A. □

The intuition for why the raider cannot make any profits when there is a minority large

[8] Notice that we now allow for $p \in [0, 1/n]$. This makes the set of the raider's strategies compact. For $p = 0$, nobody tenders in the unique symmetric equilibrium of the tender subgame. For $p = 1/n$, on the other hand, all the shares are tendered. In either case the raider's profit is zero.

[9] The existence of an equilibrium of the tender game follows from the standard result about the existence of subgame perfect equilibria.

[10] In our proof of the theorem we establish a stronger result: the raider's profits approach zero uniformly across all equilibria.

shareholder is very similar to the reason that he cannot make any profits in the absence of a large shareholder. In order for a successful takeover, the raider needs to buy a nonnegligible fraction of shares that are owned by the small shareholders, regardless of the strategy of the large shareholder. However, when the number of shares is very large, the only way that a small shareholder is willing to tender his share is if the price offer is at least the expected value of his share after the takeover activity. The raider pays the same price for every share he obtains; hence, he pays the expected post-takeover value of each share that he obtains, and makes no profits.

Note that our result depends on the assumption that the large shareholder owns a minority stake of the firm. If, however, the large shareholder would own a majority stake of the firm, then for every positive price offer, there would be an equilibrium of the subgame in which the large shareholder would sell exactly half of the firm's shares, and the small shareholders would sell none. Therefore, in equilibrium, the takeover would succeed and the raider would get half of the surplus of the takeover activity.

REMARK 1 *If, contrary to what we assume, the large shareholder holds a fraction $x > 1/2$ of the company, then the raider can facilitate a successful takeover by offering any positive price. Namely, it is in the large shareholder's interest to ensure a successful takeover. Small shareholders' holdings, on the other hand, are not necessary for a successful takeover.*

While the literature so far has emphasized the raider's inability to make a profitable tender offer for a widely dispersed company (see, for example, Grossman and Hart (1980) and Bagnoli and Lipman (1988)), Theorem 1 shows that profitable tender offers are impossible even in the presence of a large shareholder as long as he does not hold a majority stake. In light of Theorem 1, the inability of the raider to make a profitable tender offer is not a consequence of the dispersed ownership of the whole firm, but is rather a consequence of the fact that the majority stake is widely dispersed.

3. Incomplete Information Model

We now introduce the model with incomplete information. In this part, unlike in the complete-information model of Section 2, we study the takeover game with a continuum of shares.[11] The raider who wants to acquire the firm makes an unconditional price offer p for the shares of the firm. There is a continuum of shares represented by the interval $[0, 1]$, where a mass of $(1 - x)$ is held by a continuum of small shareholders of size $(1 - x)$, and a mass

[11] In Section 9 we explain why we present the complete-information model assuming a discrete number of shares, while the incomplete-information model is presented with a continuum of shares.

x of the shares is held by a large shareholder. The large shareholder is not, however, large enough to facilitate the takeover by selling all of his shares, i.e. $x < 1/2$.[12]

There are two states of the world, $\omega \in \{h, l\}$, and the common prior belief that $\omega = h$ is $\lambda \in (0, 1)$. The value of the firm is 1 if the state of the world is h and the takeover is successful. The value of the firm is 0 if the state of the world is l or if the takeover is unsuccessful. Each small shareholder observes a signal drawn from a conditionally i.i.d. $F(s|\omega)$ with support $S := [0, 1]$, and with the density function $f(s|\omega)$, where ω is the true state of the world. Large shareholder observes a signal $s \in S := [0, 1]$ drawn from distribution $H(s|\omega)$, with the density $h(s|\omega)$ and his signal is assumed to be independent of the small shareholders' signals. We assume that the monotone likelihood ratio property (MLRP) holds for the shareholders' distributions, i.e., $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are strictly increasing on the interval $[0, 1]$.[13,14]

3.1. Strategies A strategy for the raider is a price offer $p \in [0, \infty)$. Every price offer induces a tender subgame that we describe below. In a tender subgame with a fixed price offer $p \geq 0$, a small shareholder either sells his share or keeps it, while the large shareholder decides what fraction of his shares to sell. In particular, a mixed strategy for a small shareholder specifies the probability that he tenders his share for each signal:

$$\sigma : S \to [0, 1].$$

A strategy for the large shareholder is a right-continuous and weakly increasing mapping $\sigma_L : S \times [0, 1] \to [0, 1]$, which denotes the cumulative distribution function of the fraction of the shares he tenders, and whose marginal on its first coordinate coincides with the distribution of signals. The first argument is the signal, and s denotes a generic element. The second argument is the fraction, and r denotes a generic element. Modeling the strategy as a cumulative distribution function ensures that payoffs are well-defined (see Milgrom and Weber (1985)). The strategy σ_L is weakly increasing in both of its arguments, and for every $s \in [0, 1]$, it satisfies the following equation:

$$\sigma_L(s, 1) = \lambda H(s|h) + (1 - \lambda)H(s|l). \tag{1}$$

[12]Note that in light of Remark 1, we look at the case in which there is the free-riding problem, i.e., $x < 1/2$.

[13]Notice that the strictly increasing condition implies that all the densities are larger than zero and finite for all $s \in (0, 1)$.

[14]We assume strict MLRP solely for expositional purposes. All of our results go through if we assume the weak version of MLRP, i.e., $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are weakly increasing in s. The weak MLRP accommodates, among other things, a formulation with finitely many signals.

The above condition ensures that the marginal distribution of σ_L on its first coordinate is equal to the signal distribution.[15] The set of strategies of the large shareholder, Σ_L, is the set of all strategies (distributions) satisfying equality 1. In addition, we introduce the conditional distributional strategies, $\sigma_L(s,r|\omega)$, which we derive from $\sigma_L(\cdot,\cdot)$ as follows:

$$\sigma_L(\bar{s},\bar{r}|\omega) := \int_{s=0}^{\bar{s}}\int_{r=0}^{\bar{r}} \frac{h(s|\omega)}{\lambda h(s|h)+(1-\lambda)h(s|l)} d\sigma(s,r).$$

Note that $\sigma_L(s,1|\omega) = H(s|\omega)$, for every $s \in [0,1]$.

A strategy for a small shareholder, σ, is a threshold strategy if there exists a signal $s^* \in S$ such that $\sigma(s) = 1$ for every $s < s^*$ and $\sigma(s) = 0$ for every $s > s^*$.

A strategy profile in a tender subgame is a collection $\{\sigma_i\}_{i\in[0,1-x]\cup\{L\}}$. A strategy profile is *symmetric* if $\sigma_i = \sigma_j$ for every $i,j \in [0,1-x]$, and (σ,σ_L) denotes a typical symmetric strategy profile.

3.2. Payoffs A small shareholder's payoff from tendering his share is the price p. The expected payoff from keeping his share depends on his belief $q \in [0,1]$ that the takeover is successful in state h and his belief that the state is h.[16] Let

$$\beta(s) := \frac{\lambda f(s|h)}{\lambda f(s|h)+(1-\lambda)f(s|l)},$$

be the posterior belief of a small shareholder that the state is h, given his signal s. Then, the payoff function of a small shareholder is given by the following equalities:

$$U(p,s,q,keep) = \beta(s)q,$$

and

$$U(p,s,q,sell) = p.$$

Similarly, let the large shareholder's posterior belief that the state is h when he observes signal s be $\beta_L(s) := \frac{\lambda h(s|h)}{\lambda h(s|h)+(1-\lambda)h(s|l)}$. For a collection of beliefs $q_L := q(r)_{r\in[0,1]}$, the expected payoff from tendering a fraction $r \in [0,1]$ of his shares is given by:

$$U_L(p,s,q(r),r) = x\left(rp+(1-r)q(r)\beta_L(s)\right),$$

[15] We could also model the strategy of the small shareholder as a distributional strategy. However, as we will see later, in all equilibria the small shareholders' strategies have a threshold structure. Hence, we do not have to assume that the strategy of the small shareholder be representable by a distribution function.

[16] The role and meaning of the concepts, such as the belief q introduced here, will be clearer when we define the equilibrium concept below.

with the interpretation that $q(r)$ is the belief that the large shareholder attaches to the takeover succeeding in the high state when he tenders a fraction r of his shares.

Finally, the raider's payoff (i) when he offers the price p, (ii) the shareholders use the symmetric strategy profile (σ, σ_L), and (iii) he believes the probability of takeover success as a function of the large shareholder's behavior is determined by the collection q_L, is given by:

$$
\begin{aligned}
U_R(p, \sigma, \sigma_L, q_L) &= \lambda \int_{r,s} q(r) \left(xr + (1-x) \int_{s\in[0,1]} \sigma(s) f(s|h) ds \right) d\sigma_L(s, r|h) \\
&- p \left[(1-x) \int_{s\in[0,1]} \sigma(s)[\lambda f(s|h) + (1-\lambda) f(s|l)] ds + x \int_{s,r} r d\sigma_L(s,r) \right].
\end{aligned}
$$

The first term represents the raider's benefits when the takeover is successful and the state is high. The second term represents the total payment the raider makes to acquire the shares. As a reminder, small shareholders' strategy σ is a standard behavior strategy which prescribes for each signal s the probability with which a small shareholder tenders his share. On the other hand, σ_L is a distributional strategy.

3.3. Equilibrium A tuple $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ is a *symmetric* equilibrium of a tender subgame with a price offer p if the following conditions hold:

(2) $$U(p, s, q, \sigma(s)) \geq U(p, s, q, a), \; \forall a \in \{keep, sell\}, \; \forall s \in [0,1].$$

(3) $$\int_{s\in[0,1], r\in[0,1]} U_L(p, s, q(r), r) d\sigma_L(s, r) \geq \int_{s\in[0,1], r\in[0,1]} U_L(p, s, q(r), r) d\bar{\sigma}_B(s, r), \; \forall \bar{\sigma}_B \in \Sigma_L.$$

(4) $$q(r) = \begin{cases} 0, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr < 1/2 \\ 1, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr > 1/2 \\ \in [0,1], & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr = 1/2. \end{cases}$$

(5) $$q = \int_{s\in[0,1], r\in[0,1]} q(r) d\sigma_L(s, r|h).$$

The first two conditions above are the standard conditions requiring that the shareholders' behavior is optimal given their beliefs. Condition (4) describes the large shareholder's beliefs about the probability of the successful takeover in the high state when he tenders a fraction r of his shares, given the fixed behavior of the small shareholders. The fraction of shares tendered by the small shareholders in the high state, given strategy σ, is $(1-x)\int_0^1 \sigma(s) dF(s|h)$. Therefore, if the large shareholder tenders fraction r of his shares and if $(1-x)\int_0^1 \sigma(s) dF(s|h) + xr$ is larger (smaller) than $1/2$, then the takeover succeeds (fails) with certainty. The indeterminate case is when $(1-x)\int_0^1 \sigma(s)dF(s|h) + xr = 1/2$. We leave the large shareholder's beliefs, in such a knife-edge case, to be determined in equilibrium. Finally, condition (5) requires that the small shareholders' belief q about the success of the takeover in the high state be derived from q_L, using the large shareholder's strategy σ_L.

Remark 2 *Our equilibrium concept is different than Nash equilibria or refinements thereof, in that our equilibrium contains variables such as the probability of a successful takeover. This is in the spirit of the rational-expectations equilibrium concept, and allows for the probability of a successful takeover when the fraction of shares acquired is one-half to be determined endogenously. We show later that our model has a unique equilibrium outcome. Moreover, we show in Section 8 that the unique equilibrium outcome of the model with a continuum of shares is precisely the limit of the equilibrium outcomes of the takeover model with finitely many shares.*

The raider's continuation payoff from offering p when the tuple $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$ is played in the tender subgame is denoted by:

$$\Pi(p, T) := U_R(p, \sigma, \sigma_L, q_L).$$

Note that a price $p > 1$ can never result in a positive payoff for the raider; we therefore restrict his price offers to the interval $[0, 1]$. We say that the collection $(p, T(p')_{p'\in[0,1]})$ is an equilibrium of the takeover game if each $T(p')$ is an equilibrium of the tender subgame with price offer p', and if $p \in argmax_{p'\in[0,1]}\Pi(p', T(p'))$.

When there is a unique equilibrium of a tender subgame for a given price p, we write $\Pi(p)$ for the raider's profit when all other players play the unique equilibrium of the tender subgame.

4. Equilibrium Characterization

In this section we characterize the equilibrium behavior of the shareholders for both on and off the equilibrium price offers. In particular, in Theorem 2 we show that there is a unique equilibrium of each tender subgame, and we calculate the raider's profits in each of these equilibria. In Theorem 3, we show that it is possible that the raider makes a positive price offer, the takeover is successful in the high state, and the raider makes strictly positive profits. We start with a preliminary observation that the small shareholders use threshold strategies in any tender subgame.

Lemma 1 *In any equilibrium of the tender subgame where $p > 0$, the small shareholders* *use a threshold strategy, i.e., there is a* $\sigma \in [0,1]$ *such that the small shareholder tenders his share if* $s < \sigma$ *and keeps it if* $s > \sigma$*.*

PROOF: The MLRP condition implies that a small shareholder's belief $\beta(s)$ is a strictly increasing function. Fix an equilibrium of the tender subgame for some $p > 0, T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$. The small shareholder's payoff from tendering a share is p, while keeping the share yields $q\beta(s)$. Therefore, if $\sigma(s) > 0$ for some s, then for every $s' < s$, it follows that $q\beta(s') < p$ and hence $\sigma(s') = 1$. Similarly, if $\sigma(s) < 1$ for some s, then $q\beta(s') > p$ for all $s' > s$, and therefore $\sigma(s') = 0$ for every $s' > s$.[17] □

In the following development, we identify the small shareholders' equilibrium strategy with the threshold signal σ. When there is no confusion, $\sigma \in [0,1]$ denotes both the threshold signal and the small shareholders' equilibrium strategy.

We call a signal $s^* \in [0,1]$ *pivotal* if it has the property that when the small shareholders use the threshold s^*, and the large shareholder tenders all his shares, then the fraction of tendered shares is $1/2$. Since $x < 1/2$, the pivotal type $s^* \in (0,1)$ is uniquely defined[18] by

$$F(s^*|h)(1-x) + x = 1/2.$$

The *critical price* $\bar{p}$ is the price that would keep the pivotal type indifferent between tendering his share and keeping it, if he believed that the takeover would be successful with probability one in state h. In particular,

$$\bar{p} := \beta(s^*). \quad (6)$$

REMARK 3 *Note that both* s^* *and* $\bar{p}$ *are decreasing in* x*.*

We now present the unique equilibrium of each tender subgame with a price offer $p > 0$. The structure of equilibria depends on whether the price offer is below or above the critical price.

THEOREM 2 *(Characterization) For any* $p > 0$*, there is a unique equilibrium of the tender subgame,* $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$*.*

(i) If $p \le \bar{p}$*, then*

a) $\sigma = s^*$*.*

[17] Here we use a convention that if q and p are such that $q\beta(s) < p$ for all s, then the small shareholders use the threshold $\sigma = 1$; in other words, they tender irrespective of their signal. Similarly, if $q\beta(s) > p$ for all s, the threshold $\sigma = 0$, which means that they never tender.

[18] We assume continuity of $F(\cdot|h)$ through assuming that it admits a density function.

b) $\sigma_L(s,r) = 0$ *for every* $s \in [0,1]$ *and every* $r < 1$.
c) $q = \frac{p}{\beta(s^*)}$, $q(1) = \frac{p}{\beta(s^*)}$ *and* $q(r) = 0$ *for all* $r < 1$.
Moreover, the raider's profit is given by:

$$\Pi(p) = \lambda \frac{q}{2} - p\left(\lambda \frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right). \tag{7}$$

(ii) If $p > \bar{p}$, *then*
a) $\sigma = 1$ *if* $p \geq \beta(1)$, *and otherwise is the unique solution to the equality* $\beta(\sigma) = p$.
b) There is a signal $s_L \in [0,1]$ *and a fraction* $a < 1$ *such that, if the large shareholder's signal* $s > s_L$, *then he tenders fraction* a *of his shares; and if* $s < s_L$, *then he tenders all of his shares.*
c) $q = 1$, $q(r) = 0$ *for* $r < a$, *and* $q(r) = 1$ *for* $r \geq a$.
Moreover, the raider's profit is given by:

$$\begin{aligned} \Pi(p) = {} & \lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ & -p\lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ & -p(1-\lambda)[(1-x)F(\sigma|l) + x(a(1-H(s_L|l)) + H(s_L|l))]. \end{aligned} \tag{8}$$

PROOF: See Appendix B. □

The theorem characterizes the unique equilibrium of tender subgames under two cases. The first one is when the price p is smaller than or equal to the critical price $\bar{p}$. In this case, the small shareholders' equilibrium threshold is s^*, and is independent of the exact value of p. The probability of a successful takeover in the high state (q) is determined endogenously so that a shareholder receiving the signal s^* is indifferent between tendering and not tendering his share. If he tenders his share, he receives the price p, whereas if he keeps it, then it is worth zero in the low state, and one in the high state but only if the takeover succeeds. Such a shareholder believes that the state is high and the takeover succeeds with probability $\beta(s^*)q$. Therefore, the probability of a successful takeover is linear in the price offer (see Figure 1 for a depiction).

The large shareholder, on the other hand, tenders all of his shares regardless of his signal. His behavior is optimal because by tendering anything less, he would cause the takeover to fail in the high state with certainty, by the definition of the pivotal signal, s^*. Such a takeover failure would render the shares he held back valueless. In equilibrium, the large shareholder has no reason to withhold any of his shares. Indeed, in the low state he wants to sell all his shares, while in the high state the takeover fails if he withholds any fraction of his shares. His behavior is therefore independent of his signal.

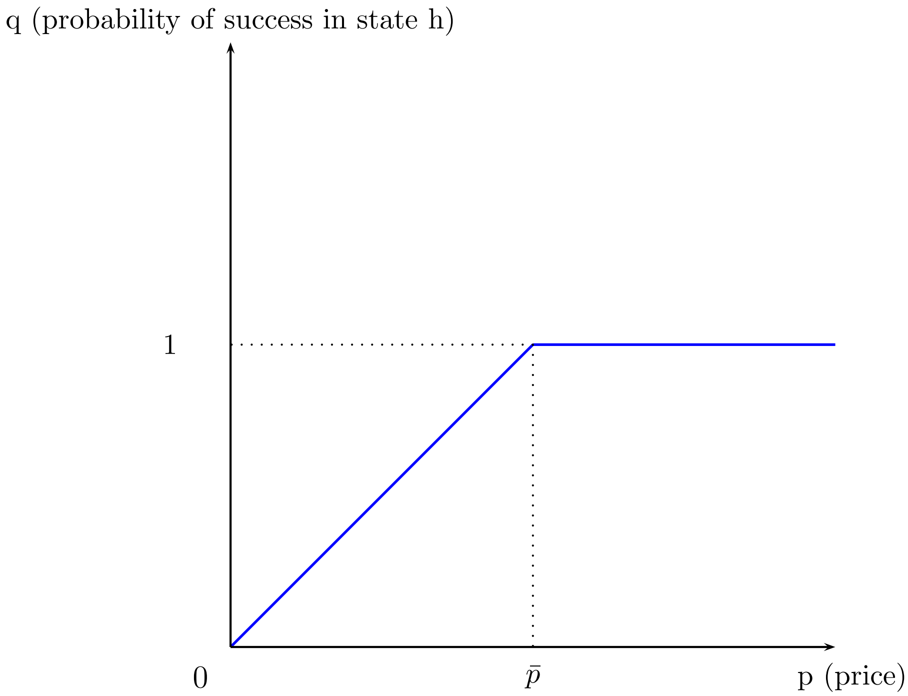

Figure 1: This figure shows how the equilibrium probability of a successful takeover in state h, denoted by q, varies with the raider's price offer. In particular, $q = \frac{p}{\bar{p}}$ for $p \leq \bar{p}$ and $q = 1$ for $p > \bar{p}$.

The shareholders' behavior, as described for $p \leq \bar{p}$, and the definition of s^* imply that exactly half of the shares are sold in the high state. The raider's payoff can now be decomposed into two parts. He benefits only from the shares that he holds in the high state conditional on the takeover succeeding, as captured in the first term in equation 7. Since (i) the probability of the high state is λ, (ii) the probability of the success of the takeover in the high state is q, and (iii) exactly half of the shares are being sold in the high state, this yields $\lambda q/2$. The second term in equation 7 represents the expected amount the raider pays for shares in the equilibrium. It is the price p times the expected quantity of shares he has bought. Notice that more shares are sold in the low state than in the high. This is because the large shareholder's tendering decision is independent of his signal, and the small shareholders tender their shares when they observe the low signals, which are in turn more likely in the low state. This is a so-called lemons problem. Only the more pessimistic small shareholders are willing to tender their shares. Therefore, the raider who wants to induce a successful takeover in the high state must accept losses on the shares he buys from the small shareholders.

The second case is when the price offer p is larger than $\bar{p}$. In this case, the small shareholder's equilibrium threshold σ is greater than the pivotal signal, s^*. Namely, the small shareholder with a signal s^* is indifferent between tendering and keeping his share even when the probability of success is one and the price is $\bar{p}$. He, therefore, strictly prefers to tender when the higher price is offered and the probability of success is less than or equal to one. Consequently the equilibrium threshold must be above s^*. An argument similar to the one above implies that the large shareholder either tenders a fraction a of his shares, which is barely sufficient to ensure a successful takeover in the high state, or he tenders all of his shares. He certainly tenders all the shares when he deems the high state unlikely, in which case even a successful takeover does not generate a high post-takeover share value. On the other hand, he tenders only fraction a of his shares, the smallest fraction which renders success in the high state certain, when his signal favors the high state. More importantly, when $p > \bar{p}$, the takeover succeeds with probability one in both states.

Before we proceed to the characterization of the raider's equilibrium price offers, we show that the raider's profit from offering a price zero is zero.

Lemma 2 $\Pi(0, T) = 0$ *for any equilibrium of the tender subgame where* $p = 0$.

Proof: When $p = 0$, $q = 0$. If on the contrary $q > 0$, it would be optimal for the small shareholders not to tender their shares and to obtain a positive payoff, which contradicts $q > 0$. Since the probability of success in the high state is zero, the raider expects a payoff of zero regardless of how many shares are tendered.[19] □

In the next theorem, we characterize the raider's optimal behavior. The raider, depending on the parameters of the environment, either offers the price zero and makes zero profits, or the price $\bar{p}$, in which case he makes a positive profit.

Theorem 3 *The raider's profit is maximized at either* $p = 0$ *or* $p = \bar{p}$. *If*

$$\Pi(\bar{p}) := \lambda\frac{1}{2} - \bar{p}\left(\lambda\frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) > 0,$$

then the raider offers the price $\bar{p}$, *and the takeover is successful with probability one in both states. If instead* $\Pi(\bar{p}) < 0$, *then the raider offers price zero, and the takeover fails with certainty in the high state.*

Proof: See Appendix B. □

Only one of two prices may arise in equilibriium. Either the raider is not willing to offer a positive price for the shares, or he pays the lowest price that ensures a successful takeover in the high state.

[19] Shareholders' equilibrium behavior for $p = 0$ is restricted only by $q = 0$. In the low state the takeover could succeed with any probability.

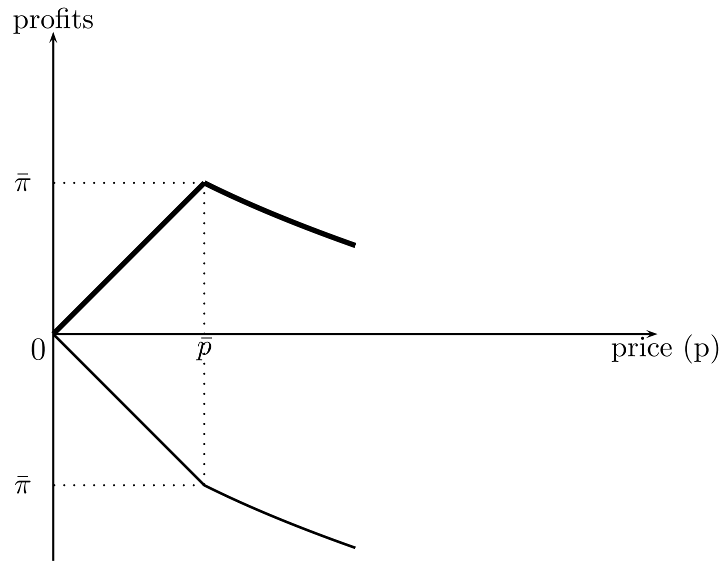

Figure 2: This figure shows the relationship between the profits of the raider and the price offer he makes. If $\lambda\frac{1}{2} - \beta(s^*)\left(\lambda\frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) > 0$, then the profit function has the shape of the thick curve (higher curve); otherwise, it has the shape of the thin curve (lower curve). In either case, the profit function is strictly decreasing in the price offer in the range $p \geq \bar{p}$.

To see that no price strictly between zero and $\bar{p}$ is offered, notice that for any such price the threshold type of the small shareholders is s^*. Moreover, the indifference condition for the type s^* implies that $p = q\beta(s^*)$. Hence, the price and the probability of a successful takeover in the high state are linearly related to each other. The fraction of shares that the raider acquires does not depend on the price, as long as $p \leq \bar{p}$, because the large shareholder tenders all of his shares at any such price. Therefore, if offering a positive price is a better strategy than offering a zero price, then the marginal cost of increasing the probability of success in the high state is strictly smaller than the marginal benefit from increasing such a probability. Consequently, offering $\bar{p}$ dominates any offer below $\bar{p}$.

The raider does not want to make an offer larger than $\bar{p}$. Note that the total surplus is equal to the probability of a successful takeover in the high state. This probability is 1 as long as the price is at least $\bar{p}$. Therefore, the total surplus is independent of the price offer in that range. However, the surplus that goes to the shareholders strictly increases with the price offer. Hence, the raider's profits are lower with higher price offers. (See figure 2, which shows that the raider's profits are linear in price until the price reaches $\bar{p}$ and are decreasing when the price is above $\bar{p}$.) Below, we argue that both asymmetric information and a large shareholder are necessary for the raider to make positive profits in our setup.

Remark 4 *(The roles of asymmetric information and a large shareholder)*

- *Suppose that the small shareholders hold no private information, and their signals are uninformative about the state of the world. Suppose, in addition that they are using a threshold strategy. Then $\bar{p} = \lambda$, and $F(s^*|h) = F(s^*|l)$. Therefore, $\Pi(\bar{p}) = 0$, and the raider makes no profits.*
- *Suppose that there was no large shareholder, i.e., suppose that $x = 0$. In that case, rearranging the equation for $\Pi(\bar{p})$ and using the MLRP condition, we find that $\Pi(\bar{p}) < 0$.*

5. Multiple large shareholders

We now show that our results extend to a setting with multiple large shareholders. Suppose there are K large shareholders, $j \in \{1, 2..., K\}$. Large shareholder j holds a fraction x_j of the total number of shares. We assume that $x := \sum_{j=1}^{K} x_j < 1/2$.[20]

First we consider the complete-information setup, where the value of the company in the case in which the takeover succeeds is 1. In a tender subgame of the game with n shares and a price offer p, the large shareholder j's strategy is $\sigma_{L,j} \in \Delta(0, 1, ..., nx_j)$, i.e., a probability distribution over the number of shares he tenders to the raider. As the number of total shares, n, grows large, the raider's equilibrium payoff approaches zero. For the intuition that follows, consider a very large number of shares. In a symmetric equilibrium the small shareholders use mixed strategies,[21] i.e., they are indifferent between tendering and not tendering their shares. This means that the probability they assign to the takeover succeeding must be equal to the price p. In turn, the ex ante expected surplus is equal to p, and the small shareholders' surplus is $(1 - x)p$. On the other hand, each of the large shareholders can guarantee himself the payoff px_j by selling all of his shares; thus the large shareholders together get at least xp. But then there is no surplus left for the raider.

Now suppose that there is incomplete information about the state of the world, and a large shareholder j receives a private signal, which is distributed according to the probability distribution function $H_j(s|\omega)$. In this case, there is a multiplicity of equilibria in the continuum model due to the possibility of coordination failures among large shareholders. However, there also exists an equilibrium in which, after the price offer $\bar{p} = \beta(s^*)$, the large shareholders tender all their shares, and the takeover is successful with certainty. In this equilibrium, the raider's profit is the same as in the unique equilibrium of the tender subgame with price offer $\bar{p}$ in the model with a single large shareholder who holds fraction x of all the shares. Hence, if $\Pi(\bar{p}) > 0$, the raider offers price $\bar{p}$ and the takeover succeeds with certainty.

[20]The number of large shareholders, K, is allowed to be any finite number. Note that the small shareholders are infinitesimal compared to any of the large shareholders, since each large shareholder owns a constant fraction of the firm.

[21]Symmetry here means that all the small shareholders use the same strategy.

6. Private benefits

Suppose that the raider obtains some private benefit, $B > 0$, if the takeover is successful, irrespective of the state of the world.

In the complete-information scenario, the raider's equilibrium payoff converges to B, and the probability that the takeover succeeds goes to one as the number of shares, n, grows. This is very similar to the result obtained by Marquez and Yılmaz (2008) in an environment without a large shareholder. The reason is that, given that the raider can in the limit not make a profit on the shares, he makes certain that he receives the private benefit B. He can always achieve this by offering the shareholders the full post-takeover value of the company.

In the incomplete-information game with a continuum of shares, the shareholders' behavior is the same regardless of the size of B; therefore, the characterization of the shareholders' behavior carries over from the environment with $B = 0$. For any strictly positive price offer, the raider receives at least half of all the shares in the high state of the world. As in the case in which $B = 0$, prices above $\bar{p}$ are dominated for the raider by $\bar{p}$.[22] In the remainder of the analysis we focus on the case in which $p \in [0, \bar{p}]$. For the prices in $(0, \bar{p}]$, exactly half the shares are sold in the high state. This in turn means that the raider receives a fraction of shares that strictly exceeds one-half, and therefore the takeover succeeds with probability one, in the low state of the world.

There is a multiplicity of equilibria in the tender subgame after $p = 0$, with the common feature that the takeover succeeds with probability zero in the high state, and that all the shareholders have payoff zero. However, the equilibrium outcome is unique, because when the raider offers any positive price, the takeover succeeds in the low state with probability one.

In equilibrium the raider offers either price $\bar{p}$ or zero. Prices above zero increase the probability of a successful takeover in the high state, but do not affect the probability of a successful takeover in the low state or, for that matter, the fraction of shares that the raider acquires. In particular, for any price $p \leq \bar{p}$, the probability that the takeover succeeds in the high state is $p/\bar{p}$. Hence, if the raider prefers to offer a positive price to offering a zero price, then he also prefers to offer $\bar{p}$ to offering any price strictly less than $\bar{p}$. The equilibrium price is $\bar{p}$ if $\Pi(\bar{p}) + \lambda B > 0$. In other words, there is a threshold $\bar{B} \geq 0$ such that if $B \geq \bar{B}$, then the takeover succeeds with probability one in both states. Otherwise, it is only successful in the low state. Moreover, this threshold is strictly smaller than the threshold identified by Marquez and Yılmaz (2008) if $x > 0$, and coincides with their threshold if $x = 0$.

[22] For all those prices, the takeover in the high state occurs with probability 1; thus, the expected surplus is $\lambda + B$. Increasing the price from $\bar{p}$ can only increase shareholders' payoffs, and thus, decrease the raider's payoff.

Some care is required when $\Pi(\bar{p}) + \lambda B < 0$; (equivalently, $\Pi(\bar{p}) + B < (1 - \lambda)B$). In this case, the raider offers price zero, the takeover succeeds with probability one in the low state and with probability zero in the high state. While we pointed out that there are equilibria of the tender subgame after price offer zero in which the takeover succeeds in the low state with any probability, the tender equilibrium requires that after the price offer zero, an equilibrium of the tender subgame is played in which the takeover succeeds with probability one in the low state. Otherwise, the raider would have a profitable deviation to a price just slightly above zero, which would ensure that he obtains the private benefit B at least in the low state.

To sum up, we state the results argued above in the form of a Theorem, without providing a formal proof.

THEOREM 4 *In an incomplete-information model in which the raider has some commonly known private benefits $B \geq 0$, the raider offers either price $\bar{p}$ or zero. He offers price $\bar{p}$ if $\Pi(\bar{p}) + \lambda B > 0$. Moreover, given the size of the large shareholder's stake $x < 1/2$, there exists a $\bar{B}_x$ such that if $B > \bar{B}_x$, the takeover succeeds with probability one in both states and the raider makes a profit.*

7. Comparative statics

We now investigate the impact of the large shareholder's stake size on the raider's profit when he offers the price $\bar{p}$. In the next lemma, we show that the raider's payoff is strictly increasing in the large shareholder's holdings in the firm.

LEMMA 3 *Let $\Pi(x) := \Pi(\bar{p}(x))$ be the raider's profit when he offers the price $\bar{p}$ and the large shareholder holds a fraction $x < 1/2$ of the shares. Then $\frac{d\Pi}{dx} > 0$.*

PROOF: See Appendix B. □

Next we argue that $\Pi(x)$ is positive when x is close to, but smaller than, 1/2, and negative when x is just slightly above zero. Together with the fact that $\Pi(x)$ is increasing and continuous in x, this ensures the existence of $x^* < 1/2$ for which $\Pi(x^*) = 0$. Moreover, for any $x > x^*$, the raider turns a profit on the takeover.

THEOREM 5 $\lim_{x \searrow 0} \Pi(x) < 0$ *and* $\lim_{x \nearrow 1/2} \Pi(x) = \frac{1}{2}[\lambda - \beta(0)] > 0$.

PROOF: See Appendix B. □

$\Pi(x)$ is the raider's profit when the large shareholder owns a fraction x of the shares and the raider offers the price $\bar{p}$. The negative limit, when x tends towards zero, then means that by offering $\bar{p}$, the raider would incur a loss. This in turn means that the raider will prefer to offer the price zero. This result is in line with the results from Marquez and Yılmaz (2008), who have shown that when the firm is owned only by small shareholders, the raider cannot make a profit.

The surprising result is that for larger x, the profit from offering the price $\bar{p}$ is larger. One might think that if there were a large shareholder, the raider would have a much more formidable opponent to deal with, which would lower his profit. However, the large shareholder gets in his own way and sells all of his shares. In fact, the larger his stake, the more shares the raider obtains from him. For example, suppose $x = 0.49$. Knowing that he will get all of the shares from the large shareholder, the raider needs to acquire one percent of the company from the small shareholders. But then, by offering a low price, he can target precisely the most pessimistic small shareholders who jointly own one percent, because they are willing to sell for a low price.

This brings us to another part of the result. The limit of the raider's profit as x goes toward one-half depends on $\beta(0)$, that is, on the belief of the small shareholders who receive the lowest possible signal. Since the prior probability of the high state is λ and we assume MLRP, it follows that $\beta(0) < \lambda$. However, $\beta(0)$ can be anywhere in $[0, \lambda)$. If it is closer to zero, the agent who is observing the lowest signal is almost certain that he is in the low state. In such an environment the low signal is very informative about the state. If $\beta(0)$ is closer to λ, then the shareholder who observes signal 0 holds almost the same beliefs as he did prior to observing the signal. Such a signal is rather uninformative. Our result shows that as x tends to one-half, the raider's profit is higher when the bottom signal is more informative. This is not only because of the conveyed information per se, but because the greater informativeness of the bottom signal helps the raider to differentiate the shareholders. This in turn enables him to buy the shares from the most pessimistic shareholders relatively cheaply.

Finally, if the raider can reap private benefits from a successful takeover, then for every benefit $B > 0$, there is an $x(B) < 1/2$ such that if $x > x(B)$, the raider makes a strictly positive profit and the probability of a successful takeover is one. If, on the other hand, $x < x(B)$, then the raider cannot make a profit, he offers the price zero, and the probability of a successful takeover in the low state is one, while the probability of a successful takeover in the high state is zero. Moreover, $x(B)$ is (weakly) decreasing in B, $x(0) < 1/2$, and $x(\bar{B}) = 0$ for $\bar{B}$ as identified in Marquez and Yılmaz (2008).[23]

[23]The function $x(B)$ is strictly decreasing for $B \in [0, \bar{B}]$ and constant at 0 for $B \geq \bar{B}$.

8. Finite Model and Convergence

In this section we introduce the tender game with finitely many shares and incomplete information. There are n shares in total. Each one of the $(1-x)n$ small shareholders holds a single share, while the large shareholder holds xn shares.

8.1. Strategies and Payoffs A mixed strategy for the raider is a distribution over a set of prices, $[0,1]$. In a tender subgame after the raider's price offer, a strategy for a small shareholder is a mapping from his signal to the probability with which he tenders his share, denoted by $\sigma^n(s)$. A strategy for the large shareholder describes the number of shares he tenders for every realization of his private signal. It will be convenient to describe his strategy as a joint probability distribution function over his signals and the fraction of shares that he is tendering. Remembering that Σ_L is the set of all strategies for the large shareholder in the model with continuum shares, let the set $\Sigma_L^n \subset \Sigma_L$ be the set of strategies such that for any $\sigma_L^n \in \Sigma_L^n$, for every $s \in [0,1]$, and for every $i \in \{0,1,...,nx-1\}$, the strategy $\sigma_L^n(s,r)$ is constant in the interval $r \in \left[\frac{i}{nx}, \frac{i+1}{nx}\right)$. A strategy for the large shareholder in the game with n shares is an element in Σ_L^n, and a typical strategy is σ_L^n. The large shareholder's strategy induces a probability distribution on the fraction of tendered shares conditional on state h, denoted by $g^n : \{0,1,...,nx\} \to [0,1]$, and defined as:

$$g^n(i) := \int_{s\in[0,1]} (\sigma_L^n(s, i/nx) - \sigma_L^n(s, i-1/nx))dH(s|h) \text{ , for } i > 0;$$

$$g^n(0) := \int_{s\in[0,1]} \sigma_L^n(s,0)dH(s|h).$$

We specify the payoffs of the small shareholders and the large shareholder in the same way that we did for the continuum shares case in Section 3. In particular, the small shareholders' payoffs are

$$U(p,s,q,keep) = \beta(s)q,$$

$$U(p,s,q,sell) = p,$$

where p denotes the price offered for the firm, s denotes a shareholder's signal, and q represents the probability of the success of the takeover in the high state, conditional on the shareholder keeping his share.

Remark 5 *We interpret the raider's price offer p as the price he offers for the whole company, which converts into offering a price p/n per share. The small shareholders' payoffs are then p/n if they tender and $\beta(s)q/n$ if they do not tender. The latter payoff is due to the fact that in the high state the company is worth 1, with a per share value of $1/n$. But* *now notice that the small shareholders' behavior is determined by the ratio of* p *and* $\beta(s)q$*. Therefore, the payoffs defined above capture the relevant behavior.*

The large shareholder's payoff when tendering a fraction r of his shares is

$$U_L(p, s, q(r), r) = x\left[rp + (1-r)q(r)\beta_L(s)\right],$$

where $q(r)$ is the probability he attaches to the takeover succeeding in the high state, given that he is selling a fraction r of his shares.

8.2. Equilibrium We say that a couple $T^n = (\sigma_L^n, \sigma^n)$ is a symmetric Nash equilibrium of the tender subgame, when there are n shares, and when the price offer for the firm's total shares is p, if the following two conditions hold:

$$U(p, s, q_{-1}^n, \sigma(s)) \geq U(p, s, q_{-1}^n, a),\ \forall a \in \{keep, sell\}\ , \forall s \in [0,1],$$

and

$$\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p, s, q^n(i/nx), i/nx)d\sigma_L^n(s, i/nx) \geq$$
$$\sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p, s, q^n(i/nx), i/nx)d\bar{\sigma}_L^n(s, i/nx),\ \forall \bar{\sigma}_L \in \Sigma_L^n,$$

where

$$q^n(i) := \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k},$$

$$q_{-1}^n := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n-1} \binom{(1-x)n-1}{k} \phi_n^k (1-\phi_n)^{(1-x)n-1-k}.$$

The term $q^n(i/nx)$ is the probability of a successful takeover in the high state when the large shareholder tenders exactly i shares, while q_{-1}^n is the probability of a successful takeover in the high state, conditional on a particular small shareholder keeping his share. The term ϕ_n refers to the probability that a small shareholder tenders in the high state, and is defined as follows:

$$\phi_n := \int_{s\in[0,1]} f(s|h)d\sigma^n(s).$$

The probability of a successful takeover in the high state is:

$$q^n := \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}.$$

For any $p \geq 0$ and $T^n = (\sigma^n, \sigma^n_L)$, the raider's payoff function, $\Pi(p,T)$, is defined similarly to the payoff function in the model with continuum shares. Specifically, its definition is as follows:

$$\begin{aligned}\Pi(p,T) \quad &= \quad \lambda\left(\sum_{i=0}^{nx} g^n(i)\left(\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}(k+i)\right)\right) \\ &\quad - \quad p\left[(1-x)\int_{s\in[0,1]}\sigma(s)[\lambda f(s|h)+(1-\lambda)f(s|l)]ds + x\int_{s,r} r d\sigma_L(s,r)\right].\end{aligned}$$

We say that a tuple $(p^n, T^n(p')_{p'\in[0,1]})$ is an equilibrium of the tender game if each $T^n(p')$ is a symmetric Nash equilibrium of the tender subgame with price offer p', and $\Pi(p,T(p)) = \sup_{p'\in[0,1]}\Pi(p',T(p'))$.

Remark 6 *Note that our equilibrium concept is equivalent to a perfect Bayesian equilibrium when the strategies are defined as measurable with respect to the prices. In fact, even after off-equilibrium price offers by the raider, the shareholders' beliefs about the state of the world are unchanged and equal to their posterior belief obtained by Bayesian update using the common prior λ and their signal.*[24]

8.3. Convergence Let an outcome $\theta := (p,\pi,q) \in [0,1]\times[0,1]\times[0,1]$ be a tuple, where p is a price, π is the raider's profit, and q is the probability of success in the high state. Every equilibrium of a finite game induces an equilibrium outcome. Notice that the model with continuum shares has a unique equilibrium outcome as long as $\Pi(\bar{p}) \neq 0$, and let $\bar{\theta}$ denote this outcome.

Theorem 6 *Let $\{\theta^n\}_{n=0,1,\ldots}$ be a sequence of equilibrium outcomes, where each θ^n denotes an equilibrium outcome of the tender game with n shares. If the equilibrium outcome of the model with continuum shares is unique, then $\lim_{n\to\infty}\theta^n = \bar{\theta}$.*

Proof: See Appendix C. □

The proof proceeds by showing that the equilibrium strategies of the small shareholders, σ^n, and that of the large shareholder, σ^n_L, as well as prices, p^n, and probabilities of success in the high state, q^n, converge to their counterparts, (σ,σ_L,p,q), in the game with a continuum of shares. From these limiting objects we derive a mapping $q(r)$, representing the large shareholder's beliefs about the success of the takeover in the high state when selling a fraction r of his shares. We conclude by showing that $(p,\sigma,\sigma_L,q,q(r)_{r\in[0,1]})$ is an equilibrium of the tender game with a continuum of shares.

[24] For a precise statement of PBE, see Fudenberg and Tirole (1991, Definition 8.2).

Remark 7 *Although in Theorem 6 we only state that the equilibrium outcomes of the finite share model converge to the unique equilibrium outcome of the model with continuum shares, we also prove that equilibrium strategies in any subgame after a positive price offer converge to the unique equilibrium strategy profile of the model with continuum shares. Therefore, the off-path equilibrium behavior of the model with continuum shares is also approximated by the off-path equilibrium behavior of the finite-shares model.*

9. Discussion

We have analyzed the impact of a large shareholder (or shareholders) on takeovers. The main finding of the paper is that, without any informational asymmetries, the raider cannot facilitate profitable takeovers even with a minority large shareholder. However, when the shareholders are privately and asymmetrically informed, and the raider is not, the presence of a large shareholder can facilitate profitable takeovers.

In this paper, we focus on a simple mechanism in which the raider offers a price per share. As Bagnoli and Lipman (1988) note, an alternative mechanism which specifies that the raider will pay a price per share if and only if all the shares are tendered extracts the full surplus from the takeover by making every share pivotal. However, such offers are highly impractical and unobserved in practice. In a related paper, we show that when the shareholders have private information about the state of the world, conditional offers such that the seller pays a price p per share if at least half of the shares are tendered, do not increase the raider's expected profit in the continuum shares model (Ekmekci and Kos (2012)).[25]

Lastly, we should point out that there is a slight disparity between the presentations of the complete and incomplete information models. The former model is presented in the setup with finitely many shares, while the latter is presented in the setup with a continuum of shares. We chose to present the complete-information model using a finite number of shares in order to better compare our results to the results obtained previously by Bagnoli and Lipman (1988) and Marquez and Yılmaz (2008). Alternatively, we could have adapted the model in the incomplete-information section. This, however, would deliver a multiplicity of equilibria. In particular, any price offer not exceeding one would be an equilibrium, since the raider's profit is zero in every such subgame. We could then show that the equilibria of finite games converge to an equilibrium of the continuum game, which in turn implies that the raider's equilibrium profits in the finite model approach zero as the number of shares grows. This approach, however, is expositionally more involved. In contrast, the incomplete-information

[25] A similar result was established in an environment without a large shareholder in Marquez and Yılmaz (2007).

model with a continuum of shares has a unique equilibrium outcome. We therefore found it much more suitable for developing the results and the corresponding intuition than countless taking of limits.

A. Proof of Theorem 1

We break down the proof of the theorem into a sequence of lemmata. We start by rewriting the raider's payoff in a form that will prove convenient later. For ease of exposition we fix n and omit it from the labeling of prices and shareholders' strategies, until further notice.

Lemma 4 *Let $\{\phi, \sigma\}$ be a symmetric equilibrium of a tender subgame after the raider offers a price per share $p \in (0, 1/n)$. The raider's expected payoff in the equilibrium can be written as*

$$\Pi(p) = \pi_S(p) + \pi_L(p),$$

where

$$\pi_S(p) = v_1 \sum_k \sigma(k) \left(\frac{n}{2} - k\right) \binom{n(1-x)}{\frac{n}{2} - k} \phi^{\frac{n}{2}-k} (1-\phi)^{n(1-x)-\frac{n}{2}+k}$$

is the expected profit he makes on the sales of the small shareholders, and

$$\pi_L(p) = v_1 \sum_k \sigma(k) \left[[k - E_\sigma[k]] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i} + E[k] \binom{n(1-x)-1}{\frac{n}{2}-k-1} \phi^{\frac{n}{2}-k} (1-\phi)^{n(1-x)-\frac{n}{2}+k} \right],$$

the expected profit he makes on the large shareholder.

Proof: As argued in the main text, in any symmetric equilibrium the small shareholders use mixed strategies, i.e. $\phi \in (0, 1)$. Given that they are mixing they must be indifferent between keeping the share and selling it at the price p, assuming that everybody else is using the equilibrium strategy:

$$p = v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i} \phi^i (1-\phi)^{n(1-x)-1-i}. \tag{9}$$

The raider only obtains a positive payoff from the shares he buys when the takeover succeeds, while he has to pay a price p for every tendered share. His profit is therefore

$$\pi(p) = v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i}(i+k) - [(1-x)n\phi + E[k]]\, p, \tag{10}$$

where $(1-x)n\phi + E[k]$ is the expected number of tendered shares: $(1-x)n\phi$ by the small shareholders and $E[k]$ by the large shareholder.

The expected profit from the small shareholders is

$$\pi_S(p) = v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} i - (1-x)np,$$

and the profit from the large shareholder

$$\pi_L(p) = v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} k - E[k]p.$$

We will use the following two equalities:

$$\sum_{i=k}^{n} \binom{n}{i} \phi^i (1-\phi)^{n-i} i = \phi n \sum_{i=k-1}^{n-1} \binom{n-1}{i} \phi^i (1-\phi)^{n-1-i}, \tag{11}$$

and

$$\sum_{i=k}^{n} \binom{n}{i} \phi^i (1-\phi)^{n-i} = \sum_{i=k}^{n-1} \binom{n-1}{i} \phi^i (1-\phi)^{n-1-i} + \binom{n-1}{k-1} \phi^k (1-\phi)^{n-k}. \tag{12}$$

The derivation of the first can be found in the appendix of Marquez and Yılmaz (2008). The intuition for the second one is easily explained in terms of coin-flips. Suppose, ϕ is the probability of heads. Then the left-hand side of the equation is the probability that at least k out of n flips are heads. This is equivalent to the probability that at least k out of $n-1$ flips are heads plus the probability that exactly $k-1$ out of $n-1$ are heads times the probability that one additional coin comes up heads: $\binom{n-1}{k-1}\phi^{k-1}(1-\phi)^{n-k}\phi$.

Now,

$$\begin{aligned}
\pi_S(p) &= v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} i - (1-x)np \\
&= v_1 \sum_k \sigma(k) \left[\sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} i - (1-x)n\phi \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i} \phi^i (1-\phi)^{n(1-x)-1-i} \right] \\
&= v_1 \sum_k \sigma(k) \left(\frac{n}{2}-k\right) \binom{n(1-x)}{\frac{n}{2}-k} \phi^{\frac{n}{2}-k} (1-\phi)^{n(1-x)-\frac{n}{2}+k},
\end{aligned}$$

where the second equality is obtained by substituting in for p from (9) and the third by using (11). Notice that the profit from the small shareholders is nonnegative due to the assumption that the large shareholder owns less than half of all shares, rendering $\sigma(k)(\frac{n}{2}-k) \geq 0$.

On the other hand

$$\begin{aligned}
\pi_L(p) &= v_1 \sum_k \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} k - E[k]p \\
&= v_1 \sum_k \sigma(k) \left[\sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i} \phi^i (1-\phi)^{n(1-x)-i} k - E[k] \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i} \phi^i (1-\phi)^{n(1-x)-1-i} \right] \\
&= v_1 \sum_k \sigma(k) \left[[k-E[k]] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i} + E[k] \binom{n(1-x)-1}{\frac{n}{2}-k-1} \phi^{\frac{n}{2}-k} (1-\phi)^{n(1-x)-\frac{n}{2}+k} \right],
\end{aligned}$$

where the second equality is obtained by substituting in for p from (9), while the third by using (12). Profit from the shares tendered by the large shareholder is also positive. The second summand is clearly positive. So is

$$\sum_k \sigma(k) \left[k - E[k]\right] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i},$$

which is easy to see from observing that $\sum_k \sigma(k) \left[k - E[k]\right] = 0$ and that $\sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i}$ is non-negative and increasing in k. □

LEMMA 5 *Let $\{\sigma, \phi\}$ be a symmetric equilibrium of the tender subgame after p. Then*

$$\sum_k \sigma(k) \left[k - E[k]\right] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i} \leq (nx - E[k]) \sum \sigma(k) \left[\binom{(1-x)n-1}{\frac{n}{2}-k-1} \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \right].$$

PROOF: The large shareholder's payoff in the specified equilibrium is

$$\begin{aligned}
u_L(p) &= \sum \sigma(k)\left[kp + (nx-k)v_1 \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i}\right] \\
&= v_1 \sum \sigma(k) nx \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} \\
&- v_1 \sum \sigma(k)\left[k \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - E[k] \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i}\phi^i(1-\phi)^{n(1-x)-1-i}\right].
\end{aligned}$$

With probability $\sigma(k)$ the large shareholder tenders k shares for which he receives kp. The remaining $nx-k$ shares are each worth v_1 in the case the takeover succeeds which occurs with probability $\sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i}$. The second equality is obtained after substituting for p from (9) and rearranging.

Now

$$\begin{aligned}
v_1 \sum \sigma(k)&\left[k \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - E[k] \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i}\phi^i(1-\phi)^{n(1-x)-1-i}\right] \\
&= v_1 nx \sum \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - u_L(p) \\
&\leq v_1 nx \sum \sigma(k) \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - nxp \\
= nxv_1 \sum \sigma(k)&\left[\sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i}\phi^i(1-\phi)^{n(1-x)-1-i}\right] \\
&= nxv_1 \sum \sigma(k)\left[\binom{(1-x)n-1}{\frac{n}{2}-k-1}\phi^{\frac{n}{2}-k}(1-\phi)^{(1-x)n-\frac{n}{2}+k}\right],
\end{aligned}$$

where the first inequality follows because the large shareholders can always guarantee himself the payoff of nxp by tendering all the shares, implying $u_L(p) \geq nxp$. The equality in the fourth line is obtained by substituting for p from (9), and the last equality follows by using (12).

Also

$$\begin{aligned}
&\sum \sigma(k)\left(k \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n(1-x)}{i}\phi^i(1-\phi)^{n(1-x)-i} - E[k] \sum_{i=\frac{n}{2}-k}^{n(1-x)-1} \binom{n(1-x)-1}{i}\phi^i(1-\phi)^{n(1-x)-1-i}\right) \\
&= \sum_k \sigma(k)\left([k-E[k]] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i}\phi^i(1-\phi)^{n(1-x)-i} + E[k]\left[\binom{(1-x)n-1}{\frac{n}{2}-k-1}\phi^{\frac{n}{2}-k}(1-\phi)^{(1-x)n-\frac{n}{2}+k}\right]\right),
\end{aligned}$$

using (12).

The last two (in)equalities together yield

$$\sum_k \sigma(k)\,[k - E[k]] \sum_{i=\frac{n}{2}-k}^{n(1-x)} \binom{n}{i} \phi^i (1-\phi)^{n(1-x)-i} \leq (nx - E[k]) \sum \sigma(k) \left[\binom{(1-x)n-1}{\frac{n}{2}-k-1} \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \right].$$

□

LEMMA 6 *Let $\{\sigma, \phi\}$ be a symmetric equilibrium of the tender subgame after p. Then*

$$\pi(p) \leq V_1 \left(\frac{1}{2} + x\right) \binom{n(1-x)}{\frac{n(1-x)}{2}} 2^{-n(1-x)}.$$

PROOF:

$$\begin{aligned}
\pi(p) &\leq v_1 \sum_k \sigma(k) \left[\left(\frac{n}{2} - k\right) \binom{n(1-x)}{\frac{n}{2}-k} + nx \binom{(1-x)n-1}{\frac{n}{2}-k-1} \right] \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \\
&= v_1 \sum_k \sigma(k) \left[\left(\frac{n}{2} - k + nx\right) \binom{n(1-x)}{\frac{n}{2}-k} - nx \binom{(1-x)n-1}{\frac{n}{2}-k} \right] \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \\
&\leq v_1 \sum_k \sigma(k) \left(\frac{n}{2} - k + nx\right) \binom{n(1-x)}{\frac{n}{2}-k} \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \\
&\leq n v_1 \left(\frac{1}{2} + x\right) \sum_k \sigma(k) \binom{n(1-x)}{\frac{n}{2}-k} \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \\
&\leq V_1 \left(\frac{1}{2} + x\right) \binom{n(1-x)}{\frac{n(1-x)}{2}} 2^{-n(1-x)},
\end{aligned}$$

where the first line follows from the above Lemmata, the second from the equality $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n}{k-1}$ and the last from observing that $\binom{n(1-x)}{\frac{n}{2}-k} \phi^{\frac{n}{2}-k} (1-\phi)^{(1-x)n-\frac{n}{2}+k} \leq \binom{(1-x)n}{\frac{(1-x)n}{2}} 2^{-(1-x)n}$.
□

Finally, Stirling's approximation yields

$$\binom{n(1-x)}{\frac{n(1-x)}{2}} = \sqrt{\frac{2}{\pi(1-x)n}}\, 4^{\frac{n(1-x)}{2}}.$$

Therefore, for large n,

$$\pi(p) \leq \left(\frac{1}{2} + x\right) \sqrt{\frac{2}{\pi(1-x)}} \frac{1}{\sqrt{n}}, \tag{13}$$

for every $p \in [0, 1/n]$ and every equilibrium (ϕ, σ) of the tender subgame with price p. Remember that, while for the above lemmate we assumed $p \in (0, 1/n)$, we also know, as stated in the main text, $\pi(0) = \pi(1/n) = 0$. Since the bound (13) is uniform over all p and equilibria of the tender subgames generated by p, the limit supremum of the raider's profit goes to zero along any sequence of equilibria of tender games. Clearly so does limit infimum. This concludes the proof of Theorem 1.

B. Proofs for the Incomplete Information Model

Proof of Theorem 2: Suppose that there exists an equilibrium of the tender subgame after some price offer $p > 0$, and denote it by $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$. In the following development, we will fix this candidate equilibrium, and characterize its properties. Then we will verify that such an equilibrium exists.

Before we characterize the equilibrium, we remind the reader of two definitions from the main text. The pivotal type, $s^* \in [0,1]$ is the type such that if the small shareholders use the threshold strategy s^*, and if the large shareholder tenders all his shares, the fraction of tendered shares is $1/2$. Since $x < 1/2$, there is indeed an interior threshold signal $s^* \in (0,1)$, which satisfies the equality, $F(s^*|h)(1-x) + x = 1/2$. Price $\bar{p}$ is the price that makes the small shareholder observing the threshold signal $s^* \in (0,1)$ indifferent between tendering her share or keeping it, if she believes that the takeover is successful in state h with probability one. In particular, $\bar{p} := \beta(s^*)$.

The first claim shows that the probability of the success of the takeover is larger than zero.

Claim 1 $\quad$ *$q > 0$.*

Proof: Suppose, on the way to a contradiction that $q = 0$. Then, not tendering yields an expected payoff of zero, while tendering yields $p > 0$, therefore $\sigma = 1$. Consequently, at least $1-x$ of shares are sold regardless of what the large shareholder does, resulting in $q(r) = 1$ for every $r \in [0,1]$. In turn $q = 1$, which contradicts the hypothesis that $q = 0$. □

Next we establish a lower bound on the threshold used in an equilibrium by small shareholders.

Claim 2 $\quad$ *$(1-x)F(\sigma|h) + x \geq 1/2$.*

Proof: If the claim was not true, less than half the shares in total would be sold in the high state, even if the large shareholder were to tender all of his shares. Implying $q(r) = 0$ for every $r \in [0,1]$. Therefore, $q = 0$ which would contradict Claim 1. □

The above claim establishes that in any equilibrium the strategy of the small shareholders is such that the large shareholder could guarantee that at least half of the shares are sold in the high state, if he wanted to.

In what follows we define the notation for the share of the large shareholder that needs to be tendered so that exactly half of the shares are tendered in the high state, given the equilibrium strategy of the small shareholders. Notice that such a share of the large shareholder exists in an equilibrium due to the previous claim.

Definition 1 *Let*

$$a := \max\{0, \frac{1/2 - (1-x)F(\sigma|h)}{x}\}.$$

In the next claim we show, roughly speaking, that the large shareholder never sells less than fraction a of his shares. Moreover, if he is selling precisely a and if $q(a) < 1$ then it should be that $a = 1$. Indeed, if the large shareholder were to sell $a < 1$ with positive probability and $q(a) < 1$, then he would be better off by selling just slightly more than a which would ensure the success of the takeover in the high state and yield a significantly larger payoff on the shares the large shareholder keeps.

Claim 3 *If $\sigma_L(s, r) > 0$ for some $r < 1$ and $s \in [0, 1]$, then $q(r) = 1$.*

Proof: The proof is in two steps. First, if $\sigma_L(s, r) > 0$ for some $r < 1$, then $r \geq a$. To see this, note that $q(r) = 0$ for $r < a$, in which case the large shareholder gets p for the shares he tendered and zero for the ones he keeps. Therefore, tendering all shares does strictly better than $r < a$. Hence, $\sigma_L(s, r) = 0$ for every $r < a$.

For the second step, we show that the claim is true under the two remaining cases: i) $r > a$ and ii) $r = a$.

i) When $r > a$, $q(r) = 1$ by the definition of a and by the equilibrium requirement on $q(r)$.

ii) Now suppose that $r = a$, $\sigma_L(s, a) > 0$, $r < 1$, and suppose contrary to the assertion of the claim, that $q(a) < 1$. Then the large shareholder has a profitable deviation by tendering a fraction arbitrarily close to but above a, by which he pushes the probability of the success in the high state to 1. This contradicts the equilibrium condition that σ_L maximizes the large shareholder's payoff. □

If the small shareholders expect the success of the takeover in the high state to be less than certain, then it must be the case that the large shareholder is selling all of his shares. Otherwise he could increase the probability to one by selling just slightly more shares.

Claim 4 *If $q < 1$, then $\sigma_L(s, r) = 0$ for every $r < 1$ and $s \in [0, 1]$.*

Proof: Let $q < 1$. If $a = 1$, then the claim is true because $\sigma_L(s, r) = 0$ for every $r < a = 1$ as shown in Claim 3.

Let $a < 1$, and on the way to a contradiction assume that $\sigma_L(s, r) > 0$ for some $r < 1$. Then $q(r) = 1$ for all such r, by Claim 3. Moreover, if $a < 1$, then $q(1) = 1$. Therefore $q = 1$, contradicting the supposition that $q < 1$. □

So far we have established several properties of the equilibrium. In what follows, we will complete the characterization under two cases: i) when $p < \bar{p}$ and ii) when $p \geq \bar{p}$.

CASE 1: Suppose that $p < \bar{p}$.

When the price p is below the threshold $\bar{p}$ the takeover cannot succeed with certainty in the high state.

CLAIM 5 *If $p < \bar{p}$, then $q < 1$.*

PROOF: Suppose on the way to a contradiction that $q = 1$. Then, either $\sigma = 0$ or $\sigma > 0$. If $\sigma > 0$, then $p = q\beta(\sigma)$, $p < \bar{p} = \beta(s^*)$, and $\beta(\cdot)$ is strictly increasing imply $\sigma < s^*$. Therefore,

$$(1 - x)F(\sigma|h) + x < (1 - x)F(s^*|h) + x = 1/2,$$

hence $q(r) = 0$ for every $r \in [0, 1]$. Consequently $q = 0$, which contradicts the assumption we started with. A similar analysis yields a contradiction when $\sigma = 0$. □

Claim 1 and 5 put together yield that in any equilibrium, it has to be the case that $q \in (0, 1)$. From the previous analysis we also know that in such an equilibrium, the large shareholder would try to tip the scale in the favor of the certain success, if he could. The only way he can be prevented from doing so is if he is already selling all of his shares.

CLAIM 6 *$\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$.*

PROOF: From Claim 5, we know that $q < 1$. The result is then implied by Claim 4. □

In words, the large shareholder tenders all his shares regardless of his information.

CLAIM 7 *$a = 1$, $\sigma = s^*$ and $q = \frac{p}{\beta(s^*)}$.*

PROOF: If $a < 1$, then $q(1) = 1$, and since the large shareholder would be tendering all his shares by claim 6, $q = 1$. This contradicts the statement of Claim 5 that $q < 1$. Therefore $a = 1$, and using the definition of a, $\sigma = s^*$. Finally, since σ is the threshold type, $q = \frac{p}{\beta(\sigma)} = \frac{p}{\beta(s^*)}$, completing the proof. □

Below, we summarize the characterization of the equilibrium of the tender subgame when $p < \bar{p}$.

Summary 1 *If* $0 < p < \bar{p}$*, then*

$$\sigma = s^*, \qquad q = \frac{p}{\beta(s^*)}, \qquad \sigma_L(s, r) = 0,$$

for every $s \in [0, 1]$ *and* $r < 1$*, and*

$$q(1) = q, \qquad q(r) = 0,$$

for every $r < 1$*. It is straightforward to verify that this is an equilibrium. We conclude that there is a unique equilibrium for* $0 < p < \bar{p}$*.*

CASE 2: Suppose that $p \geq \bar{p}$.

First we establish that if the price offer is high, then in equilibrium, the takeover succeeds with probability one in the high state.

Claim 8 *If* $p \geq \bar{p}$ *then* $q = 1$*.*

Proof: On the way to a contradiction, assume that $q < 1$. Then by Claim 4, $\sigma_L(1, r) = 0$ for every $r < 1$, i.e., the large shareholder tenders all his shares. Moreover, $p \geq \bar{p}$ and $q < 1$ imply $\beta(\sigma) = \frac{p}{q} > \bar{p} = \beta(s^*)$ and therefore $\sigma > s^*$. But then $q = 1$, because the large shareholder tenders all his shares and $(1 - x)F(\sigma|h) + x > 1/2$. This contradicts the initial hypothesis that $q < 1$. □

Remark 8 *Since* $q = 1$*,* σ *is found by the identity* $\beta(\sigma) = p$*. Since* $p \geq \bar{p}$*,* $\sigma \geq s^*$*.*[26]

At low prices, $p < \bar{p}$, the takeover succeeds in the high state with a probability q smaller than one. The only way to prevent the large shareholder from increasing this probability to one was if he was already tendering all of his shares in the equilibrium. For a high price $p \geq \bar{p}$, however, the probability of the takeover succeeding in the high state must be one, as shown in Claim 8. This leaves scope for the large shareholder to keep some of the shares if he deems them more valuable then the price the raider is offering.

Definition 2 *Let* s_L *be the signal that satisfies* $\beta_L(s_L) = p$*, if such a signal exists. Let* $s_L := 0$ *if* $\beta_L(0) > p$ *and let* $s_L := 1$ *if* $\beta_L(1) < p$*.*

s_L is the signal at which the large shareholder's expected value for each of his shares is equal to the price, when he is expecting the takeover to succeed with probability one.

[26]If $p \geq \beta(1)$, we set $\sigma = 1$, in which case the small agents tender their shares for all the signals.

The following claim shows that the large shareholder tenders fraction a of his shares when his signal is high and all of his shares when the signal is low.

CLAIM 9 *i)* $\sigma_L(s, r) = 0$ *for* $r < a$ *and any* $s \in [0, 1]$. *In words, the large shareholder does not tender a fraction less than* a.

ii) For $s < s_L$ *and* $r < 1$: $\sigma_L(s, r) = 0$. *For* $s > s_L$ *and* $r \in [a, 1)$:

$$\sigma_L(s, r) = \lambda[F(s|h) - F(s_L|h)] + (1 - \lambda)[F(s|l) - F(s_L|l)].$$

Notice that $\sigma_L(s, 1) = \lambda F(s|h) + (1 - \lambda)F(s|l)$ *for all* $s \in [0, 1]$, *by the definition of distributional strategies.*

In words, the large shareholder tenders exactly fraction a *if his signal is above the threshold signal* s_L. *He tenders all his shares if his signal is below* s_L.

iii) If $s_L < 1$, *then* $q(a) = 1$.

PROOF: Part i) follows directly from Claim 3.

We will argue part ii) by considering two cases: $\sigma = s^*$ and $\sigma > s^*$. If $\sigma = s^*$, then $a = 1$ by the definition of a and s^*. Since the large shareholder never tenders a fraction smaller than a, as in part i), $\sigma_L(s, r) = 0$ for every $r < 1$ and every $s \in [0, 1]$. $q = 1$ then implies that $q(a) = 1$.

If $\sigma > s^*$, then $a < 1$. When $s < s_L$, the price p is greater than $\beta_L(s)$, by the definition of s_L. Therefore, it is optimal for the large shareholder to tender all his shares. Hence, if $s < s_L$, then $\sigma_L(s, r) = 0$ for every $r < 1$.

If $s_L = 1$, then the proof is complete. So, let $s_L < 1$. A direct calculation shows that tendering any fraction $r > a$ is dominated by tendering fraction $\frac{a+r}{2}$ of shares. Moreover, if $q(a) < 1$, then tendering a fraction arbitrarily close to a from above is a profitable deviation. Therefore, the optimality condition for the large shareholder's strategy delivers that $q(a) = 1$. Since $\sigma_L(s, r) = 0$ for $r < a$, in any equilibrium from part i), $\sigma_L(s, a) = \lambda[F(s|h) - F(s_L|h)] + (1 - \lambda)[F(s|l) - F(s_L|l)]$ for $s > s_L$, which concludes the proof. □

We have proven that all equilibria of the tender subgame have the following structure:

i) If $0 < p < \bar{p}$, then $\sigma = s^*$, $q = \frac{p}{\beta(s^*)}$, $\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$. Moreover, the raider's profit is:

$$\Pi(p) = \lambda(q/2 - p/2) + (1 - \lambda)(-p[(1 - x)F(s^*|l) + x]).$$

ii) If $p = \bar{p}$, then $\sigma = s^*$, $q = 1$, $\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$; the raider's profit is:

$$\Pi(p) = \lambda(1/2 - p/2) + (1-\lambda)(-p[(1-x)F(s^*|l) + x]).$$

iii) If $p > \bar{p}$, then σ satisfies $\beta(\sigma) = p$, $q = 1$, $\sigma_L(s,r)$ satisfies the findings above; the raider's profit is:

$$\begin{aligned}\Pi(p) = \lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p\lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p(1-\lambda)[(1-x)F(\sigma|l) + x(a(1-H(s_L|l)) + H(s_L|l))].\end{aligned}$$

□

Proof of Theorem 3: We start by rewriting $\Pi(p)$ for $p \leq \bar{p}$. Inserting $q = p/\beta(s^*)$ into (7), we obtain

$$\Pi(p) = p\left[\frac{\lambda}{2}\left(\frac{1}{\beta(s^*)} - 1\right) - (1-\lambda)[(1-x)F(s^*|l) + x]\right].$$

Since the profit function is linear in p, for $p \in [0,\bar{p}]$, if $\lambda/2(1/\beta(s^*) - 1) - (1-\lambda)((1-x)(F(s^*|l)) + x$ is non-negative, then any price less that $\bar{p}$ is weakly dominated by $\bar{p}$. If on the other hand, the expression is negative, then any price $p \in (0,\bar{p}]$ is dominated by zero.

Next we will provide four inequalities to argue that any price $p > \bar{p}$ is dominated by $\bar{p}$. Before we proceed notice that $p > \bar{p}$ implies $\sigma > s^*$.

For the first inequality, $\sigma > s^*$ and $p = \frac{\lambda f(\sigma|h)}{\lambda f(\sigma|h)+(1-\lambda)f(\sigma|l)}$ imply:

$$\lambda(1-p)F(\sigma|h) + (1-\lambda)(-p)F(\sigma|l) < \lambda(1-p)F(s^*|h) + (1-\lambda)(-p)F(s^*|l).$$

To see this, we rewrite the inequality by plugging in the value of p and rearranging to obtain:

$$\frac{F(\sigma|h)}{f(\sigma|h)} - \frac{F(\sigma|l)}{f(\sigma|l)} < \frac{F(s^*|h)}{f(\sigma|h)} - \frac{F(s^*|l)}{f(\sigma|l)}.$$

The above inequality holds because the derivative of $K(s) := \frac{F(s|h)}{f(\sigma|h)} - \frac{F(s|l)}{f(\sigma|l)}$ with respect to s is negative when $s < \sigma$, due to MLRP.

The second inequality follows directly from $p > \bar{p}$:

$$\lambda(1-p)F(s^*|h) + (1-\lambda)(-p)F(s^*|l) < \lambda(1-\bar{p})F(s^*|h) + (1-\lambda)(-\bar{p})F(s^*|l).$$

The first two inequalities together yield

$$\lambda(1-\bar{p})F(s^*|h) + (1-\lambda)(-\bar{p})F(s^*|l) > \lambda(1-p)F(\sigma|h) + (1-\lambda)(-p)F(\sigma|l). \tag{14}$$

The third inequality follows from the MLRP condition in a similar fashion as the first inequality, but by using the condition $p = \beta_L(s_L)$ (The case of $p > \beta_L(1)$ leads to $s_L = 1$, and the inequality follows similarly.):

$$\lambda(1-p)[a(1-H(s_L|h)) + H(s_L|h)] + (1-\lambda)(-p)[a(1-H(s_L|l)) + H(s_L|l)] \\ \leq \lambda(1-p) + (1-\lambda)(-p).$$

Indeed, rearranging shows that the above inequality is true if and only if:

$$\frac{1-H(s_L|h)}{1-H(s_L|l)} \geq \frac{h(s_L|h)}{h(s_L|l)}.$$

This last inequality follows directly from MLRP.

The fourth inequality again follows directly from $p > \bar{p}$

$$\lambda(1-p) + (1-\lambda)(-p) < \lambda(1-\bar{p}) + (1-\lambda)(-\bar{p})$$

The last two inequalities yield

(15)
$$\lambda(1-\bar{p})+(1-\lambda)(-\bar{p}) > \lambda(1-p)[a(1-H(s_L|h))+H(s_L|h)]+(1-\lambda)(-p)[a(1-H(s_L|l))+H(s_L|l)].$$

Now, for $p > \bar{p}$:

$$\begin{aligned}
&\Pi(\bar{p}) - \Pi(p) = \\
&\lambda(1-\bar{p})F(s^*|h) + (-\bar{p})(1-\lambda)(1-x)F(s^*|l) - \lambda(1-p)(1-x)F(\sigma|h) + p(1-\lambda)(1-x)F(\sigma|l) \\
&+ x\left[\lambda(1-\bar{p}) - p(1-\lambda) - \lambda(1-p)[a(1-H(s_L|h)) + H(s_L|h)] + p(1-\lambda)[a(1-H(s_L|l)) + H(s_L|l)]\right] \\
&> 0,
\end{aligned}$$

where the first equality uses the identity $1/2 - x = F(s^*|h)$, the term in the second line is larger than zero due to (14) and the term in the third line due to (15). □

Proof of Lemma 3: $s^*(x)$ is strictly increasing in x, which is readily seen from

$$(1-x)F(s^*|h) + x = 1/2. \tag{16}$$

Therefore $\beta(s^*(x))$ is strictly decreasing in x. $\Pi(x)$ can be rewritten as follows:

$$\Pi(x) = \frac{\lambda}{2} - \beta(s^*)[\frac{\lambda}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]].$$

The first term is decreasing in s^*, hence increasing in x. Next we show that the second term is decreasing in x. First, $\beta(s^*)$ is decreasing in x. Second, as we will show below, $(1-x)F(s^*|l)+x$ is decreasing in x, finishing the proof. Differentiating (16) with respect to x yields

$$0 = 1 - F(s^*|h) + (1-x)f(s^*|h)\frac{ds^*}{dx}.$$

Now:

$$\begin{aligned}\frac{d((1-x)F(s^*|l)+x)}{dx} &= 1 - F(s^*|l) + (1-x)f(s^*|l)\frac{ds^*}{dx} \\ &= 1 - F(s^*|l) - f(s^*|l)\frac{1-F(s^*|h)}{f(s^*|h)} \\ &< 0,\end{aligned}$$

where the second line follows from (16) and the third from MLRP. This concludes the proof. □

Proof of Theorem 5: Let $s^*(x)$ be defined by

$$(1-x)F(s^*(x)|h) + x = \frac{1}{2},$$

for all $x \in [0, 1/2]$. Then

$$\begin{aligned}\lim_{x\searrow 0}\Pi(x) &= \lim_{x\searrow 0}\left(\frac{\lambda}{2} - \beta(s^*(x))\left[\frac{\lambda}{2} + (1-\lambda)[(1-x)F(s^*(x)|l) + x]\right]\right) \\ &= \frac{\lambda}{2} - \beta(s^*(0))\left[\frac{\lambda}{2} + (1-\lambda)F(s^*(0)|l)\right] \\ &= (1-\lambda)f(s^*(0)|l)\beta(s^*(0))\left[\frac{F(s^*(0)|h)}{f(s^*(0)|h)} - \frac{F(s^*(0)|l)}{f(s^*(0)|l)}\right] \\ &< 0,\end{aligned}$$

where the third line follows by observing $1/2 = F(s^*(0)|h)$ and rearranging, and the inequality is due to MLRP.

On the other hand,

$$
\begin{aligned}
\lim_{x\nearrow 1/2}\Pi(x) &= \lim_{x\nearrow 1/2}\left(\frac{\lambda}{2}-\beta(s^*(x))\left[\frac{\lambda}{2}+(1-\lambda)[(1-x)F(s^*(x)|l)+x]\right]\right)\\
&=\frac{1}{2}\left[\lambda-\beta(0)\right]\\
&>0,
\end{aligned}
$$

where the second line follows after observing $s^*(1/2)=0$. □

C. Finite Model and Convergence: Proof of Theorem 6

We prove Theorem 6 via a sequence of Lemmata. In particular, we show that equilibrium outcomes of the finite shares model converge to the unique equilibrium outcome of the model with a continuum shares. In the following development, we denote the strategies in the finite shares model with n shares using the superscript n.

C.1. Threshold Strategies The next lemma is a preliminary observation showing that the equilibrium strategies of small shareholders in finite shares model have a threshold structure.

Lemma 7 *Let $T^n=(\sigma_L^n,\sigma^n)$ be a symmetric Nash Equilibrium of the tender subgame with price offer p when there are n shares. Then the small shareholders' strategy σ^n is a threshold strategy.*

Proof: The MLRP property on the signal distribution implies that $\beta(s)$ is strictly increasing in s. A small shareholder's payoff from tendering a share is p, whereas the payoff from keeping it is $\beta(s)q_{-1}^n$. Therefore, if $U(p,s,q_{-1}^n,keep)\geq U(p,s,q_{-1}^n,sell)$ for a signal $s\in[0,1]$, then $U(p,s',q_{-1}^n,keep)>U(p,s',q_{-1}^n,sell)$ for every $s'<s$. Similarly, if $U(p,s,q_{-1}^n,sell)\geq U(p,s,q_{-1}^n,keep)$ for a signal $s\in[0,1]$, then $U(p,s',q_{-1}^n,sell)>U(p,s',q_{-1}^n,keep)$ for every $s'>s$. □

Henceforth, the equilibrium strategy of a small shareholder will denote the threshold type of the strategy, i.e., the term σ^n denotes the threshold type of the strategy. Hence

$$\phi_n=F(\sigma^n|h).$$

C.2. Convergence Let the collection $\{p^n, T^n, q^n\}_{n=1,2...}$ be a sequence where each $p^n \in [0,1]$ is a price, T^n is a symmetric Nash equilibrium of the tender subgame with price offer p^n and n shares, and q^n is derived from T^n as described above. In the following development, we fix this sequence.

C.2.1. *Method of proof* We first prove that, the equilibrium outcomes of the sequence $\{p^n, T^n, q^n\}_n$ converge to the unique equilibrium outcome of the continuum game we identified in the main text. On the way to the result, we first argue that there is a strategy $\sigma_L \in \Sigma_L$ to which the sequence σ_L^n converges. Moreover, σ^n converges to a threshold σ, prices, p^n, converge to a price p, and q^n converges to q. From these limit objects, we *derive* a new mapping $q(r)_{r\in[0,1]}$ and show that $(\sigma_L, \sigma, q, q(r)_{r\in[0,1]})$ is an equilibrium of the tender subgame with a price offer p in the continuum game. Finally we show that the equilibrium prices of the finite games, p^n, converge to the equilibrium price, p, of the continuum game.

The lemma below shows that every collection $\{p^n, T^n, q^n\}_n$ has a convergent subsequence.

LEMMA 8 *There exists a subsequence of* $\{p^n, \sigma^n, \sigma_L^n, q^n\}_n$ *and an increasing and right-continous function* $\sigma_L \in \Sigma_L$ *such that* $\sigma_L^n(s,r) \to \sigma_L(s,r)$ *at every continuity point of* $\sigma_L(s,r)$, $p^n \to p$, $q^n \to p$, $\sigma^n \to \sigma$.

PROOF: Since $p^n, \sigma^n, q^n \in [0,1]$ for all n, the sequence $\{p^n, \sigma^n, \sigma_L^n, q^n\}_n$ has a subsequence $\{p^{nk}, \sigma^{nk}, \sigma_L^{nk}, q^{nk}\}_{nk}$ along which the object $\{p^{nk}, \sigma^{nk}, q^{nk}\}_{nk}$ converges. Sequence $\{\sigma_L^{nk}\}_{nk}$ has a subsequence $\{\sigma_L^{nkj}\}_{nkj}$ which converges to a distribution due to Helly's theorem; distributions here have a bounded support (see Billingsley (1986), Thm 25.10). Moreover, since all the distributional strategies along the sequence $\{\sigma_L^{nkj}\}_{nkj}$ satisfy the property (1) so does the limit. Therefore, the limit distribution of $\{\sigma_L^{nkj}\}_{nkj}$ is in Σ_L and $\{p^{nkj}, \sigma^{nkj}, \sigma_L^{nkj}, q^{nkj}\}_{nkj}$ is a convergent subsequence of $\{p^n, \sigma^n, \sigma_L^n, q^n\}_n$. □

From this point on, we use the term $lim_{n\to\infty}$ to take the limit over the convergent subsequence identified in the previous lemma. We denote the limit point to which the subsequence converges with the collection of the price p, threshold signal σ, large shareholder's strategy σ_L and the probability of success in the high state, q.

So far we identified a limit of a sequence of equilibria of finite games. In the following development, we establish that the limiting strategies constitute, a part of, an equilibrium of the continuum game. Note, however, that an equilibrium in the continuum game is identified by two strategies, one for small shareholders and one for the large shareholder, a probability q and a mapping $\{q(r)\}_{r\in[0,1]}$. In our description of the equilibrium of the continuum game we will use the limit distributions for the strategies, the limit of q^n for q, while in the next definition we describe how to specify the mapping $\{q(r)\}_{r\in[0,1]}$. Let $a := min\{a^*, 1\} \in [0,1]$

where a^* is the solution to $(1-x)\sigma + xa^* = 1/2$.

Definition 3 *Let $q(r) = 0$ for all $r < a$, $q(r) = 1$ for all $r > a$. If $\sigma_L(1, a|h) - \lim_{y \to a^-} \sigma_L(1, y|h) > 0$, then let*

$$q(a) = \frac{q - (1 - \sigma_L(1, a|h))}{\sigma_L(1, a|h) - \lim_{y \to a^-} \sigma_L(1, y|h)},$$

otherwise let $q(a)$ be an arbitrary number between 0 and 1.

First we establish that $q(a)$ as defined above can actually be interpreted as a probability.

Lemma 9 $q(a) \in [0, 1]$.

Proof: We start by showing $q(a) \geq 0$. On the way to the result, note that for every $\epsilon > 0$,

$$1 - \sigma^n(1, a + \epsilon|h) = \sum_{i=(a+\epsilon)nx}^{nx} g^n(i).$$

Definition of a and the fact that ϕ_n converges to ϕ imply

$$\sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1 - \phi_n)^{(1-x)n-k} \to_{n \to \infty} 1,$$

uniformly over all $i \geq (a+\epsilon)nx$. Indeed, a was defined so that whenever the large shareholder tenders more than fraction a of his shares in the continuum game, given the fixed behavior of the small shareholders, he expects the takeover in the high state to succeed with probability one. Now

$$\sum_{i=(a+\epsilon)nx}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1 - \phi_n)^{(1-x)n-k} - \sum_{i=(a+\epsilon)nx}^{nx} g^n(i) \to 0.$$

The above observations can be put together to show that there exists an N such that for

$n > N$,

$$\begin{aligned}(1-\epsilon)(1-\sigma^n(1,a+\epsilon|h)) &= (1-\epsilon)\sum_{i=(a+\epsilon)nx}^{nx} g^n(i)\\ &\leq \sum_{i=0}^{nx} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}\\ &= q^n.\end{aligned}$$

Since the inequality is true for every large n, it has to be true in the limit:

$$q \geq (1-\epsilon)(1-\sigma_L(1,a+\epsilon|h)).$$

Moreover, the last inequality holds for every $\epsilon > 0$, and σ_L is right continuous, therefore

$$q \geq 1-\sigma_L(1,a|h).$$

Next we argue that $q(a) \leq 1$. For this, it suffices to show that

$$1-q \geq \lim_{y\to a^-} \sigma_L(1,y|h).$$

Suppose, to the contrary that $1-q < \lim_{y\to a^-}\sigma_L(1,y|h)$. Then there exists an $\epsilon_1 > 0$ such that

$$1-q < \sigma_L(1,a-\epsilon_1|h). \tag{17}$$

$1-q$ is the probability the small agents attach in the limit to the failure of the takeover in the high state. It is easy to verify that

$$1-q = \lim_{n\to\infty} \sum_{i=0}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k}.$$

Fix an ϵ_2 such that $0 < \epsilon_2 \leq \epsilon_1$ and notice that definition of a and the fact that ϕ_n converges to ϕ imply

$$\sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \to 1,$$

uniformly for all $i \leq (a-\epsilon_2)nx$. The idea is, in the limit game a is the fraction of shares that the large shareholder needs to sell, so that exactly half of the shares are sold, given the small shareholders' strategy. Thus, if he is selling a fraction smaller than a, and the small shareholders' strategies are converging to the limit strategy, it has to be the case that for large n the takeover is failing with probability 1 in the high state.

But then there exist an $\epsilon_3 > 0$ and N such that for $n > N$

$$\begin{aligned}
1 - q &= \lim_{n\to\infty} \sum_{i=0}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \\
&\geq \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i) + \sum_{i=(1-\epsilon_2)nx}^{nx} g^n(i) \sum_{k=0}^{\frac{n}{2}-i-1} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} - \epsilon_3 \\
&\geq \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i) - \epsilon_3.
\end{aligned}$$

Since the above inequality holds for every ϵ_3 and $\sigma_L(1, a - \epsilon_2|h) = \lim_n \sum_{i=0}^{(1-\epsilon_2)nx} g^n(i)$,

$$1 - q \geq \sigma_L(1, a - \epsilon_2|h),$$

which contradicts (17). Hence, $1 - q \geq \lim_{y\to a^-} \sigma_L(1, y|h)$. □

Now that we have established the limiting structure of the game we need to show that the limiting strategies, together with the beliefs, form an equilibrium of the continuum game. First we show that the limit threshold signal of the small shareholders represents the optimal strategy for them in the continuum game.

Lemma 10 *σ is such that $U(p, s, q, keep) > (<) p$ if $s > (<) \sigma$.*

Proof: Similarly as in the calculations for the complete information case

$$q^n - q_{-1}^n = \sum_{i=0}^{nx} g^n(i) \binom{(1-x)n-1}{\frac{n}{2}-1} \phi_n^{\frac{n}{2}-i} (1-\phi_n)^{(1-x)n-\frac{n}{2}+i},$$

which is easily seen to converge to 0 as n goes to infinity. Now, $q_{-1}^n \to q_n$ and $q_n \to q$ imply $q_{-1}^n \to q$.

Let $s < \sigma$. Since $\sigma^n \to \sigma$, there is a N such that for every $n > N$, $s < \sigma^n$. Observing the threshold signal σ^n makes a small shareholder indifferent between tendering and not tendering, in the game with n shares. Therefore, $U(p^n, s, q_{-1}^n, keep) < p^n$. U continuous in q_{-1}^n, $q_{-1}^n \to q$, and $p^n \to p$ imply $U(p, s, q, keep) \leq p$. Since this is true for every $s < \sigma$, and since $\beta(s)$ is increasing in s, it follows that $U(p, s, q, keep) < p$. A similar argument shows that $U(p, s, q, keep) > p$ for $s > \sigma$. □

The last piece of the puzzle in the limit result for the equilibria of the tender subgames is to establish that the large shareholder's limiting strategy σ_L is a best response to the small shareholders' limiting strategies in the continuum game, given the limiting price and beliefs.

LEMMA 11

$$\int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) \geq \int_{s,r} U_L(p, s, q(r), r) d\tilde{\sigma}_L(s, r),$$

for every $\tilde{\sigma}_L \in \Sigma_L$.

PROOF: Suppose, contrary to the assertion of the lemma, that there exists a $\tilde{\sigma}_L \in \Sigma_L$ such that

$$\int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) < \int_{s,r} U_L(p, s, q(r), r) d\tilde{\sigma}_L(s, r).$$

We will consider two cases: $a < 1$ or $a = 1$.

Case 1: $a < 1$. There exist an $\epsilon > 0$, and a $\bar{\sigma}_L \in \Sigma_L$ such that:

$$(18) \qquad \int_{s,r} U_L(p, s, q(r), r) d\sigma_L(s, r) < \int_{s,r} U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r),$$
$$\bar{\sigma}_L(1, a + \epsilon) = \bar{\sigma}_L(1, a - \epsilon),$$

and $\bar{\sigma}_L(1, y) = \tilde{\sigma}_L$, for $y \in [0, a - \epsilon)$. The idea is to take the strategy $\tilde{\sigma}_L$ and construct a new strategy $\bar{\sigma}_L$ by shifting the probability mass that $\tilde{\sigma}_L(1, \cdot)$ assigns to the interval $[a - \epsilon, a + \epsilon]$ toward the endpoint of the interval, $a + \epsilon$. The existence of a $\bar{\sigma}_L$ satisfying the above inequality is guaranteed because, given $a < 1$, shifting the shares slightly above a cannot decrease the payoff discontinuously.

Let for every $r \in [0, 1]$,

$$\bar{\sigma}_L^n(s, r) := \bar{\sigma}_L(s, i/nx),$$

for the unique i that satisfies $i - 1 < rnx \leq i$.

In what follows, we will show that the large shareholder's equilibrium payoffs in the finite games converge to his payoffs in the continuum game with price p and tuple T. We will use this finding together with the hypothesis that σ_L is not a best response to find a profitable deviation from σ_L^n when n is large, obtaining a contradiction.

In particular, let $U_L(p^n, \sigma^n, \sigma_L^n)$ be the large shareholder's payoff from following the strategy σ_L^n when the small shareholders follow the symmetric threshold strategy σ^n, in the finite game with n shares and a price offer p^n. We argue that:

$$\lim_{n\to\infty} U_L(p^n, \sigma^n, \sigma_L^n) = \int U_L(p, s, q(r), r) d\sigma_L(s, r), \tag{19}$$

$$\lim_{n\to\infty} U_L(p^n, \sigma^n, \bar{\sigma}_L^n) = \int U_L(p, s, q(r), r) d\bar{\sigma}_L(s, r). \tag{20}$$

We start with the first equality. Note that for every $\epsilon > 0$, $q^n(r)$ converges uniformly to 1 in the domain $r > a + \epsilon$ and converges uniformly to zero in the domain $r < a - \epsilon$. The large shareholder's payoff can now be rewritten as:

$$\begin{aligned} U_L(p^n, \sigma^n, \sigma_L^n) = & \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) \\ = & \sum_{i\in\{(a+\epsilon)nx,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) \\ + & \sum_{i\in\{0,1,\ldots,(a-\epsilon)nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) \\ + & \sum_{i\in\{(a-\epsilon)nx+1,\ldots,(a+\epsilon)nx-1\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx). \end{aligned}$$

We use the facts that $q^n(\cdot) \to q(\cdot)$ uniformly for $r \in [0, a-\epsilon] \cup [a+\epsilon, 1]$, σ_L^n converges to σ_L, and that $U_L(p, s, q(r), r)$ is continuous in $r \in [0, a-\epsilon] \cup [a+\epsilon, 1]$ and in its first argument, to argue that there is an N such that for $n > N$:

$$\begin{aligned} & \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) \geq \\ & \geq \int_{r>a+\epsilon} \int_s U_L(p, s, q(r), r) d\sigma_L(s, r) + \int_{r<a+\epsilon} \int_s U_L(p, s, q(r), r) d\sigma_L(s, r) + \\ & x(a-\epsilon)p \int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1, r) + x(1-a-\epsilon)\lambda \left(\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i) \sum_{k=n/2-i}^{(1-x)n} \binom{(1-x)n}{k} \phi_n^k (1-\phi_n)^{(1-x)n-k} \right) - \epsilon. \end{aligned} \tag{21}$$

In the above expression, the first two terms on the right-hand side are the limit payoffs in the specified regions. We obtain the third term by explicitly rewriting the U_L term in the integral, and bounding it generously. The second inequality below bounds the large shareholder's payoff from above, in a similar fasion as the first inequality did from below:

$$\sum_{i\in\{0,1,\dots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) \le \tag{22}$$
$$\int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+\int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+$$
$$x(a+\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r)+x(1-a+\epsilon)\lambda\left(\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}\right)+\epsilon.$$

Remember that:

$$q=\lim_{n\to\infty}\sum_{i=0}^{xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}.$$

The above sum can be split into three parts where the first sum is from 0 to $(a-\epsilon)xn-1$, the second from $(a-\epsilon)xn$ to $(a+\epsilon)xn$ and the third from $(a+\epsilon)xn+1$ to xn. The first sum converges, due to the definition of a, to 0, and the third to $\lim_{n\to\infty}\sum_{i=(a+\epsilon)xn+1}^{xn} g^n(i)$, which is in turn equal to $1-\sigma_L(1,a+\epsilon|h)$. Therefore:

$$q=1-\sigma_L(1,a+\epsilon|h)+\lim_{n\to\infty}\sum_{i=(a-\epsilon)xn}^{(a+\epsilon)xn} g^n(i)\sum_{k=n/2-i}^{(1-x)n}\binom{(1-x)n}{k}\phi_n^k(1-\phi_n)^{(1-x)n-k}.$$

(21) can now be rewritten as

$$\sum_{i\in\{0,1,\dots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) \le$$
$$\le\int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+\int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)$$
$$+x(a+\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r)+x(1-a+\epsilon)\lambda(q-(1-\sigma_L(1,a+\epsilon|h)))+2\epsilon.$$

and (22) as

$$\sum_{i\in\{0,1,\dots,nx\}}\int_{s\in[0,1]} U_L(p^n,s,q^n(i/nx),i/nx)d\sigma_L^n(s,i/nx) \ge$$
$$\ge\int_{r>a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)+\int_{r<a+\epsilon}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)$$
$$+x(a-\epsilon)p\int_{a-\epsilon}^{a+\epsilon} d\sigma_L(1,r)+x(1-a-\epsilon)\lambda(q-(1-\sigma_L(1,a+\epsilon|h)))-2\epsilon.$$

Since the above inequalities hold for every $\epsilon>0$, and since the cumulative distributions are right-continous functions:

$$\lim_{n\to\infty} \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) =$$
$$= \int_{r>a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r)$$
$$+xap[\sigma_L(1,a) - \lim_{y\to a^-} \sigma_L(1,y)] + x(1-a)\lambda[q-(1-\sigma_L(1,a|h))].$$

Replacing the definition of $q(a)$, and rewriting the definition of $U_L(p,s,q(a),a)$, we get:

$$\lim_{n\to\infty} \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\sigma_L^n(s, i/nx) =$$
$$= \int_{r>a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_{r<a}\int_s U_L(p,s,q(r),r)d\sigma_L(s,r) + \int_s U_L(p,s,q(a),a)d\sigma_L(s,a)$$
$$= \int_{r,s} U_L(p,s,q(r),r)d\sigma_L(s,r).$$

This completes the proof of (19). (20) can be shown using the same method, after first observing that $\bar{\sigma}_L^n$ is constructed so that it converges to $\bar{\sigma}_L$ at every continuity point of $\bar{\sigma}_L$, and that:

$$\begin{aligned} U_L(p^n,\sigma^n,\bar{\sigma}_L^n) &= \sum_{i\in\{0,1,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx) \\ &= \sum_{i\in\{(a+\epsilon)nx,\ldots,nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx) \\ &+ \sum_{i\in\{0,1,\ldots,(a-\epsilon)nx\}} \int_{s\in[0,1]} U_L(p^n, s, q^n(i/nx), i/nx) d\bar{\sigma}_L^n(s, i/nx), \end{aligned}$$

because $\bar{\sigma}_L(1,a+\epsilon) = \bar{\sigma}_L(1,a-\epsilon)$, by construction. Now the steps used in the proof of (19) can be used to show $\lim_{n\to\infty} U_L(p^n,\sigma^n,\bar{\sigma}_L^n) = \int U_L(p,s,q(r),r)d\bar{\sigma}_L(s,r)$.

(18), (19) and (20) imply

$$\begin{aligned} \lim_{n\to\infty} U_L(p^n,\sigma^n,\bar{\sigma}_L^n) &= \int U_L(p,s,q(r),r)d\bar{\sigma}_L(s,r) \\ &> \int U_L(p,s,q(r),r)d\sigma_L(s,r) \\ &= \lim_{n\to\infty} U_L(p^n,\sigma^n,\sigma_L^n). \end{aligned}$$

Therefore there exists an n such that

$$U_L(p^n,\sigma^n,\bar{\sigma}_L^n) > U_L(p^n,\sigma^n,\sigma_L^n),$$

contradicting the fact that σ_L^n is a best response.

Case 2: $a = 1$. We first argue that, if $a = 1$, and if $p > 0$, then the strategy $\bar{\sigma}_L$, according to which the large shareholder tenders all shares at every signal gives the large shareholder a strictly higher payoff then any other strategy in the continuum game. In particular, let $\bar{\sigma}_L(1,1) = 1$ and $\bar{\sigma}_L(1,r) = 0$ for every $r < 1$. Then, $\int U_L(p,s,q(r),r)d\bar{\sigma}_L(s,r) > \int U_L(p,s,q(r),r)d\sigma'_L(s,r)$ for every $\sigma'_L \neq \bar{\sigma}_L$. Indeed, when the large shareholder tenders less than fraction a in the continuum game, the probability of a successful takeover, by the definition of a, is zero. Therefore it is profitable for the large shareholder to deviate towards tendering fraction a of his shares. Moreover, the large shareholder's payoff from tendering all his shares is px.

If σ_L is suboptimal, then $\bar{\sigma}_L$ yields the large shareholder a strictly higher payoff than σ_L: $\int U_L(p,s,q(r),r)d\sigma_L(s,r) < xp$. Moreover, $\int U_L(p,s,q(r),r)d\sigma_L(s,r) = \lim_{n\to\infty} U_L(p^n,\sigma^n,\sigma^n_L)$. Since $p^n \to p$, we conclude that, for some n, $U_L(p^n,\sigma^n,\sigma^n_L) < xp^n$. But then tendering all the shares in the game with n shares is a profitable deviation from σ^n_L for the large shareholder, contradicting the assumption that σ^n_L is a best response.

The only remaining case is $p = 0$. In this case, all strategies give the large shareholder zero payoff in the continuum game, and hence all strategies are best responses. □

The above lemmata can be combined to establish that the limit $(\sigma,\sigma_L,q,q(r)_{r\in[0,1]})$ of the sequence $\{\sigma^n,\sigma^n_L,q^n\}_n$ is an equilibrium of the continuum game with the price offer p.

Lemma 12 *The tuple $T = (\sigma,\sigma_L,q,q(r)_{r\in[0,1]})$ is an equilibrium of the tendering subgame in the continuum game with price p.*

Proof: The above lemmata show how to construct the belief function $q(r)$, and establish that σ and σ_L are best responses for the small shareholders and the large shareholder, respectively. In particular, lemma 11 shows that σ_L is a best response to $q(r)$ and p, lemma 10 shows that small shareholder's strategy is a best response to q and p. In definition 3, we construct $q(r)$ in a way that it satisfies the equilibrium conditions for $q(r)$. Moreover, $q(r)$ integrates to q using σ_L, by construction. The only caveat that the definition doesn't deliver is if the large shareholder's strategy doesn't have a mass on fraction a. In this case, $q = 1 - \sigma_L(1,a)$, and this follows from the two inequalities above inequality 17. □

The last thing to argue is that the limit of equilibrium prices coincides with the equilibrium price of the continuum game.

Lemma 13 *Let $\{p^n,\sigma^n,\sigma^n_L\}_n$ be a sequence of prices and equilibrium strategies of the tender subgames with price offer p^n in the finite shares model with n shares. If $\lim_n p^n = p$, then, $\lim_n \Pi(p^n,\sigma^n,\sigma^n_L) = \Pi(p,T(p))$, where $T(p)$ is the equilibrium of the tender subgame with a price offer p in the model with continuum shares.*

Proof: We omit the formal proof to this result, as it is very similar to showing how the large shareholder's payoff in the finite shares model converges to his payoff in the model with a continuum of shares; as in equality (19). $\square$

LEMMA 14 *If* $\Pi(\bar{p}) \neq 0$*, then* p^n*,* $\Pi^n(p^n)$ *and* q^n *converge to the unique equilibrium price, profit and the probability of success in the high state in the continuum model.*

PROOF: Let p be the equilibrium price in the model with continuum shares. We will first show that $p^n \to p$. By the previous lemma, $\lim_n \Pi(p, \sigma^n, \sigma^n_L) = \Pi(p, T(p))$, where (σ^n, σ^n_L) is an equilibrium of the tender subgame after the price offer p. Since $\Pi(\bar{p}) \neq 0$, $\Pi(p, T(p)) > \Pi(p', T(p'))$ for every $p' \neq p$. Now, suppose on the way to a contradiction that $\lim_n p^n = p' \neq p$. then again by the previous lemma, $\lim \Pi(p^n, \sigma'^n, \sigma'^n_L) = \Pi(p', T(p')) < \Pi(p, T(p))$ where (σ'^n, σ'^n_L) is an equilibrium of the tender subgame after price offer p^n. This is a contradiction to $\Pi(p^n, \sigma'^n, \sigma'^n_L) \geq \Pi(p, \sigma^n, \sigma^n_L)$. Now that we have established that $p^n \to p$, we can use the previous lemma to conclude that $\lim_n \Pi(p^n, \sigma^n, \sigma^n_L) = \Pi(p, T(p))$.

Now we'll argue that q^n converges to q. In every tender sub game, q^n converges to q, as delivered by Lemma 12. On the other hand, in any equilibrium the raider offers either price 0 or $\bar{p}$. Moreover, Theorem 3 implies his choice is unique unless $\Pi(\bar{p}) = 0$. For both prices $p = 0$ and $p = \bar{p}$ the probability of success in the high state, q is uniquely defined as shown in Lemma 2 and Theorem 2, respectively. Therefore, unless $\Pi(\bar{p}) = 0$, q^n converges to the unique probability of success in the high state in the continuum model.

Finally, notice that $\Pi^n(p^n)$ converges to the unique profit, even if $\Pi(\bar{p}) = 0$. While in the latter case there might be a multiplicity of prices in the limit, 0 and $\bar{p}$, both of those prices deliver profit 0. $\square$

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